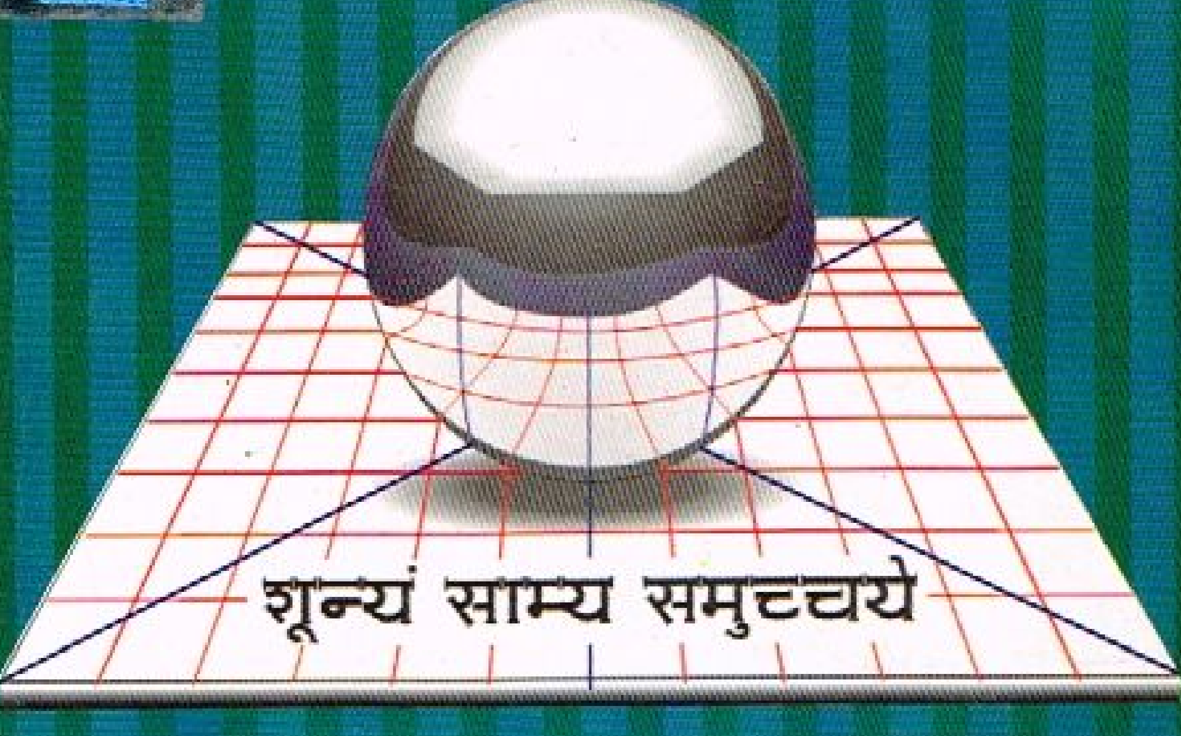
New
Fully Revised Edition

MAGICAL BOOK ON
QUICKER MATHS



MIRACULOUS FOR COMPETITIONS, STIMULATING FOR GENERAL USE

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THE UNDISPUTED BESTSELLER

Contents

Foreword

When did you get up this morning? At 6.35. Did you reach your destination in time? Yes, I arrived 10 minutes earlier even though I got up five minutes late. I usually walk but today I took a bus. Why not a taxi? Isn't it faster? Faster, yes. But in my case the speed of the bus was sufficient. Besides, I compared the fares of the two means of transport and concluded that taking a taxi would mean incurring unnecessary expenditure.

Wow! what a clever guy! Saves money, saves time. Does he know some magic? Is he a student of occult sciences? No, dear. He is an ordinary mortal like any of us. But he exudes an uncommon confidence, thanks to his ability to compute at a magical pace.

In other words, Quicker Maths is an asset at every step of one's life. By Quicker Maths, the book means speed and accuracy at both simple numerical operations and complex problems. And it is heartening that the book takes into account all kinds of problems that one may encounter in the ordinary run of life.

There are several books one comes across which take care of problems asked in examinations. But they are conventional and provide solutions with the help of detail method. This book provides you both the detail as well as the quicker methods. And it is in the latter that the book is irresistible. That the book provides you both the methods is a pointer to the fact that you are not being led into a faith where you have to blindly follow what the *guru* says. The conclusions have been rationally drawn. The book therefore serves as a guide - the true function of a *guru* - and once you get well-versed with the book, you will feel empowered enough to evolve formulas of your own. I think the real magic lies there!

At the same time, a careful study of the book endows you with the magical ability to arrive at an answer within seconds. There may be some semi-educated persons who sneer at this value of the book. Can you beat a computer? - a contemptuous question is asked. Absurd question. For one, the globe is yet to get sufficiently equipped with computers. Two, even when we enter a full-fledged hi-tech age, let us hope it is not at the cost of our minds. Let not computers and calculators become the proverbial Frankenstein's monster. The mind is a healthy

organ and computing a healthy function. Computers and calculators were devised by the brain to aid it, not to consume it.

That the mathematical ability of the brain be in tact is a concern of every individual. In most of the examinations even calculators are not allowed, let alone computers. And it is here that this magical book proves immensely useful. If you are a reader of this book, you can definitely feel more confident - miles ahead of others.

There is no doubt that there has been a lot of labour involved in the book. For all those students who are gearing up to drive their Mathematics Marutis at an amazing 200 kmph, the book definitely provides the requisite infrastructure. And what is more, the methods are accident-free with proper cautions at necessary places. Here is a book that will help exam-takers glide and enthusiastic students enjoy the ride.

In an age when speed is being maniacally pursued, a careful study of the book will serve as a powerful accelerator. At the same time, its simple language makes it easily accessible across the linguistic barriers. Besides, the fact that you vividly see the Quicker Method makes things very interesting.

And so the book provides you speed not at the cost of joy. It is not merely a mechanical device, but has an organic charm. One concludes: fast driving is fun.

Chetananand Singh
Editor, *Banking Services Chronicle*

Author's Preface

We at *Banking Services Chronicle (BSC)* analyse students' problems. If a student is not able to perform well at an exam, our research group members try to penetrate the student's psyche and get at the roots of the problems. In the course of our discussions we found that the mathematics section often proves to be the Achilles' heel for most of the students. Letters from our students clearly indicated that their problem was not that they could not solve the questions. No, the questions asked in general competitions are in fact so easy that most of the students would secure a cent per cent score, if it were not for the time barrier. The problem then is: INABILITY TO SOLVE the question IN TIME.

Unfortunately, there was hardly any book available to the student which could take care of the time aspect. And this prompted the *BSC* members to action. We decided to offer a comprehensive book with our attention targeted at the twin advantage factors: *speed* and *accuracy*. Sources were hunted for: Vedic Mathematics to computer programmings. Our aim was to get everything beneficial from wherever possible. The most-encountered questions were categorised. And Quicker Methods were intelligently arrived at and diligently verified.

How does the book help save your time? Probably all of you learnt by heart the multiplication-tables as children. And you have also been told that multiplication is the quicker method for a specific type of addition. Similarly, there exists a quicker method for almost every type of problems provided you are well-versed with some key determinants and formulas.

For the benefit of understanding we have also given the detail method and how we arrive at the Quicker Method. However, for practical purposes you need not delve too much into the theory. Concentrate on the working formula instead.

For the benefit of non-mathematics students, the book takes care to explain the oft-used terms in an ordinary language. So that even if you are vaguely familiar with numbers, the book will prove beneficial for it is self-explanatory.

The mathematics students are relatively in a comfortable position. They do not have to make an effort to understand the concepts. But even in their case, there are certain aspects of questions asked in the

competitive exams which have been left by them untouched since school days. So a revision is desirable.

In the case of every student, however, the unique selling proposition of the book lies in its ability to increase the student's problem-solving speed. Due caution has been observed to proceed methodically. A gradual progress has been made from simple to complex examples. There are theorems and solved examples followed by exercises. A systematic, chapter-by-chapter study will definitely result in a marked improvement of the student's mathematical speed. The students are requested to send their responses to the book and suggestions for further improvement.

And, finally, I would extend my thanks to all those who have played a role in making the book available to the reader. I specially thank Mr Madhukar Pandey for having played a key role in promoting the endeavour, Mr Chetananand Singh for the meticulous editing of the book and Mr Niranjan Bharti for having carefully verified the results. Mr Niranjan Singh's all-round assistance cannot be forgotten. Friends kept on encouraging me at every step. The inspiration I received from Mr Sanjay, Mr Deven Bharti, Mr Nagendra Kumar Sinha, Mr Sandeep Varma, Mr Manoj Kumar, Mr Vijay Kumar, Mr Rajeev Raman, Mr Anil Kumar and Mr JK Singh, to name a few, has been invaluable. And thanks to Mr Pradeep Gupta for printing.

Preface to the Second Edition

It is a great pleasure to note that *Magical Book on Quicker Maths* continues to be popular among the students who are looking for better results in this cut-throat world of competition. This book has brought the new concept of time-saving quicker method in mathematics. So many other publications have tried to publish similar books but none could reach even close to it. The reason is very simple. It is the first and the original book of its kind. Others can only be duplicate and not the original. Some people can even print the duplicate of the same book. It will prove dangerous to our publication as well as to our readers. So, we suggest our readers to confirm the originality of this book before buying. The confirmation is very simple. You can find a three-dimensional HOLOGRAM on the cover page of this book.

This edition has been extensively revised. Mistakes in its first edition have been corrected. Some new chapters like Permutation-Combination, Probability, Binary System, Quadratic Expression etc. have been introduced. Some old chapters have been rewritten. Hope you will now find this book more comprehensive and more useful.

Preface to the Third Edition

The pattern of question paper as well as the standard of questions have changed over the past couple of years. Besides Permutation-Combination and Probability, questions from trigonometry - in the form of Height and Distance - have also been introduced. In chapters like Data Analysis, Data Sufficiency, and Series, new types of question are being asked.

With the above context in mind, a few new chapters have been introduced and a few old ones enlarged. Important Previous Exam questions have been added to almost all the chapters. But they have been added in larger numbers in the chapters specially mentioned above.

An introductory chapter has been added on "How to Prepare for Maths". I suggest going through this chapter before setting any targets.

A revision in the cover price was long due. The first edition (1995), which cost Rs 200, had only 612 pages. The price remained the same even in the second edition (1999) in spite of the number of pages being increased to 749. But the third edition (2000) has gone into 807 voluminous pages. So the price is being increased to Rs 225. Kindly bear with us.

How to Prepare for MATHS

(Using this book for Competitive Exams)

1. Importance of Maths paper (PO)

Quantitative Aptitude is a compulsory paper. You can't neglect. So make sure you are ready to improve your mathematical skills. Each question values 1.2 marks whereas each question of Reasoning values only 1.066 marks in PO exam. So, if you devote relatively more time on this paper you get more marks. Also, the answers of Maths questions are more confirmed than answers of Reasoning questions, which are often confusing. Most of you feel it is a more time-consuming paper, but if you follow our guidelines, you can save your valuable time in examination hall.

Other exams: There are very few competitive exams without maths paper. SSC exams have different types of Maths paper. The mains exam of SSC contains Subjective Question paper. Keeping this in mind, I have also given the detail method of each short-cut or Quicker Method given in this book. Each theorem, which gives you a direct formula also contains proof of the theorem, which is nothing but a general form (denoting numerical values by letters say X, Y, Z etc) of detail method.

2. Preparation for this paper

(A) How to start your preparation

Maths is a very interesting subject. If you don't find it interesting, it simply means you havn't tried to understand it. Let me assure you it is very simple and 100% logical. There is nothing to be assumed and nothing to be confused about. So nothing to worry if you come forward with firm determination to learn maths.

The most basic things in Maths are:

a) Addition - Subtraction b) Multiplication - Division

All these four things are most useful. At least one of these four things is certainly used in any type of mathematical question. So, if we do our basic calculations faster we save our valuable time in each question. To calculate faster, I suggest the following tips:

(i) Remember the TABLE upto 20 (at least):

You should know that tables have been prepared to make calculations faster. You can see the use of table in the following example:

Evaluate: 16×18

If you don't remember the table of either 16 or 18 you will proceed like this:

$$\begin{array}{r} 16 \\ \underline{18} \\ 128 \\ \underline{16} \\ \underline{288} \end{array}$$

But if you know the table of 16, your calculations would be:

$16 \times 18 = 16\,(10 + 8) = 16 \times 10 + 16 \times 8 = 160 + 128 = 288$

Or, if you know the table of 18; your calculation would be

$18 \times 16 = 18(10 + 6) = 18 \times 10 + 18 \times 6 = 180 + 108 = 288$

If you can, you remember the table upto 30 or 40. It will be precious for you.

Note: (1) You should try the above two methods on some more examples to realise the beauty of tables. Try to evaluate:

19×13; 17×24; 18×32 $(18 \times 30 + 18 \times 2 = 540 + 36 = 576)$; 19×47; 27×38; 33×37 etc.

(2) All the above calculations should be done mentally. Try it.

(ii) LEARN the one-line Addition or Subtraction method from this book

In the first chapter we have given some methods of faster addition and subtraction. Suppose you are given to calculate:

$789621 - 32169 + 4520 - 367910 = ...$

If you don't follow this book you will do like:

$$\begin{array}{r} 789621 \\ \underline{+4520} \\ 794141 \end{array} \qquad \begin{array}{r} 32169 \\ \underline{+367910} \\ 400079 \end{array}$$

$$\begin{array}{r} 794141 \\ \underline{-400079} \\ 394062 \end{array}$$

The above method takes three steps, i.e. (i) add the two +ve values; (ii) add the two -ve values; (iii) subtract the second addition from the first addition.

But you can see the one-step method given in the chapter. Have mastery over this method. It takes less writing as well as calculating time.

(iii) Learn the one-line Multiplication or Division method from this book

Method of faster multiplication is given in the second chapter. I think it is the most important chapter of this book. Multiplication is used in almost all the questions, so if your multiplication is faster you can save at least 35% of your usual time. You should learn to use the faster one-line method. It needs some practice to use this method frequently. The following example will show you how this method saves your valuable time.

Ex. Multiply: 549×36

If you don't follow the one-line method of multiplication, you will calculate like:

$$\begin{array}{r} 549 \\ \underline{36} \\ 3294 \\ \underline{1647} \\ 19764 \end{array}$$

If this method takes 30 seconds I assure you that one-line method given in this book will take at the most 15 seconds. Try it.

One-line method of calculation for **Division** is also very much useful. You should learn and try it if you find it interesting. But, as division is less used, some of you may avoid this chapter.

(iv) Learn the Rule of Fractions

In the chapter **Ratio and Proportion** on Page No. 239, I have discussed this rule. It is the faster form of unitary method. It is nothing but simplified form of Rule of Three and Rule of Proportional Division. No doubt, it works faster and is used in almost all the mathematical questions where unitary method (*Aikik Niyam*) is used. See the following example:

Ex. If 8 men can reap 80 hectares in 24 days, how many hectares can 36 men reap in 30 days?

Soln: I don't know how much time you will take to answer the question but if we follow the rule of fraction our calculation would be:

$80 \times \frac{36}{8} \times \frac{30}{24} = 450$ hectares.

In this book this method is used very frequently. It is only when you go through the various chapters of this book that you will find how wonderful the method is.

It saves at least half of your usual time. I think this method should be adopted by all of you at any cost. First learn it and then use it wherever you can.

So, these four points are necessary for your strong and firm start. And only strong and firm start is the key to sure success.

(B) Clear the Fundamentals behind each chapter

There are 36 chapters in this book. Each chapter has some important basic fundas. Those fundas should be clear to you. Any doubt with the basics will hamper your further steps. Now the question arises - what are those basic fundas? Naturally, for each chapter, there are different fundas. I will discuss one chapter and its basic fundas. You can understand and find the same with different chapters. My chapter is - **PARTNERSHIP (on page 265)**

Partnership

Simple (Different investments for the same period)	Compound [Different investments (P & Q); Diff periods (t_1 & t_2)]
Ratio of profit = P : Q.	Ratio of profit = $P \times t_1 : Q \times t_1$

It is natural that your fundas of ratio should be clear before going through this chapter. Now, you can understand what I actually mean by the fundas. In a similar way, you can collect all the fundas and basic formulae at one place.

(C) More and More Examples

You are suggested to go through as many examples as possible. Each question given in examples has some uniqueness. Mark it and keep it in mind. To collect more examples of different types you may consult different books available in the market.

(D) Use of Quicker Formulae

Before going for quicker formula I suggest you to know the detail method of the solution as well. So see the proofs of all the formulae carefully. Once you get familiar with the detail solution, you find it easier to understand the quicker method. Direct Formulae or Quicker Methods save your valuable time but they have very high potential of creating confusion in their usage. So you should know where the particular formula should be used. A little change in the questions may lead you to wrong solution. So be careful before using them. In case of any confusion, you are suggested to solve the questions without using direct formula or quicker method.

Only frequent use of the quicker methods can make you perfect in Quicker Maths.

After covering all the chapters and knowing all the methods, you should be prepared for practice.

3. Practice of Maths Paper

(A) Pattern of paper

You should know the pattern and style of the question paper of the exam for which you are going to appear. Suppose you are preparing for Bank PO exam. You should know that maths paper consists of 50 questions. Out of which 15-20 questions are from Data Analysis, 5 Questions from Data Sufficiency, 5-10 are from Numericals (Calculation based), and 20-25 are from Mathematical Chapters (like Profit & Loss, Percentage, Partnership, Mensuration, Time & Work, Train, Speed etc.). In other exams it may be different. The pattern can be known from previous papers.

(B) Collection of Previous Papers as well as sample papers

If you can, you should arrange as many as possible numbers of previous and sample papers. There are many sources: Guides, Books, Magazines etc. The most standard and reliable sets are available in the magazine *Banking Services Chronicle*. Also, with our Correspondence Course we give at least 60 sets of math papers separately and 60 sets of maths with full-length Practice Sets (of 225 Questions).

(C) Now start your practice:

From the beginning to the end, the complete session of practice should be divided into five parts.

Part (i): Take your first test with previous paper without taking time into consideration. Try to solve all questions. Note down the total time and score in your **performance diary**. Also note down the questions which took more than one minute. Now you have to find out the reason of your low performance, if it is so. Naturally, you would find the following reasons:

- Some questions were difficult and time-consuming.
- Some questions were unsolvable for you.
- You lost your concentration.
- You lost your patience.
- You did more writing job.
- You could not use Quicker Methods.

Try to find out the solutions to all the above problems. If any of the questions was difficult for you, it means your initial preparation was not good. But don't worry. Go through that chapter again and clear your basic concept. Because the standard of question is always within your reach. If you have passed your 10th exam with maths, you can solve all the questions. You should take at least 10-12 tests in this part.

Part (ii): This time the paper may be either previous or model (sample). Fix your alloted period (say 50 minutes for PO). And solve as many questions as possible within that period. Once you have completed your test, count the number of correct questions. Note down the number of questions solved by you and the no. of correct solutions in your **performance diary**. Now you can find the reasons for your low scoring. If the reasons are the same as in Part (i), you try to resolve the problem again. Take at least 5-6 tests in this part. After analysing your performance and problems you should be ready for your third part of test.

Part (iii): This time you try to solve all the questions within a time period fixed by you in advance. This period should be less than that for the tests in part (ii). If you couldn't do it, try it again on another test paper. Part (iii) should not be considered completed unless you have achieved your goal.

Part (iv): After completion of part (iii), you need to increase your speed. This part is penultimate stage of your final achievement. You should try to solve the complete paper of maths within a minimum possible time (say, 30-35 minutes for PO paper). This part may take 3 to 4 months. Keep patience and go on practicing.

Part (v): This is the last part of your practice. After part (iv), you should take your test with complete full-length paper (225 questions for PO).

Addition

In the problem of addition we have two main factors (*speed* and *accuracy*) under consideration. We will discuss a method of addition which is faster than the method used by most people and also has a higher degree of accuracy. In the latter part of this chapter we will also discuss a method of checking and double-checking the results.

In using conventional method of addition, the average man cannot always add a fairly long column of figures without making a mistake. We shall learn how to check the work by individual columns, without repeating the addition. This has several advantages:

1) We save the labour of repeating all the work;
2) We locate the error, if any, in the column where it occurs; and
3) We are *certain* to find error, which is not necessary in the conventional method.

This last point is something that most people do not realise. Each one of us has his own weaknesses and own kind of proneness to commit error. One person may have the tendency to say that 9 times 6 is 56. If you ask him directly he will say "54", but in the middle of a long calculation it will slip out as "56". If it is his favourite error, he would be likely to repeat it when he checks by repetition.

Totalling in columns

As in the conventional method of addition, we write the figures to be added in a column, and under the bottom figure we draw a line, so that the total will be under the column. When writing them we remember that the mathemetical rule for placing the numbers is to align the right-hand-side digits (when there are whole numbers) and the decimal points (when there are decimals). For example:

Right-hand-side-digits alignment	Decimal alignment
4234	13.05
8238	2.51
646	539.652
5321	2431.0003
350	49.24
9989	

The conventional method is to add the figures down the right-hand column, 4 plus 8 plus 6, and so on. You can do this if you wish

in the new method, but it is not compulsory; you can begin working on any column. But for the sake of convenience, we will start on the right-hand column.

We add as we go down, but we "never count higher than 10". That is, when the running total becomes greater than 10, we reduce it by 10 and go ahead with the reduced figure. As we do so, we make a small tick or check-mark beside the number that made our total higher than 10. For example:

4
8 4 plus 8, 12 : this is more than 10, so we subtract 10 from 12. Mark a tick and start adding again.
6 6 plus 2, 8
1 1 plus 8, 9
0 0 plus 9, 9
9 9 plus 9, 18: mark a tick and reduce 18 by 10, say 8.

The final figure, 8, will be written under the column as the "running total".

Next we count the ticks that we have just made as we dropped 10's. As we have 2 ticks, we write 2 under the column as the "tick figure". The example now looks like this :

```
                4234
                8238
                 646
                5321
                 350
                9989
                ----
running total :    8
ticks :            2
```

If we repeat the same process for each of the columns we reach the result:

```
                4234
                8238
                 646
                5321
                 350
                9989
                ----
running total : 6558
ticks :         2222
```


Now we arrive at the final result by adding together the running total and the ticks in the way shown in the following diagram.

```
running total :   0 6 5 5 8 0
ticks :           0 2 2 2 2 0
                  -----------
Total :            2 8 7 7 8
```

Save more time : We observe that the running total is added to the ticks below in the immediate right column. This addition of the ticks with immediate left column can be done in single step. That is, the number of ticks in the first column from right is added to the second column from right, the number of ticks in the 2nd column is added to the third column, and so on. The whole method can be understood in the following steps.

```
          4234
          8238
           646
Step I.   5321
           350
          9989
          ----
Total :      8
```

[4 plus 8 is 12, mark a tick and add 2 to 6, which is 8; 8 plus 1 is 9; 9 plus 0 is 9; 9 plus 9 is 18, mark a tick and write down 8 in the first column of total-row.]

```
          4234
          8238
           646
Step II.  5321
           350
          9989
          ----
Total:      78
```

[3 plus 2 (number of ticks in first column) is 5; 5 plus 3 is 8; 8 plus 4 is 12, mark a tick and carry 2; 2 plus 2 is 4; 4 plus 5 is 9; 9 plus 8 is 17, mark a tick and write down 7 in 2nd column of total-row.]

In a similar way we proceed for 3rd and 4th columns.

$$
\begin{array}{r}
4234 \\
\cancel{8}23\cancel{8} \\
\cancel{6}\cancel{4}6 \\
5321 \\
350 \\
\cancel{9}\cancel{9}8\cancel{9} \\
\hline
\end{array}
$$

Total: 28778

Note : We see that in the leftmost column we are left with 2 ticks. Write down the number of ticks in a column left to the leftmost column. Thus we get the answer a little earlier than the previous method.

One more illustration :

Q: 707.325 + 1923.82 + 58.009 + 564.943 + 65.6 = ?

Solution :

$$
\begin{array}{r}
707.325 \\
19\cancel{2}3.\cancel{8}2 \\
5\cancel{8}.00\cancel{9} \\
5\cancel{6}4.\cancel{9}43 \\
\cancel{6}5.6 \\
\hline
\end{array}
$$

Total: 3319.697

You may raise a question : is it necessary to write the numbers in column-form? The answer is 'no'. You may get the answer without doing so. Question written in a row-form causes a problem of alignment. If you get command over it, there is nothing better than this. For initial stage, we suggest you a method which would bring you out of the alignment problem.

Step I. "Put zeros to the right of the last digit after decimal to make the no. of digits after decimal equal in each number."

For example, the above question may be written as

707.325 + 1923.820 + 58.009 + 564.943 + 65.600

Step II. Start adding the last digit from right. Strike off the digit which has been dealt with. If you don't cut, duplication may occur. During running total, don't exceed 10. That is, when we exceed 10, we mark a tick anywhere near about our calculation. Now, go ahead with the number exceeding 10.

$$707.32\cancel{5} + 1923.82\cancel{0} + 58.00\cancel{9} + 564.94\cancel{3} + 65.60\cancel{0} = \_\_\_\_.\_\_7$$

5 plus 0 is 5; 5 plus 9 is 14, mark a tick in rough area and carry over 4; 4 plus 3 is 7; 7 plus 0 is 7, so write down 7. During this we strike off all the digits which are used. It saves us from confusion and duplication.

Step III. Add the number of ticks (in rough) with the digits in 2nd places. and erase that tick from rough.

$$707.3\cancel{2}\cancel{5} + 1923.8\cancel{2}\cancel{0} + 58.0\cancel{0}\cancel{9} + 564.9\cancel{4}\cancel{3} + 65.6\cancel{0}\cancel{0} = \_\_\_\_.\_97$$

1 (number of tick) plus 2 is 3; 3 plus 2 is 5; 5 plus 0 is 5; 5 plus 4 is 9 and 9 plus 0 is 9; so write down 9 in its place.

Step IV.

$$707.\cancel{3}\cancel{2}\cancel{5} + 1923.\cancel{8}\cancel{2}\cancel{0} + 58.\cancel{0}\cancel{0}\cancel{9} + 564.\cancel{9}\cancel{4}\cancel{3} + 65.\cancel{6}\cancel{0}\cancel{0} = \_\_\_\_.697$$

3 plus 8 is 11; mark a tick in rough and carry over 1; 1 plus 0 is 1; 1 plus 9 is 10, mark another tick in rough and carry over zero; 0 plus 6 is 6, so put down 6 in its place.

Step V.

Last Step : Following the same way get the result:

$$\cancel{7}\cancel{0}\cancel{7}.\cancel{3}\cancel{2}\cancel{5} + \cancel{1}\cancel{9}\cancel{2}\cancel{3}.\cancel{8}\cancel{2}\cancel{0} + \cancel{5}\cancel{8}.\cancel{0}\cancel{0}\cancel{9} + \cancel{5}\cancel{6}\cancel{4}.\cancel{9}\cancel{4}\cancel{3} + \cancel{6}\cancel{5}.\cancel{6}\cancel{0}\cancel{0} = 3319.697$$

Addition of numbers (without decimals) written in a row form

Q. 53921 + 6308 + 86 + 7025 + 11132 = ?

Soln : Step I: 53921 + 6308 + 86 + 7025 + 11132 = ____2

Step II : 53921 + 6308 + 86 + 7025 + 11132 = ___72

Step III: 53921 + 6308 + 86 + 7025 + 11132 = __472

Step IV: 53921 + 6308 + 86 + 7025 + 11132 = _8472

Step V: 53921 + 6308 + 86 + 7025 + 11132 = 78472

Note : One should get good command over this method because it is very much useful and fast-calculating. If you don't understand it, try again and again.

Addition and subtraction in a single row

Ex. 1: 412 - 83 + 70 = ?

Step I: For units digit of our answer add and subtract the digits at units places according to the sign attached with the respective numbers. For example, in the above case the unit place of our temporary result is

2 - 3 + 0 = -1

So, write as:
412 - 83 + 70 = ___ (-1)
Similarly, the temporary value at tens place is
1 - 8 + 7 = 0. So, write as:
412 - 83 + 70 = __ (0) (-1)
Similarly, the temporary value at hundreds place is 4. So, we write as:
412 - 83 + 70 = (4) (0) (-1)

Step II: Now, the above temporary figures have to the changed into real value. To replace (-1) by a +ve digit we borrow from digits at tens or hundreds.

As the digit at tens is zero, we will have to borrow from hundreds. We borrow 1 from 4 (at hundreds) which becomes 10 at tens leaving 3 at hundreds. Again we borrow 1 from tens which becomes 10 at units place, leaving 9 at tens. Thus, at units place 10 - 1 = 9. Thus our final result = 399.

The above explanation can be represented as

(-1)	(10) (-1)	(10)
(4)	(0)	(-1)
(3)	(9)	(9)

Note: The above explanation is easy to understand. And the method is more easy to perform. If you practise well, the two steps (I & II) can be performed simultaneously.
The second step can be performed in another way like:
(4) (0) (-1) = 400 - 1 = 399

Ex. 2: 5124 - 829 + 731 - 435
Soln: According to step I, the temporary figure is:
(5) (-4) (0) (-9)
Step II: Borrow 1 from 5. Thousands place becomes 5 - 1 = 4. 1 borrowed from thousands becomes 10 at hundreds. Now, 10 - 4 = 6 at hundreds place, but 1 is borrowed for tens. So digit at hundreds becomes 6 - 1 = 5. 1 borrowed from hundreds becomes 10.
Again we borrow 1 from tens for units place, after which the digit at tens place is 9. Now, 1 borrowed from tens becomes 10 at units place. Thus the result at units place is 10 - 9 = 1.
Our required answer = 4591
Note: After step I we can perform like:
5 (-4) (0) (-9) = 5000 - 409 = 4591

But this method can't be combined with step I to perform simultaneously. So, we should try to understand steps I & II well so that in future we can perform them simultaneously.

Ex. 3: 73216 - 8396 + 3510 - 999 = ?
Soln: Step I gives the result as:
(7) (-2) (-5) (-16) (-9)
Step II: Units digit = 10 - 9 = 1 [1 borrowed from (-16) results - 16 - 1 = -17]
Tens digit = 20 - 17 = 3 [2 borrowed from (-5) results -5 - 2 = -7]
Hundreds digit = 10 - 7 = 3 [1 borrowed from -2 results -2 - 1 = - 3]
Thousands digit = 10 - 3 = 7 [1 borrowed from 7 results 7 - 1 = 6]
So, the required value is 67331.
The above calculations can also be started from leftmost digit as done in last two examples. We have started from rightmost digit in this case. The result is the same in both cases. But for the combined operation of two steps you will have to start from rightmost digit (i.e. units digit). See Ex. 4.
Note: Other method for step II: (-2)(-5)(-16)(-9) = (-2)(-6)(-6)(-9) = -(2669)
∴ Ans = 70000 - (2669) = 67331

Ex. 4: 89978 - 12345 - 36218 = ?
Soln: **Step I:** (4) (1) (4) (2) (-5)
Step II: 4 1 4 1 5

Single step solution:

Now, you must learn to perform the two steps simultaneously. This is the simplest example to understand the combined method.
At units place: 8 - 5 - 8 = (-5). To make it positive we have to borrow from tens. You should remember that we can't borrow from -ve value i.e., from 12345. **We will have to borrow from positive value i.e. from 89978.** So, we borrowed 1 from 7 (tens digit of 89978):
(-1)
8 9 9 7 8 - 12345 - 36218 = _ _ _ _ 5
Now digit at tens: (7 - 1 =) 6 - 4 - 1 = 1
Digit at hundreds: 9 - 3 - 2 = 4
Digit at thousands: 9 - 2 - 6 = 1
Digit at ten thousands: 8 - 1 - 3 = 4
∴ the required value = 41415

Ex. 5: 28369 + 38962 - 9873 = ?
Soln: Single step solution: Units digit = 9 + 2 - 3 = 8
Tens digit = 6 + 6 - 7 = 5 Hundreds digit = 3 + 9 - 8 = 4

Thousands digit = 8 + 8 - 9 = 7

Ten thousands digit = 2 + 3 = 5 ∴ required value = 57458

Ex. 6: Solve Ex. 2 by single-step method.

Soln: 5124 - 829 + 731 - 435 =

Units digit: 4 - 9 + 1 - 5 = (-9). Borrow 1 from tens digit of the positive value. Suppose we borrowed from 3 of 731. Then

$5124 - 829 + 7\overset{-1}{3}1 - 435 = \_\,\_\,\_\,1$

Tens digit: 2 - 2 + 2 - 3 = (-1). Borrow 1 from hundreds digit of +ve value. Suppose we borrowed from 7 of 731. Then

$5124 - 829 + \overset{-1}{7}\,\overset{-1}{3}\,1 - 435 = \_\,\_\,9\,1$

Hundreds digit: 1 - 8 + 6 - 4 = (-5). Borrow 1 from thousands digit of +ve value. We have only one such digit, i.e. 5 of 5124. Then

$\overset{-1}{5}\,1\,2\,4 - 829 + \overset{-1}{7}\,\overset{-1}{3}\,1 - 435 = 4591$

(Thousands digit remains as 5 - 1 = 4)

Now you can perform the whole calculation in a single step without writing anything extra.

Ex. 7: Solve Ex. 3 in a single step without writing anything other than the answer. Try it yourself. Don't move to next example until you can confidently solve such questions within seconds.

Ex. 8: 10789 + 3946 - 2310 - 1223 = ?

Soln: Whenever we get a value more than 10 after addition of all the units digits, we will put the units digit of the result and carry over the tens digit. **We add the tens digit to +ve value, not to the -ve value.** Similar method should be adopted for all digits.

$1\,\overset{+1}{0}\,\overset{+1}{7}\,\overset{+1}{8}\,9 + 3946 - 2310 - 1223 = 11202$

Note: 1. We put +1 over the digits of +ve value 10789. It can also be put over the digits of 3946. But it can't be put over 2310 and 1223.

2. In the exam when you are free to use your pen on question paper you can alter the digit with your pen instead of writing +1, +2, -1, -2 .... over the digits. Hence, instead of writing 8, you should write 9 over 8 with your pen. Similarly, write 8 in place of $\overset{+1}{7}$.

Ex. 9: 765.819 - 89.003 + 12.038 - 86.89 = ?

Soln: First, equate the number of digits after decimals by putting zeroes at the end. So, ? = 765.819 - 89.003 + 12.038 - 86.890

Now, apply the same method as done in Ex. 4, 5, 6, 7 & 8.

$\overset{-1}{7}\,\overset{-1}{6}\,\overset{-1}{5}.\overset{-1}{8}\,\overset{+1}{1}\,9 - 89.003 + 12.038 - 86.890 = 601.964$

Method of checking the calculation : Digit-sum Method

This method is also called the **nines-remainder method.** The concept of digit-sum consists of this :

I. We get the digit-sum of a number by "adding across" the number. For instance, the digit-sum of 13022 is 1 plus 3 plus 0 plus 2 plus 2 is 8.

II. We always reduce the digit-sum to a single figure if it is not already a single figure. For instance, the digit-sum of 5264 is 5 plus 2 plus 6 plus 4 is 8 (17, or 1 plus 7 is 8).

III. In "adding across" a number, we may drop out 9's. Thus, if we happen to notice two digits that add up to 9, such as 2 and 7, we ignore both of them; so the digit-sum of 990919 is 1 at a glance. (If we add up 9's we get the same result.)

IV. Because "nines don't count" in this process, as we saw in III, a digit-sum of 9 is the same as a digit-sum of zero. The digit-sum of 441, for example, is zero.

Quick Addition of Digit-sum : When we are "adding across" a number, as soon as our running total reaches two digits we add these two together, and go ahead with a single digit as our new running total.

For example : To get the digit-sum of 886542932851 we do like: 8 plus 8 is 16, a two-figure number. We reduce this 16 to a single figure: 1 plus 6 is 7. We go ahead with this 7 ; 7 plus 6 is 4 (13, or 1+3=4), 4 plus 5 is 9, forget it. 4 plus 2 is 6. Forget 9 .... Proceeding this way we get the digit-sum equal to 7.

For decimals we work exactly the same way. But we don't pay any attention to the decimal point. The digit-sum of 6.256, for example, is 1.

Note : It is not necessary in a practical sense to understand why the method works, but you will see how interesting this is. The basic fact is that the reduced digit-sum is the same as the remainder when the number is divided by 9.

For example : Digit-sum of 523 is 1. And also when 523 is divided by 9, we get the remainder 1.

Checking of Calculation

Basic rule : Whatever we do to the numbers, we also do to their digit-sum; then the result that we get from the digit-sum of the numbers must be equal to the digit-sum of the answer.

For example :

The number : $23 + 49 + 15 + 30 = 117$

The digit-sum : $5 + 4 + 6 + 3 = 0$

Which reduces to : $0 = 0$

This rule is also applicable to subtraction, multiplication and upto some extent to division also. These will be discussed in the coming chapters. We should take another example of addition.

$1.5 + 32.5 + 23.9 = 57.9$

digit-sum: $6 + 1 + 5 = 3$

or, $3 = 3$

Thus, if we get LHS = RHS we may conclude that our calculation is correct.

Sample Question : Check for all the calcutions done in this chapter.

Note : Suppose two students are given to solve the following question: 1.5+ 32.5 + 23.9 = ?

One of them gets the solution as 57.9. Another student gets the answer 48.9. If they check their calculation by this method, both of them get it to be correct. Thus this method is not always fruitful. If our luck is against us, we may approve our wrong answer also.

Addition of mixed numbers

Q. $3\frac{1}{2} + 4\frac{4}{5} + 9\frac{1}{3} = ?$

Solution : A conventional method for solving this question is by converting each of the numbers into pure fractional numbers first and then taking the LCM of denominators. To save time, we should add the whole numbers and the fractional values separately. Like here,

$$3\frac{1}{2} + 4\frac{4}{5} + 9\frac{1}{3} = (3+4+9) + \left(\frac{1}{2} + \frac{4}{5} + \frac{1}{3}\right)$$

$$= 16 + \frac{15 + 24 + 10}{30} = 16 + 1\frac{19}{30}$$

$$= (16 + 1) + \frac{19}{30} = 17 + \frac{19}{30} = 17\frac{19}{30}$$

Q. $5\frac{2}{3} - 4\frac{1}{6} + 2\frac{3}{4} - 1\frac{1}{4}$

Soln : $(5 - 4 + 2 - 1) + \left(\frac{2}{3} - \frac{1}{6} + \frac{3}{4} - \frac{1}{4}\right)$

$$= 2 + \left(\frac{8 - 2 + 9 - 3}{12}\right) = 2 + \frac{12}{12} = 2 + 1 = 3$$

Multiplication

Special Cases

We suggest you to remember the tables up to 30 because it saves some valuable time during calculation. Multiplication should be well commanded, because it is needed in almost every question of our concern.

Let us look at the case of multiplication by a number more than 10.

MULTIPLICATION BY 11

Step I: The last digit of the multiplicand (number multiplied) is put down as the right-hand figure of the answer.

Step II: Each successive digit of the multiplicand is added to its neighbour at the right.

Ex.1. Solve $5892 \times 11 = ?$

Soln: Step I: Put down the last figure of 5892 as the right hand figure of the answer:

$$\begin{array}{r} \underline{5892 \times 11} \\ 2 \end{array}$$

Step II: Each successive figure of 5892 is added to its right-hand neighbour. 9 plus 2 is 11, put 1 below the line and carry over 1. 8 plus 9 plus 1 is 18, put 8 below the line and carry over 1. 5 plus 8 plus 1 is 14, put 4 below the line and carry over 1.

$$\begin{array}{r} \underline{5892 \times 11} \\ 12 \end{array}$$ (9+2 = 11, put 1 below the line and carry over 1)

$$\begin{array}{r} \underline{5892 \times 11} \\ 812 \end{array}$$ (8+9+1 =18, put 8 below the line and carry over 1)

$$\begin{array}{r} \underline{5892 \times 11} \\ 4812 \end{array}$$ (5+8+1 = 14, put 4 below the line and carry over 1)

Step III: The first figure of 5892, 5 plus 1, becomes the left-hand figure of the answer:

$$\begin{array}{r} \underline{5892 \times 11} \\ 64812 \end{array}$$

The answer is 64812.

As you see, each figure of the long number is used twice. It is first used as a "number", and then, at the next step, it is used as a neighbour. Looking carefully, we can use just one rule instead of three rules, And this only rule can be called as **"add the right neighbour"** rule.

We must first write a zero in front of the given number, or at least imagine a zero there.

Then we apply the idea of adding the neighbour to every figure of the given number in turn:

$\frac{05892 \times 11}{2}$ As there is no neighbour on the right, so we add nothing.

$\frac{05892 \times 11}{4812}$ -------- As we did earlier

$\frac{05892 \times 11}{64812}$ ----------- zero plus 5 plus carried-over 1 is 6

This example shows why we need the zero in front of the multiplicand. It is to remind us not to stop too soon. Without the zero in front, we might have neglected the last 6, and we might then have thought that the answer was only 4812. The answer is longer than the given number by one digit, and the zero in front takes care of that.

Sample Problems : Solve the following :

1) 111111 × 11 2) 23145 × 11 3) 89067 × 11 4) 5776800 × 11 5)1122332608 × 11

Ans : 1) 1222221 2) 254595 3) 979737 4) 63544800 5) 12345658688

MULTIPLICATION BY 12

To multiply any number by 12, we

"Double each digit in turn and add its neighbour".

This is the same as multiplying by 11 except that now we double the "number" before we add its "neighbour".

For example :

Ex 1: Solve : 5324 × 12

Soln : Step I. $\frac{05324 \times 12}{8}$ (double the right hand figure and add zero, as there is no neighbour)

Step II. $\frac{05324 \times 12}{88}$ (double the 2 and add 4)

Step III. $\frac{05324 \times 12}{888}$ (double the 3 and add 2)

Step IV. $\frac{05324 \times 12}{3888}$ (double the 5 and add 3 (=13), put 3 below the line and carry over 1)

Last Step. $\frac{05324 \times 12}{63888}$ (zero doubled is zero, plus 5 plus carried-over 1)

The answer is 63,888. If you go through it yourself you will find that the calculation goes very fast and is very easy.

Practice Question

Solve the following:

1)35609 × 12 2)11123009 × 12 3)456789 × 12 4)22200007 × 12 5)444890711 × 12

Ans : 1) 427308 2) 133476108 3) 5481468 4) 266400084 5) 5338688532

MULTIPLICATION BY 13

To multiply any number by 13, we

"Treble each digit in turn and add its right neighbour".

This is the same as multiplying by 12 except that now we "treble" the "number" before we add its "neighbour".

If we want to multiply 9483 by 13, we proceed like this :

Step I. $\frac{09483 \times 13}{9}$ (treble the right hand figure and write it down as there is no neighbour on the right)

Step II. $\frac{09483 \times 13}{79}$ (8×3+3 = 27, write down 7 and carry over 2)

Step III. $\frac{09483 \times 13}{279}$ (4×3+8+2 = 22, write down 2 and carry over 2)

Step IV. $\frac{09483 \times 13}{3279}$ (9×3+4+2 = 33, write down 3 and carry over 3)

Last Step. $\frac{09483 \times 13}{123279}$ (0×3+9+3 = 12, write it down)

The answer is 1,23,279.

In a similar way, we can define rules for multiplication by 14,15,.... But, during these multiplications we will have to get four or five times of a digit, which is sometimes not so easy to carry over. We have an easier method of multiplication for those large values.

Can you get similar methods for multiplication by 21 and 31? It is not very tough to define the rules. Try it.

Multiplication by 9

Step I: Subtract the right-hand figure of the long number from 10. This gives the right-hand figure of the answer.

Step II: Taking the next digit from right, subtract it from 9 and add the neighbour on its right.

Step III: At the last step, when you are under the zero in front of the long number, subtract one from the neighbour and use that as the left-hand figure of the answer.

Ex 1: $8576 \times 9 = ?$

Soln : $\underline{08576 \times 9}$

77184

Step I. Subtract the 6 of 8576 from 10, and we have 4 of the answer.

Step II. Subtract the 7 from 9 (we have 2) and add the neighbour 6; the result is 8.

Step III. $(9-5)+7 = 11$; put 1 under the line and carry over 1.

Step IV. $(9-8)+5+1$ (carried over) = 7, put it down.

Step V (Last step). We are under the left-hand zero, so we reduce the left-hand figure of 8576 by one, and 7 is the left-hand figure of the answer.

Thus answer is 77184.

Here are a few questions for you.

1) $34 \times 9 = ?$ 2) 569×9 ? 3) $1328 \times 9 = ?$ 4) $56493 \times 9 = ?$

5) $89273258 \times 9 = ?$

Answers

1) 306 2) 5121 3) 11952 4) 508437 5) 803459322

We don't suggest you to give much emphasis on this rule. Because it is not very much easy to use. Some times it proves very lengthy also.

Another method :

Step I : Put a zero at the right end of the number; ie, write 85760 for 8576.

Step II: Subtract the original number from that number. Like

$85760 - 8576 = 77184$

MULTIPLICATION BY 25

Suppose you are given a large number like 125690258. And someone asks you to multiply that number by 25. What will you do? Probably you will do nothing but go for simple multiplication. Now, we suggest you to multiply that number by 100 and then divide by 4.

To do so remember the two steps :

Step I: Put two zeroes at the right of the number (as it has to be multiplied by 100).

Step II: Divide it by 4.

So, your answer is $12569025800 \div 4 = 3142256450$. Is it easier than your method?

General Rule for Multiplication

Having dealt in fairly sufficient detail with the application of special cases of multiplication, we now proceed to deal with the "General Formula" applicable to all cases of multiplication. It is sometimes not very convenient to keep all the above cases and their steps in mind, so all of us should be very much familiar with "General Formula" of multiplication.

Multiplication by a two-digit number

Ex. 1 Solve (1) $12\times13 = ?$ (2) $17\times18 = ?$ (3) $87\times92 = ?$

Soln : (1) $12 \times 13 = ?$

Step I. Multiply the right-hand digits of multiplicand and multiplier (unit-digit of multiplicand with unit-digit of the multiplier).

1 2

1 3

6 (2×3)

Step II. Now, do cross-multiplication, ie, multiply 3 by 1 and 1 by 2. Add the two products and write down to the left of 6.

1 2

1 3

5 6 $(3\times1+2\times1)$

Step III. In the last step we multiply the left-hand figures of bo[th] multiplicand and multiplier.

```
1   2
|
1   3

  1 5 6   (1×1)
```

(2) 17 × 18 = ?

```
Step I.      1 7
             1 8
             ---
               6 (7 × 8 = 56, write down 6 and carry over [5])

Step II.     1 7
             1 8
             ---
             0 6   (1×8+7×1+5 = 20, write down 0
                    and carry over 2)

Step III.    1 7
             1 8
             ---
             306  (1×1+2 = 3, write it down)
```

(3) 87 × 92 = ?

```
Step I.     8 7
            9 2
            ---
              4  (7×2 = 14, write down 4 and carry over 1)

Step II.    8 7
            9 2
            ---
             04 (8×2+9×7+1 = 80, write down 0 and carry ove[r 8])

Step III.   8 7
            9 2
            ---
           8004  (8×9+8 = 80)
```

Practice questions

1) 57 × 43 2) 51 × 42 3) 38 × 43 4) 56 × 92 5) 81 × 19
6) 23 × 99 7) 29 × 69 8) 62 × 71 9) 17 × 37 10) 97 × 89

ANSWERS

1) 2451 2) 2142 3) 1634 4) 5152 5) 1539 6) 2277 7) 2001 8) 4402
9) 629 10) 8633

Ex 2. Solve (1) 325 × 17 = ? (2) 4359 × 23 = ?

Soln (1): Step I.

```
        3 2 5
            |
          1 7
          ---
            5 ( 5 × 7 = 35, put down 5 and carry over 3)

Step II.  3 2 5
             X
           1 7
           ---
           2 5 (2×7+5×1+3 = 22, put down 2 and carry over 2)

Step III. 3 2 5
           X
           1 7
          -----
          5 2 5   ( 3×7+2×1+2 = 25, put down 5 and
                    carry over 2)
```

Note : Repeat the cross-multiplication untill all the consecutive pairs of digits exhaust. In step II, we cross-multiplied 25 and 17 and in step III, we cross-multiplied 32 and 17.

```
Step IV.   3 2 5
           \
             1 7
          -------
          5 5 2 5  (3×1+2 = 5, put it down).
```

(2) Step I.

```
    4 3 5 9
          |
        2 3
    -------
          7  (9×3 = 27, put down 7, carry over 2).
```

Step II. 4 3 5 9

2 3

5 7 (5×3+9×2+2 = 35, put down 5 and carry over 3)

Step III. 4 3 5 9

2 3

2 5 7 (3×3+5×2+3 = 22, put down 2, carry over 2)

Step IV. 4 3 5 9

2 3

0 2 5 7 (4×3+3×2+2 = 20, put down 0, carry over 2)

Step V. 4 3 5 9

2 3

1 0 0 2 5 7 (4×2+2 = 10, put it down)

We can write all the steps together :

4 3 5 9
2 3

4×2/ 4×3+2×3 /3×3+5×2/ 5×3+9×2/ 9×3
= 10 20 22 35 27
= 100257

Or, we can write the answer directly without writing the intermediate steps. The only thing we should keep in mind is the "carrying numbers".

4359 × 23 = 10 20 22 35 27 or, 4 3 5 9 × 2 3 = 1 0 0 2 5 7

Note : You should try for this direct calculation. It saves a lot of time. It is a very systematic calculation and is very easy to remember. Watch the above steps again and again until you get that systematic pattern of cross-multiplication.

Multiplication by a 3-digit number

Ex: 1. Solve (1) 321 × 132 = ? (2) 4562 × 345 = ? (3) 69712 × 641 = ?

Soln : (1) **Step I.** 3 2 1

1 3 2

2 (1×2 = 2)

Step II. 3 2 1

1 3 2

7 2 (2×2+3×1 = 7)

Step III. 3 2 1

1 3 2

3 7 2 (2×3+3×2+1×1 = 13, write down 3 and carry over 1)

Step IV. 3 2 1

1 3 2

2 3 7 2 (3×3+1×2+1 = 12, write down 2 and carry over 1)

Step V. 3 2 1

1 3 2

4 2 3 7 2 (1×3+1 = 4)

or,

3 2 1
1 3 2

1×3 /3×3+1×2/ 2×3+3×2+1×1/ 2×2+3×1/ 1×2
= 4 12 13 7 2
= 42372

(2) 4 5 6 2
3 4 5

4×3/ 4×4+3×5/ 5×4+4×5+3×6/ 5×5+4×6+3×2/ 5×6+4×2/ 2×
= 15 37 63 58 39 10
= 1573890

(3) 6 9 7 1 2
6 4 1

6×6 /4×6+6×9/ 1×6+4×9+6×7/ 1×9+4×7+6×1/ 1×7+4×1+6×2/ 1×1+4×2/ 1
= 44 86 88 45 23 9 2
= 44685392

Note : Did you get the clear concept of cross-multiplication and carrying-cross-multiplication? Did you mark how the digits in cross-multiplication increase, remain constant, and then decrease ? Take a sharp loo at question (3). In the first row of the answer, if you move from right t left, you will see that there is only one multiplication (1×2) in the fir part. In the second part there are two (1×1 and 4×2), in the 3rd part thre (1×7, 4×1 and 6×2), in the 4th part three (1×9, 4×7 and 6×1), in the 5t part again three (1×6, 4×9 and 6×7), in the 6th part two (4×6 and 6×9 and in the last part only one (6×6) multiplication. The participation c digits in cross-multiplication can be shown by the following diagrams.

I. 6 9 7 1 2 II. 6 9 7 1 2 III. 6 9 7 1 2 IV. 6 9 7 1 2 V. 6 9 7 1

6 4 1 6 4 1 6 4 1 6 4 1 6 4

VI. 6 9 7 1 2 VII. 6 9 7 1 2

6 4 1 6 4 1

For each of the groups of figures, you have to cross-multiply.

Multiplication by a 4-digit number

Example : Solve 1) 4325 × 3216 2) 646329 × 8124

Soln : 1) 4325 × 3216 = ?

Step I. 4 3 2 5

3 2 1 6

0 (5×6 = 30, write down 0 and carry over 3)

Step II. 4 3 2 5

3 2 1 6

0 0 (2×6+1×5+3 = 20, write down 0 and carry over 2)

Step III. 4 3 2 5

3 2 1 6

2 0 0 (3×6+2×1+5×2+2 = 32, write down 2 and carry over 3)

Step IV. 4 3 2 5

3 2 1 6

9 2 0 0 (4×6+3×1+2×2+5×3+3 = 49, write down 9 and carry over 4)

Step V. 4 3 2 5

3 2 1 6

0 9 2 0 0 (4×1+3×2+2×3+4 = 20, write down 0 and carry over 2)

Step VI. 4 3 2 5

3 2 1 6

9 0 9 2 0 0 (4×2+3×3+2 = 19, write down 9 and carry over 1)

Step VII. 4 3 2 5

3 2 1 6

1 3 9 0 9 2 0 0 (4×3+1 = 13, write it down)

Ans : 13909200

2. 646329 × 8124 = ?

Try this question yourself and match your steps with the given diagrammatic presentation of participating digits.

Step I. 6 4 6 3 2 9
8 1 2 4

Step II. 6 4 6 3 2 9
8 1 2 4

Step III. 6 4 6 3 2 9
8 1 2 4

Step IV. 6 4 6 3 2 9
8 1 2 4

Step V. 6 4 6 3 2 9
8 1 2 4

Step VI. 6 4 6 3 2 9
8 1 2 4

Step VII. 6 4 6 3 2 9
8 1 2 4

Step VIII. 6 4 6 3 2 9
8 1 2 4

Step IX. 6 4 6 3 2 9
8 1 2 4

Practice Problems : Solve the following :

1) 234 × 456 2) 336 × 678 3) 872 × 431 4) 2345 × 67
5) 345672 × 456 6) 569 × 952 7) 458 × 908 8) 541 × 342
9) 666 × 444 10) 8103 × 450 11) 56321 × 672 12) 1278 × 569
13) 5745 × 562 14) 4465 × 887 15) 8862 × 341

ANSWERS

1) 106704 2) 227808 3) 375832 4) 157115 5) 157626432
6) 541688 7) 415864 8) 185022 9) 295704 10) 3646350
11) 37847712 12) 727182 13) 3228690 14) 3960455 15) 3021942

CHECKING OF MULTIPLICATION

Ex1: 15 × 13 = 195
digit-sum : 6 × 4 = 6
or, 24 = 6
or, 6 = 6. Thus, our calculation is correct.

Ex 2: 69712 × 641 = 44685392
digit-sum : 7 × 2 = 5
or, 14 = 5
or, 5 = 5 Therefore, our calculation is correct.

Ex 3: 321 × 132 = 42372
digit sum : 6 × 6 = 0
or, 36 = 0
or, 0 = 0 Thus, our calculation is correct.

But if someone gets the answer 43227, and tries to check his calculation with the help of digit-sum rule, see what happens :

321 × 132 = 43227
digit sum : 6 × 6 = 0
or, 36 = 0
or, 0 = 0

This shows that our answer is correct. But it is not true. Thus we see that if our luck is very bad, we can aprove our wrong answer.

Divisibility

We now take up the interesting question as to how one can determine whether a certain given number, however large it may be, is divisible by a certain given divisor. There is no defined general rule for checking the divisibility. For different divisors, the rules differ at large. We will discuss the rule for divisors from 2 to 19.

Divisibility by 2

Rule: *Any number, the last digit of which is either even or 0, is divisible by 2.*

For example: 12,86,130,568926 and 5983450 are divisible by 2 but 13,133 and 596351 are not divisible by 2.

Divisibility by 3

Rule: *If the sum of the digits of a number is divisible by 3, the number is also divisible by 3.*

For example:

1) 123 : 1+2+3 = 6 is divisible by 3; hence 123 is also divisible by 3.
2) 5673 : 5+6+7+3 = 21; therefore divisible by 3.
3) 89612 : 8+9+6+1+2 = 26 = 2+6 = 8 is not divisible by 3, therefore, the number is not divisible by 3.

Divisibility by 4

Rule: *If the last two digits of a number is divisible by 4, the number is divisible by 4. The number having two or more zeros at the end is also divisible by 4.*

For example :

1) 526428 : 28 is divisible by 4, therefore, the number is divisible by 4.
2) 5300 : There are two zeros at the end, so it is divisible by 4.
3) 134000 : As there are more than two zeros, the number is divisible by 4.
4) 134522 : As the last two-digit number (22) is not divisible by 4, the number is not divisible by 4.

Note: The same rule is applicable to check the divisibility by 25. That is, a number is divisible by 25 if its last two digits are either zeros or divisible by 25.

Divisibility by 5

Rule : *If a number ends in 5 or 0, the number is divisible by 5.*

For example :

1) 1345 : As its last digit is 5, it is divisible by 5.
2) 1340 : As it last digit is 0, it is divisible by 5.
3) 1343 : As its last digit is neither 5 nor 0, it is not divisible by 5.

Divisibility by 6

Rule : *If a number is divisible by both 3 and 2, the number is also divisibl by 6. So, for a number to be divisible by 6,*

1) *The number should end with an even digit or 0 and*
2) *The sum of its digits should be divisible by 3.*

For example :

1) 63924 : The first condition is fulfilled as the last digit (4) is an eve number and also (6+3+9+2+4=)24 is divisible by 3; therefor the number is divisible by 6.
2) 154 : The first condition is fulfilled but not the second, therefore, th number is not divisible by 6.
3) 261 : The first condition is not fulfilled, therefore, we don't need to check for the 2nd condition.

Special Cases

The rules for divisibility by 7, 13, 17, 19 ... are very muc unique and are found very rarely. Before going on for the rule, we shoul know some terms like **"one-more" osculator** and **negative osculato**

"One-more" osculator means the number needs one more to b a multiple of 10. For example: osculator for 19 needs 1 to become 2 (=2×10), thus osculator for 19 is 2 (taken from $\underline{2} \times 10 = 20$). Similarl osculator for 49 is 5 (taken from $\underline{5} \times 10 = 50$).

Negative osculator means the number should be reduced b one to be a multiple of 10. For example :

Negative osculator for 21 is 2 (taken from $\underline{2} \times 10 = 20$) Similarly, negative osculator for 51 is 5 (taken from $\underline{5} \times 10 = 50$).

Note : (1) What is the osculator for 7 ?

Now, we look for that multiple of 7 which is either less or mor by 1 than a multiple of 10. For example $7 \times 3 = 21$, as 21 is one mor than $\underline{2} \times 10$; our **negative osculator** is 2 for 7.

And $7 \times 7 = 49$ or 49 is one less than $\underline{5} \times 10$; our **'one-more osculator** is 5 for 7.

Similarly osculator for 13, 17 and 19 is :

For 13 : $13 \times 3 = 39$, "one more" osculator is 4 (from $\underline{4} \times 10$)

For 17 : $17 \times 3 = 51$, negative osculator is 5 (from $\underline{5} \times 10$)

For 19 : 19×1. "one-more" osculator is 2 (from $\underline{2} \times 10$)

(2) Can you define osculators for 29, 39, 21, 31, 27 and 23.

(3) Can you get any osculator for an even number or a number ending with '5' ? (No. But why ?)

Divisibility by 7

First of all we recall the osculator for 7. Once again, for your convenience, as $7 \times 3 = 21$ (one more than $\underline{2} \times 10$), our negative osculator is 2. This oscuator '2' is our key-digit. This and only this digit is used to check the divisibility of any number by 7. See how it works :

Ex 1: Is 112 divisible by 7 ?

Soln: Step I. $\underline{11}\,\underline{2}$: $11 - 2 \times 2 = 7$

As 7 is divisible by 7, the number 112 is also divisible by 7.

Ex 2: Is 2961 divisible by 7 ?

Soln: Step I: $\underline{296}\,\underline{1}$: $296 - 1 \times 2 = 294$

Step II: $\underline{29}\,\underline{4}$: $29 - 4 \times 2 = 21$

As 21 is divisible by 7, the number is also divisible by 7.

Ex 3: $\underline{5527783}\,\underline{8}$: $5527783 - 8 \times 2 = 5527767$

$\underline{552776}\,\underline{7}$: $552776 - 7 \times 2 = 552762$

$\underline{55276}\,\underline{2}$: $55276 - 2 \times 2 = 55272$

$\underline{5527}\,\underline{2}$: $5527 - 2 \times 2 = 5523$

$\underline{552}\,\underline{3}$: $552 - 3 \times 2 = 546$

$\underline{54}\,\underline{6}$: $54 - 6 \times 2 = 42$

As 42 is divisible by 7, the number is also divisible by 7.

Note : 1. In all the examples each of the numbers obtained after the equal sign (=) is also divisible by 7. Whenever you find a number which looks divisible by 7, you may stop there and conclude the result without any hesitation.

2. The above calculations can be done in one line or even mentally. Try to do so.

Divisibility by 8

Rule: *If the last three digits of a number is divisibl by 8, the number is also divisible by 8. Also, if the last three digits of a number are zeros, the number is divisible by 8.*

Ex.1. 1256 : As 256 is divisible by 8, the number is also divisible by 8.

Ex.2. 135923120 : As 120 is divisible by 8, the number is also divisible by 8.

Ex.3. 139287000 : As the number has three zeros at the end the number is divisible by 8.

Note: The same rule is applicable to check the divisibility by 125.

Divisibility by 9

Rule: *If the sum of all the digits of a number is divisible by 9, the number is also divisible by 9.*

Ex.1. 39681 : 3+9+6+8+1= 27 is divisible by 9, hence the number is also divisible by 9.

Ex.2. 456138 : 4+5+6+1+3+8= 27 is divisible by 9, hence the number is also divisible by 9.

Divisibility by 10

Rule : *Any number which ends with zero is divisible by 10. There is no need to discuss this rule.*

Divisibility by 11

Rule: *If the sums of digits at odd and even places are equal or differ by a number divisible by 11, then the number is also divisible by 11.*

Ex.1. 3245682 : S_1 = 3+4+6+2= 15 and S_2 = 2+5+8= 15

As $S_1 = S_2$, the number is divisible by 11.

Ex.2. 283712 : S_1 = 2+3+1=6 and S_2 = 8+7+2= 17. As S_1 and S_2 differ by 11 (divisible by 11), the number is also divisible by 11.

Ex.3. 84927291658 : S_1 = 8+9+7+9+6+8= 47 and S_2 = 4+2+2+1+5= 14

As (S_1 − S_2 =) 33 is divisible by 11, the number is also divisible by 11.

Divisibility by 12

Rule : *Any number which is divisible by both 4 and 3, is also divisible by 12.*

To check the divisibility by 12, we

1) first divide the last two-digit number by 4. If it is not divisible by 4, the number is not divisible by 12. If it is divisible by 4 then
2) check whether the number is divisible by 3 or not.

Ex.1. 135792 : 92 is divisible by 4 and also (1+3+5+7+9+2=)27 is divisible by 3 ; hence the number is divisible by 12.

Remark : Recall the method for calculation of digit-sum. What did you do earlier (in 1st chapter)? "Forget nine". Do the same here. For example: digit-sum of 135792 __1 plus 3 plus 5 is 9, forget it. 7 plus 2 is 9, forget it. And finally we get nothing. That means all the "forget nine" counts a number which is multiple of 9. Thus, the number is divisible by 9.

Divisibility by 13

Osculator for 13 is 4 (See note). But this time our osculator is not negative (as in case of 7). It is 'one-more' osculator. So, the working will be different now. This can be seen in the following examples.

Ex 1: Is 143 divisible by 13 ?

Soln : 14 3 : 14 + 3 × 4 = 26

Since 26 is divisible by 13, the number 143 is also divisible by 13.

or,

This working may further be simplified as :

Step I : 1 4 3

16 [4 × 3 (from 143) + 4 (from 143)]

Step II: 1 4 3

26/16 [4 × 6 (from 16) +1 (from 16) +1 (from 143) = 26]

As 26 is divisible by 13, the number is also divisible by 13.

Note : The working of second method is also very systematic. At the same time it is more acceptable because it has less writing work.

Ex 2: Check the divisibility of 24167 by 13.

2 4 1 6 7

26/6/20/34 [4×7 (from 24167) +6 (from 24167)= 34]

[4×4 (from 34) +3 (from 34) +1 (from 24167) = 20]

[4×0 (from 20) +2 (from 20) +4 (from 24167) = 6]

[4×6 (from 6) +2 (from 24167) = 26]

Since 26 is divisible by 13 the number is also divisible by 13.

Remark : Have you understood the working ? If your answer is no, we suggest you to go through each step carefully. This is very simple and systematic calculation.

Ex 3: Check the divisibility of 6944808 by 13.

Soln : 6 9 4 4 8 0 8

39/18/12/41/19/32

4×8+0 = 32

4×2+3+8 = 19

4×9+1+4 = 41

4×1+4+4 = 12

4×2+1+9 = 18
4×8+1+6 = 39

Since 39 is divisible by 13, the given number is divisible by 13.
Note : (1) This method is applicable for "one-more" osculator only. So we can't use this method in the case of 7.
(2) This is a one-line method and you don't need to write the calculations during exams. These are given merely to make you understand well.

Divisibility by 14

Any number which is divisible by both 2 and 7, in also divisible by 14. That is, the number's last digit should be even and at the same time the number should be divisible by 7.

Divisibility by 15

Any number which is divisible by both 3 and 5 is also divisible by 15.

Divisibility by 16

Any number whose last 4 digit number is divisible by 16 is also divisible by 16.

Divisibility by 17

Negative osculator for 17 is 5 (see note). The working for this is the same as in the case of 7.
Ex 1: Check the divisibility of 1904 by 17.
Soln : 190 4 : 190 - 5×4 = 170
Since 170 is divisible by 17, the given number is also divisible by 17.
Note : Students are suggested not to go upto the last calculation. Whenever you find the number divisible by the given number on right side of your calculation stop further calculation and conclude the result.
Ex 2: 957508:

95750 8 : 95750 - 5×8 = 95710
9571 0 : 9571 - 5×0 = 9571
957 1 : 957 - 5×1 = 952
95 2 : 95 - 5×2 = 85

Since 85 is divisible by 17, the given number is divisible by 17.

Ex 3: 8971563 :

897156 3 : 897156 - 5×3 = 897141
89714 1 : 89714 - 5×1 = 89709
8970 9 : 8970 - 5×9 = 8925

892 5 : 892 - 5×5 = 867
86 7 : 86 - 5×7 = 51

Since 51 is divisible by 17, the given number is also divisible by 17.

Divisibility by 18

Rule: *Any number which is divisible by 9 and has its last digit (unit-digit) even (or zero), is divisible by 18.*
Ex. 1. 926568 : Digit-sum is multiple of nine (ie, divisible by 9) and unit-digit (8) is even, hence the number is divisible by 18.
Ex. 2. 273690 : Digit-sum is multiple of nine and the number ends in zero, so the number is divisible by 18.
Note: During the calculation of digit-sum, follow the method of **"forget nine"**. If you get zero at the end of your calculation, it means the digit-sum is divisible by 9.

Divisibility by 19

If you recall, the 'one-more' osculator for 19 is 2. The method is similar to that of 13, which is well known to you. Let us take an example.
Ex 1: 149264
Soln : 1 4 9 2 6 4
19/9/12/11/14

Thus, our number is divisible by 19.
Note: You must have understood the working (see the case of 13).

Squaring

Squaring of a number is largely used in mathematical calculations. There are so many rules for special cases. But we will discuss a general rule for squaring which is capable of universal application.

This method is intimately connected with a procedure known as the "Duplex Combination" process and is of still greater importance and utility at the next step on the ladder, namely, the easy and facile extraction of square roots. We now go on to a brief study of this procedure.

Duplex Combination Process

The first one is by squaring; and the second one is by cross-multiplication. In the present context, it is used in both senses (a^2 and 2ab).

In the case of a single central digit, the square is meant; and in the case of an even number of digits equidistant from the two ends, double the cross-product is meant. A few examples will elucidate the procedure.

Ex.1: For 2, Duplex (D) = $2^2 = 4$

Ex.2: For 8, D = $8^2 = 64$

Ex.3: For 34, D = $2\times(3\times4) = 24$

Ex.4: For 79, D = $2\times(7\times9) = 126$

Ex.5: For 103, D = $2(1\times3)+0^2 = 6$

Ex.6: For 346, D = $2(3\times6)+4^2 = 52$

Ex.7: For 096, D = $2(0\times6)+81 = 81$

Ex.8: For 1342, D = $2(1\times2)+2(3\times4) = 28$

Ex.9: For 7358, D = $2(56) + 2(15) = 142$

Ex.10: For 23564, D = $2(2\times4)+2(3\times6)+5^2 = 77$

Ex.11: For 123456, D = $2(1\times6)+2(2\times5)+2(3\times4) = 56$

Now, we see the method of squaring in the following examples.

Ex1. $207^2 = ?$

Soln: 207^2 = D for 2 / D for 20 / D for 207 / D for 07 / D for 7

$= 2^2 / 2(2\times0) / 2(2\times7)+0^2 / 2(0\times7) / 7^2$

= 4 / 0 / 28 / 0 / 49

= 4 / 0 / 2_8 / 0 / 4_9

= 4 / 0 + 2 / 8 / 0 + 4 / 9 = 42849

If you have understood the duplex method and its use in squaring, you may get the answer in a line. For example: $207^2 = 42_28 4_49$

Explanations. 1. Duplex of 7 is $7^2 = 49$. Put the unit digit (9) of duplex in answer line and carry the other (4).

2. 2×0×7 + 4(carried) = 4; write it down at 2nd position.

3. $2 \times 2 \times 7 + 0^2 = 28$; write down 8 and carry over 2.

4. 2×2×0 + 2(carried over)= 2; write it down.

5. $2^2 = 4$; write it down.

Note: (1) If there are n digits in a number, the square will have either 2n or 2n-1 digits.

(2) Participation of digits follows the same systematic pattern as in multiplication.

Ex.2: $(897)^2 = 80_{16}4_{20}6_{13}0_49 = 804609$

Explanations: 1. $7^2 = 49$; write down 9 and carry over 4.

2. 2×9×7 + 4 (carried) = 130; write down 0 and carry over 13.

3. $2 \times 8 \times 7 + 9^2 + 13 = 206$; write down 6 and carry over 20.

4. 2×8×9 + 20 = 164; write down 4 and carry over 16.

5. $8^2 + 16 = 80$; write it down.

Ex. 3: $(1\,4\,3\,2)^2 = 2_10_25_30_26_124 = 2050624$

Explanations: 1. $2^2 = 4$; write it down.

2. 2×(3×2) = 12; write down 2 and carry over 1.

3. $2 \times (4 \times 2) + 3^2 + 1 = 26$; write down 6 and carry over 2.

4. 2(1×2) + 2(4×3)+2 = 30; write down 0 and carry over 3.

5. $2(1 \times 3) + 4^2 + 3 = 25$; write down 5 and carry over 2.

6. 2(1×4) + 2 = 10; write down 0 and carry over 1.

7. $1^2 + 1 = 2$; write it down.

Ex 4: $(73214)^2 = 53_46_40_32_68_29_179_16 = 5360289796$

Ex 5: $(5432819)^2 = 29_45_51_55_{11}5_{10}2_{16}2_{12}2_{12}8_66_{14}7_26_81$
$= 29515522286761$

Practice problem:

Q: Find the squares of the following numbers:

1) 835 2) 8432 3) 45321 4) 530026 5) 73010932

ANSWERS

1) 697225 2) 71098624 3) 2053993041 4) 280927560676
5) 5330596191508624

Note: To find the square of a fractional (decimal) number, we square the number without looking at decimal. After that we count the number of digits after the decimal in the original value. In the squared value, we place the decimal after double the number of digits after decimal in the original value. For example: $(12.46)^2$
$= 15_15_32_45_51_36 = 155.2516$

Some special cases derived with help of Duplex Combination Process

1. Square of numbers from 51 to 59.

We take a general representative of the numbers (from 51 to 59) say, 5A.

Now, $(5A)^2 = 5^2 / 2 \times 5 \times A / A^2 = 25 / 10 \times A / A^2$

We have; 10 × A = A0 [Like 10 × 4 = 40, 10 × 6 = 60, etc]

$\therefore\ (5A)^2 = 25 / A0 / A^2$

$= (25 + A) / A^2$; where A^2 should be written as a two-digit-number

Now, we see that our duplex combination process reduces to a simplier form. Using the above equation:

Ex. 1: $(51)^2 = 25 + 1 / (1)^2 = 26 / 01 = 2601$

Ex. 2: $(52)^2 = 25 + 2 / (2)^2 = 27 / 04 = 2704$

Ex. 3: $(54)^2 = 25 + 4 / (4)^2 = 29 / 16 = 2916$

Ex. 4: $(59)^2 = 25 + 9 / (9)^2 = 34 / 81 = 3481$

2. Square of a number with unit digit as 5.

We take a general representative of such number, say, A5

Now, $(A5)^2 = A^2 / 2 \times A \times 5 / 5^2$
$= A^2 / 10 \times A / 25$
$= A^2 / A0 / 25$ $[\because\ 10 \times A = A0]$
$= A^2 + A / 25$
$= A\,(A + 1) / 25$

Using the above equation:

Ex. 1: $(15)^2 = 1 \times (1 + 1) / 5^2 = 2 / 25 = 225$

Ex. 2: $(25)^2 = 2 \times (2 + 1) / 5^2 = 6 / 25 = 625$

Ex. 3: $(85)^2 = 8 \times (8 + 1) / 5^2 = 72 / 25 = 7225$

Ex. 4: $(115)^2 = 11 \times (12) / 5^2 = 132 / 25 = 13225$

Ex. 5: $(225)^2 = 22 \times (23) / 5^2 = 506 / 25 = 50625$

3. Square of a number wich is nearer to 10^x

We use the algebraic formula

$$x^2 = (x^2 - y^2) + y^2 = (x + y)(x - y) + y^2$$

Ex. 1: $(98)^2 = (98 + 2)(98 - 2) + 2^2 = 9600 + 4 = 9604$

Ex. 2: $(103)^2 = (103 - 3)(103 + 3) + 3^2 = 10600 + 9 = 10609$

Ex. 3: $(993)^2 = (993 + 7)(993 - 7) + 7^2 = 986000 + 49 = 986049$

Ex. 4: $(1008)^2 = (1008 - 8)(1008 + 8) + 8^2 = 1016000 + 64 = 1016064$

To check the calculation

We use the digit-sum method for checking calculations in squaring. For example:

In Ex 1: $(207)^2 = 42849$

digit-sum: $(0)^2 = (0)^2$

Hence, our calculation is correct.

In Ex 2: $(897)^2 = 804609$

digit-sum: $(6)^2 = 18$

or, $36 = 18$ or, $0 = 0$ Thus, our calculation is correct.

In Ex 3: $(1432)^2 = 2050624$

digit-sum: $1^2 = 1$ or, $1 = 1$ Thus, our calculation is correct.

Note: 1. Follow the "forget-nine" rule during the calculation of digit-sum.

2. Check all the calculations mentally.

3. Check the correctness of calculations in other examples without using pen.

Cube

Cubes of large numbers are rarely used. During our mathematical calculations, we sometimes need the cube value of two-digit numbers. So, an easy rule for calculating the cubes of 2-digit numbers is being given. In its process the cube values of the "first ten natural numbers", ie, 1 to 10, are used. Readers are suggested to remember the cubes of only these "first ten natural numbers."

$1^3 = 1$, $2^3 = 8$, $3^3 = 27$; $4^3 = 64$, $5^3 = 125$,
$6^3 = 216$ $7^3 = 343$, $8^3 = 512$, $9^3 = 729$ and $10^3 = 1000$.

To calculate the cube value of two-digit numbers we proceed like this:

Step I: The first thing we have to do is to put down the cube of the tens-digit in a row of 4 figures. The other three numbers in the row of answer should be written in a geometrical ratio in the exact proportion which is there between the digits of the given number.

Step II: The second step is to put down, under the second and third numbers, just two times of second and third number. Then add up the two rows.

For example :

Ex 1. $12^3 = ?$

Soln. Step I: We see that the ten-digit in the number is 1, so we write the cube of 1. And also as the ratio between 1 and 2 is 1 : 2, the next digits will be double the previous one. So, the first row is

1 2 4 8

Step II: In the above row our 2nd and 3rd digits (from right) are 4 and 2 respectively. So, we write down 8 and 4 below 4 and 2 respectively. Then add up the two rows.

$$\begin{array}{cccc} 1 & 2 & 4 & 8 \\ & 4 & 8 & \\ \hline 1 & 7 & {}_{1}2 & 8 \end{array} = 1728$$

Ex 2 : $11^3 = ?$ (Solve it yourself.)

Ex 3: $16^3 = ?$

$$\textbf{Soln :}\quad \begin{array}{cccc} 1 & 6 & 36 & 216 \\ & 12 & 72 & \\ \hline 4 & {}_{3}0 & {}_{12}9 & {}_{21}6 \end{array} = 4096$$

Explanations : 1^3 (from 16) = 1. So, 1 is our first digit in the first row. Digits of 16 are in the ratio 1:6, hence our other digits should be $1\times6=6$, $6\times6=36$, $36\times6=216$. In the second row, double the 2nd and 3rd number is written. In the third row, we have to write down only one digit below each column (except under the last column which may have more than one digit). So, after putting down the unit-digit, we carry over the rest to add up with the left-hand column. Here,

i) Write down 6 of 216 and carry over 21.
ii) 36 + 72 + 21 (carried) = 129, write down 9 and carry over 12.
iii) 6 + 12 + 12 (carried) = 30, write down 0 and carry over 3.
iv) 1 + 3 (carried) = 4, write down 4.

Ex 4: $18^3 = ?$

Soln :

1	8	64	512	
	16	128		
5	48	243	512	= 5832

i) Write down 2 and carry over 51 of 512.
ii) 64+128+51 = 243, write down 3 and carry over 24.
iii) 8+16+24 = 48, write down 8 and carry over 4.
iv) 1+4 = 5 write it down.

Ex 5: $17^3 = ?$ (Solve it yourself)

Ex 6: $19^3 = ?$ (Solve it yourself)

Ex 7: $21^3 = ?$

Soln :

8	4	2	1	[$8=2^3$, $8\div2=4$, $4\div2=2$, $2\div2=1$, since ratio is 2:1]
	8	4		[$4\times2=8$, $2\times2=4$, double is written below]
9	12	6	1	= 9261

Do you mark the difference? If no, go through the following explanations.

Step I: i) $2^3 = 8$ is the first figure (from left) in the first row.

ii) Ratio between the two digits is 2:1, ie, the number should be halved subsequently. Therefore, the next three numbers in the first row should be 4, 2 & 1.

Step II: It should be clear to all of you because it has nothing new.

Ex 8 : $23^3 = ?$

Soln :

8	12	18	27	
	24	36		
12	41	56	27	= 12167

Explanations:

Step I: i) $2^3 = 8$ ——— the first figure (from left) in the first row.

ii) 2:3 $\Rightarrow$ the next numbers should be $\frac{3}{2}$ of the previous ones. So, we have $8\times\frac{3}{2}=12$, $12\times\frac{3}{2}=18$, $18\times\frac{3}{2}=27$.

Ex 9: $33^3 = ?$

Soln :

27	27	27	27	
	54	54		
35	89	83	27	= 35937

Explanations:

Step i) $3^3 = 27$ ——— the first figure in first row.

ii) 3:3 = 1:1 $\Rightarrow$ the subsequent numbers should be the same.

Ex 10: $34^3 = ?$

Soln :

27	36	48	64	
	72	96		
39	123	150	64	= 39304

Explanations:

i) $3^3 = 27$ ——— the first figure (from left) in the first row.

ii) Ratio is 3:4, ie, the next numbers should be $\frac{4}{3}$ of their previous ones. Here, $27\times\frac{4}{3}=36$, $36\times\frac{4}{3}=48$, $48\times\frac{4}{3}=64$.

Ex 11: $93^3 = ?$

Soln :

729	243	81	27	
	486	162		
804	753	245	27	= 804357

Explanations:

i) $9^3 = 729$ ——— the first figure (from left) in the first row,

ii) 9:3 $\Rightarrow$ 3 :1 $\Rightarrow$ the subsequent figures should be $\frac{1}{3}$ of their previous ones.

Ex 12: 97^3 = ?

Soln :

729	567	441	343	
	1134	882		
912	1836	1357	343	= 912673

Explanations:

i) $9^3 = 729$ ------- first figure (from left) in the first row.

ii) Ratio = 9 : 7 $\Rightarrow$ Next numbers should be $\frac{7}{9}$ of the previous ones. Therefore, $729 \times \frac{7}{9} = 567$, $567 \times \frac{7}{9} = 441$, $441 \times \frac{7}{9} = 343$

Practice Problems.

Q. Find the cubes of the following numbers.

1. 17 **2.** 26 **3.** 27 **4.** 32 **5.** 41 **6.** 43 **7.** 49 **8.** 51 **9.** 53 **10.** 55 **11.** 57 **12.** 64 **13.** 67 **14.** 69 **15.** 73 **16.** 77 **17.** 88 **18.** 92 **19.** 95 **20.** 99.

ANSWERS

1) 4913 2) 17576 3) 19683 4) 32768 5) 68921 6) 79507
7) 117649 8) 132651 9) 148877 10) 166375 11) 185193
12) 262144 13) 300763 14) 328509 15) 389017 16) 456533
17) 681472 18) 778688 19) 857375 20) 970299

Note: Don't use this method for getting the cubes of 20, 30, 40,....
Do you know the other quick method?

Checking the correctness (with the help of digit-sum)

Ex. 1: $12^3 = 1728$

digit sum : $(3)^3 = 0$ (7+2 = 9, forget it. 1+8 = 9, forget it.)

or, 0 = 0 , thus the cube value is correct.

Ex 2: $16^3 = 4096$

digit-sum : $7^3 = 1$

or, (49) 7 = 1

or, $4 \times 7 = 1$

or, 28 = 1 or, 1 = 1, Thus cube value is correct.

Practice-Problem: Check all the calculations (from Ex 2 to Ex 12) done in this chapter.

HCF and LCM

Factor : One number is said to be a factor (*Gunankhand*) of another when it divides the other exactly. Thus, 6 and 7 are factors of 42.

Common Factor : A common factor of two or more numbers is a number that divides each of them exactly. Thus, 3 is a common factor of 9, 18, 21 and 33.

Highest Common Factor (HCF) : HCF of two or more numbers is the greatest number that divides each of them exactly. Thus, 6 is the HCF of 18 and 24. Because there is no number greater than 6 that divides both 18 and 24.

Note : The terms **Highest Common Divisor** and **Greatest Common Measure** are often used in the sense of Highest Common Factor (HCF).

To find the HCF of two or more numbers

Method I : Method of Prime Factors

Rule : *Break the given numbers into prime factors and then find the product of all the prime factors common to all the numbers. The product will be the required HCF.*

Ex. 1. Find the HCF of 42 and 70.

Soln : $42 = \underline{2} \times 3 \times \underline{7}$

$70 = \underline{2} \times 5 \times \underline{7}$

HCF = $2 \times 7 = 14$

Ex. 2. Find the HCF of 1365, 1560 and 1755.

Soln : $1365 = \underline{3} \times \underline{5} \times 7 \times \underline{13}$

$1560 = 2 \times 2 \times 2 \times \underline{3} \times \underline{5} \times \underline{13}$

$1755 = 3 \times 3 \times \underline{3} \times \underline{5} \times \underline{13}$

HCF = $3 \times 5 \times 13 = 195$

Note : (1) In finding the HCF, we need not break all the numbers into their prime factors. We may find the prime factors of one of the numbers. Then the product of those prime factors which divide each of the remaining numbers exactly will be the required HCF. In Ex. (1), the prime factors of 42 are $2 \times 3 \times 7$. Of these three factors, only 2 and 7 divide 70 exactly. Hence, the required HCF $= 2 \times 7 = 14$

In Ex. (2) the prime factors of 1365 are $3 \times 5 \times 7 \times 13$. Of these four factors, only 3,5 and 13 divide the other two numbers 1560 and 1755 exactly. Hence, the required HCF = $3 \times 5 \times 13 = 195$

(2) We must remember that the quotient obtained by dividing
numbers by their HCF are prime to each other.

In Ex. (1) $42 \div 14 = 3$

$70 \div 14 = 5$. We see that 3 is prime to 5, ie, 3 can't divide 5 exact

In Ex. (2), $1365 \div 195 = 7$, $1560 \div 195 = 8$ and $1755 \div 195 =$

We see that 7, 8 and 9 are prime to one another, i.e. none divid the other.

Method II: Method of Division

Rule : *Divide the greater number by the smaller number, divide t divisor by the remainder, divide the remainder by the next remaind and so on until no remainder is left. The last divisor is the required HC*

Ex. 1. 42) 70 (1
$\underline{42}$
28) 42 (1
$\underline{28}$
14) 28 (2
$\underline{28}$
0

$\therefore$ HCF = 14

Note : The above rule for finding the HCF of numbers is based on th following two principles:

(i) Any number which divides a certain number also divides any multip of that number; for example, 6 divides 18 therefore, 6 divides an multiple of 18.

(ii) Any number which divides each of the two numbers also divide their sum, their difference and the sum and difference of any multiple of that numbers.

Thus 5, being a common factor of 25 and 15, is also a factor of (25 + 15), and (25 - 15).

Again, 5 is also a factor of $(25 \times a + 15 \times b)$ and of $(25 \times a - 15 \times b)$, where a and b are integers.

In accordance with these principles,

HCF of 42 and 70
= HCF of 28 and 42 [$28 = 70 - 42$]
= HCF of 14 and 28 [$14 = 42 - 28$]
= HCF of 14 and 14 [$14 = 28 - 14$]
$\therefore$ HCF of 42 and 70 = 14.

HCF of 13281 and 15844
= HCF of 2563 and 13281 [$2563 = 15844 - 13281$]
= HCF of 466 and 2563 [$466 = 13281 - 5 \times 2563$]

= HCF of 233 and 466 [$233 = 2563 - 5 \times 466$].
= HCF of 233 and 233 [$233 = 466 - 233$]
$\therefore$ HCF of 13281 and 15844 = 233

The above discussed method is very much interesting. It gives results very quickly. But one should have a good understanding of this method.

To find the HCF of more than two numbers

Rule : *Find the HCF of any two of the numbers and then find the HCF of this HCF and the third number and so on. The last HCF will be the required HCF.*

Ex. 1. Find the HCF of 1365, 1560 and 1755.

Soln : 1365) 1560 (1
$\underline{1365}$
195) 1365 (7
$\underline{1365}$
0

Therefore, 195 is the HCF of 1365 and 1560.

Again, 195) 1755 (9
$\underline{1755}$
0

$\therefore$ the required HCF = 195

Method III: The work of finding the HCF may sometimes be simplified by the following devices :

(i) Any obvious factor which is common to both numbers may be removed before the rule is applied. Care should however be taken to multiply this factor into the HCF of the quotients.

(ii) If one of the numbers has a prime factor not contained in the other, it may be rejected.

(iii) At any stage of the work, any factor of the divisor not contained in the dividend may be rejected. This is because any factor which divides only one of the two cannot be a portion of the required HCF.

Ex. Find the HCF of 42237 and 75582.

Soln : $42237 = 9 \times 4693$
$75582 = 2 \times 9 \times 4199$

We may reject 2 which is not a common factor (by rule i). But 9 is a common factor. We, therefore, set it aside (by rule ii) and find the HCF of 4199 and 4693.

4199) 4693 (1
$\underline{4199}$
494

494 is divisible by 2 but 4199 is not. We, therefore, divide 494 by 2 and proceed with 247 and 4199 (by rule iii).

$$\begin{array}{r} 247\ )\ 4199\ (\ 17 \\ \underline{247} \\ 1729 \\ \underline{1729} \\ 0 \end{array}$$

The HCF of 4199 and 4693 is 247. Hence, the HCF of the original numbers is $247 \times 9 = 2223$.

Note : If the HCF of two numbers be unity, the numbers must be prime to each other.

To find the HCF of two or more concrete quantities

First, the quantities should be reduced to the same unit.

Ex. Find the greatest weight which can be contained exactly in 1 kg 235 gm and 3 kg 430 gm.

Soln : 1 kg 235 gm = 1235 gm
3 kg 430 gm = 3430 gm

The greatest weight required is the HCF of 1235 and 3430, which will be found to be 5 gm.

HCF of decimals

Rule : *First make (if necessary) the same number of decimal places in all the given numbers; then find their HCF as if they are integers and mark off in the result as many decimal places as there are in each of the numbers.*

Ex. 1: Find the HCF of 16.5, 0.45 and 15.

Soln : The given numbers are equivalent to

16.50, 0.45 and 15.00

Step I: First we find the HCF of 1650, 45 and 1500. Which comes to 15.

Step II: The required HCF = 0.15

Ex. 2: Find the HCF of 1.7, 0.51 and 0.153.

Soln : Step I : First we find the HCF of 1700, 510 and 153. Which comes to 17.

Step II : The required HCF = 0.017

HCF of vulgar fractions

Def: *The HCF of two or more fractions is the highest fraction which is exactly divisible by each of the fractions.*

Rule : First express the given fractions in their lowest terms:

$$\text{Then HCF} = \frac{\text{HCF of numerators}}{\text{LCM of denominators}}$$

Note : The HCF of a number of fractions is always a fraction (but this is not true with LCM).

Ex. 1: Find the HCF of $\frac{54}{9}$, $3\frac{9}{17}$ and $\frac{36}{51}$,

Soln : Here, $\frac{54}{9} = \frac{6}{1}$, $3\frac{9}{17} = \frac{60}{17}$ and $\frac{36}{51} = \frac{12}{17}$

Thus, the fractions are $\frac{6}{1}$, $\frac{60}{17}$ and $\frac{12}{17}$

$$\therefore \text{ HCF } = \frac{\text{HCF of } 6, 60, 12}{\text{LCM of } 1, 17, 17} = \frac{6}{17}$$

Note : We see that each of the numbers is perfectly divisible by $\frac{6}{17}$

Ex. 2: Find the greatest length that is contained an exact number of times in $3\frac{1}{2}$ m and $8\frac{3}{4}$ m.

Soln : $3\frac{1}{2} = \frac{7}{2}$ and $8\frac{3}{4} = \frac{35}{4}$

The greatest length will be the HCF of $\frac{7}{2}$ and $\frac{35}{4}$.

$$\therefore \text{ the required length } = \frac{\text{HCF of 7 and 35}}{\text{LCM of 2 and 4}} = \frac{7}{4} = 1\frac{3}{4}\text{ m}$$

Miscellaneous Examples on HCF

Ex. 1: What is the greatest number that will divide 2400 and 1810 and leave remainders 6 and 4 respectively ?

Soln : Since on dividing 2400 a remainder 6 is left, the required number must divide (2400 - 6) or 2394 exactly. Similarly, it must divide (1810 - 4) or 1806 exactly. Hence, the greatest required number should be the HCF of 2394 and 1806, i.e., 42.

Ex. 2. What is the greatest number that will divide 38, 45 and 52 and leave as remainders 2, 3 and 4 respectively ?

Soln : The required greatest number will be the HCF of (38 - 2), (45 - 3) and (52 - 4) or 36, 42 and 48.

$\therefore$ Ans = 6

Ex. 3: Find the greatest number which will divide 410, 751 and 1030 so as to leave remainder 7 in each case?

Soln: The required greatest number = HCF of (410 - 7), (751 - 7) and (1030–7).

$\therefore$ Ans = 31

Ex. 4: Find the greatest number which is such that when 76, 151 and 22 are divided by it, the remainders are all alike. Find also th common remainder.

Soln : Let k be the remainder, then the numbers (76 - k), (151 - k) an (226 - k) are exactly divisible by the required number. Now, w know that if two numbers are divisible by a certain number, the their difference is also divisible by that number. Hence, th numbers (151 - k) - (76 - k), (226 - k) - (151 - k) and (226 - k) (76 - k) or 75, 75 and 150 are divisible by the required number.

Therefore, the required number = HCF of 75, 75 and 150 = 7

And the remainder will be found after dividing 76 by 75, as 1

Ex. 5: The numbers 11284 and 7655, when divided by a certain numbe of three digits, leave the same remainder. Find that number of thre digits.

Soln : The required number must be a factor of (11284 - 7655) or 3629

Now, 3629 = 19 × 191

∴ 191 is the required number.

Ex 6: The product of two numbers is 7168 and their HCF is 16; find th numbers.

Soln: The numbers must be multiples of their HCF. So, let the number be 16a and 16b where a and b are two numbers prime to each other

∴ 16a × 16b = 7168 or, ab = 28

Now, the pairs of numbers whose product is 28 are 28, 1; 14, 2 7, 4.

14 and 2 which are not prime to each other should be rejected.

Hence, the required numbers are 28 × 16, 1 × 16; 7 × 16, 4 × 16 or, 448, 16; 112, 64

Common multiple: A common multiple of two or more numbers is a number which is exactly divisible by each of them. Thus, 30 is a common multiple of 2, 3, 5, 6, 10 and 15.

Least common multiple (LCM): The LCM of two or more given numbers is the least number which is exactly divisible by each of them.

Thus, 15 is a common multiple of 3 and 5.

30 is a common multiple of 3 and 5.

45 is a common multiple of 3 and 5.

But 15 is the least common multiple (LCM) of 3 and 5.

To find the LCM of two or more given numbers

Method I: Method of Prime Factors

Rule: *Resolve the given numbers into their prime factors and then find the product of the highest power of all the factors that occur in the given numbers. This product will be the LCM.*

Ex. 1: Find the LCM of 8,12, 15, and 21.

Soln:

$$8 = 2 \times 2 \times 2 = 2^3$$
$$12 = 2 \times 2 \times 3 = 2^2 \times 3$$
$$15 = 3 \times 5$$
$$21 = 3 \times 7$$

Here, the prime factors that occur in the given numbers are 2, 3, 5 and 7 and their highest powers are respectively 2^3, 3, 5 and 7.

Hence, the required LCM = $2^3 \times 3 \times 5 \times 7 = 840$

Ex. 2: Find the LCM of 18, 24, 60 and 150.

Soln :

$$18 = 2 \times 3 \times 3 = 2 \times 3^2$$
$$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$$
$$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$$
$$150 = 2 \times 3 \times 5 \times 5 = 2 \times 3 \times 5^2$$

Here, the prime factors that occur in the given numbers are 2, 3 and 5, and their highest powers are 2^3, 3^2 and 5^2 respectively.

Hence, the required LCM = $2^3 \times 3^2 \times 5^2 = 1800$

Note : The LCM of two numbers which are prime to each other is their product.

Thus, the LCM of 15 and 17 is 15 × 17 = 255

Method II: The LCM of several small numbers can be easily found by the following rule:

Write down the given numbers in a line separating them by commas. Divide by any one of the prime numbers 2,3,5,7, etc., which will exactly divide at least any two of the given numbers. Set down the quotients and the undivided numbers in a line below the first. Repeat the process until you get a line of numbers which are prime to one another. The product of all divisors and the numbers in the last line will be the required LCM.

Note : To simplify the work, we may cancel, at any stage of the process, any one of the numbers which is a factor of any other number in the same line.

Ex. 1: Find the LCM of 12, 15, 90, 108, 135, 150.

Soln :

2	$\cancel{12}$, $\cancel{15}$, 90, 108, 135, 150	------(1)
3	$\cancel{45}$, 54, 135, 75	------(2)
3	18, 45, 25	-----(3)
5	6, 15, 25	-----(4)
	6, $\cancel{3}$, 5	-----(5)

$\therefore$ the required LCM = $2 \times 3 \times 3 \times 5 \times 6 \times 5 = 2700$

In line (1), 12 and 15 are the factors of 108 and 90 respectively, therefore, 12 and 15 are struck off.

In line (2), 45 is a factor of 135, therefore 45 is struck off.

In line (5), 3 is a factor of 6, therefore 3 is struck off.

Note: The product of two numbers is equal to the product of their HCF and LCM.

For example: The LCM and HCF of 12 and 15 are 60 and 3 respectively.

Multiplication of two numbers = $12 \times 15 = 180$

HCF × LCM = $3 \times 60 = 180$

Thus, we see that the product of two numbers is equal to the product of their LCM and HCF.

LCM of decimals

Rule: *First make (if necessary) the same number of decimal places in all the given numbers; then find their LCM as if they were integers, and mark in the result as many decimal places as there are in each of the numbers.*

Ex. Find the LCM of 0.6, 9.6 and 0.36.

Soln: The given numbers are equivalent to 0.60, 9.60 and 0.36.

Now, find the LCM of 60, 960 and 36. Which is equal to 2880.

$\therefore$ the required LCM = 28.80

LCM of fractions

The LCM of two or more fractions is the least fraction or integer which is exactly divisible by each of them.

Rule: First express the fractions in their lowest terms, then

$$\text{LCM} = \frac{\text{LCM of numerator}}{\text{HCF of denominator}}$$

Ex. 1: Find the LCM of

a) $\frac{1}{2}, \frac{5}{8}$ b) $\frac{108}{375}, 1\frac{17}{25}, \frac{54}{55}$ c) $4\frac{1}{2}, 3, 10\frac{1}{2}$

Soln: a) The required LCM = $\frac{\text{LCM of 1 and 5}}{\text{HCF of 2 and 8}} = \frac{5}{2} = 2\frac{1}{2}$

b) $\frac{108}{375} = \frac{36}{125}$, $1\frac{17}{25} = \frac{42}{25}$

Thus, the fractions are $\frac{36}{125}, \frac{42}{25}$ and $\frac{54}{55}$

$\therefore$ the required LCM = $\frac{\text{LCM of } 36, 42 \text{ and } 54}{\text{HCF of } 125, 25, 55} = \frac{756}{5} = 151\frac{1}{5}$

(c) $4\frac{1}{2} = \frac{9}{2}$, $10\frac{1}{2} = \frac{21}{2}$

Thus, the fractions are $\frac{9}{2}, \frac{3}{1}$ and $\frac{21}{2}$

$\therefore$ the required LCM = $\frac{\text{LCM of } 9, 3 \text{ and } 21}{\text{HCF of } 2, 1, 2} = \frac{63}{1} = 63$

Note: In Ex. 1 (c), we see that the LCM of fractions is an integer. Thus, we may conclude that LCM of fractions may be a fraction or an integer.

Miscellaneous Examples on LCM

Ex. 1: The LCM of two numbers is 2079 and their HCF is 27. If one of the numbers is 189, find the other.

Soln: The required number = $\frac{\text{LCM} \times \text{HCF}}{\text{First number}} = \frac{2079 \times 27}{189} = 297$

Ex. 2: Find the least number which, when divided by 18, 24, 30 and 42, will leave in each case the same remainder 1.

Soln: Clearly, the required number must be greater than the LCM of 18, 24, 30 and 42 by 1.

Now, $18 = 2 \times 3^2$

$24 = 2^3 \times 3$

$30 = 2 \times 3 \times 5$

$42 = 2 \times 3 \times 7$

$\therefore$ LCM = $3^2 \times 2^3 \times 5 \times 7 = 2520$

$\therefore$ the required number = $2520 + 1 = 2521$

Ex 3. What is the least number which, when divided by 52, leaves 33 as the remainder, and when divided by 78 leaves 59, and when divided by 117 leaves 98 as the respective remainders?

Soln: Since 52 - 33 = 19, 78 - 59 = 19, 117 - 98 = 19

We see that the remainder in each case is less than the divisor by 19. Hence, if 19 is added to the required number, it becomes exactly divisible by 52,78 and 117. Therefore, the required number is 19 less than the LCM of 52, 78 and 117.

The LCM of 52,78 and 117 = 468

$\therefore$ the required number = 468 - 19 = 449

Ex 4: Find the greatest number of six digits which, on being divided by 6, 7, 8, 9 and 10, leaves 4, 5, 6, 7 and 8 as remainders respectively.

Soln: The LCM of 6, 7, 8, 9 and 10 = 2520

The greatest number of 6 digits = 999999

Dividing 999999 by 2520, we get 2079 as remainder. Hence, the 6-digit number divisible by 2520 is (999999 - 2079), or 997920.

Since 6 - 4 = 2, 7 - 5 = 2, 8 - 6 = 2, 9 - 7 = 2, 10 - 8 = 2, the remainder in each case is less than the divisor by 2.

$\therefore$ the required number = 997920 - 2 = 997918

Ex 5: Find the greatest number less than 900, which is divisible by 8, 12 and 28.

Soln: The least number divisible by 8,12 and 28 is 168. Clearly, any multiple of 168 will be exactly divisible by each of the numbers 8, 12 and 28. But since the required number is not to exceed 900, it is 168 × 5 = 840.

Ex 6: Find the least number which, upon being divided by 2, 3, 4, 5, 6 leaves in each case a remainder of 1, but when divided by 7 leaves no remainder.

Soln: The LCM of 2, 3, 4, 5, 6 = 60

$\therefore$ the required number must be = 60k +1, where k is a positive integer.

$= (7 \times 8 + 4)k + 1 = (7 \times 8k) + (4k + 1)$

Now, this number is to be divisible by 7. Whatever may be the value of k, the portion (7 × 8k) is always divisible by 7. Hence, we must choose that least value of k which will make 4k + 1 divisible by 7. Putting k = 1, 2, 3, 4, 5 etc. in succession, we find that k should be 5.

$\therefore$ the required number = 60k + 1 = 60 × 5 + 1 = 301

Note: The above example could also be worded as follows. A person had a number of toys to distribute among children. At first, he gave 2 toys to each child, then 3, then 4, then 5, then 6, but was always left with one. On trying 7 he had none left. What is the smallest number of toys that he could have had ?

Ex. 7: What least number must be subtracted from 1936 so that the remainder when divided by 9, 10, 15 will leave in each case the same remainder 7?

Soln: The LCM of 9, 10 and 15 = 90

On dividing 1936 by 90, the remainder = 46

But 7 is also a part of this remainder

$\therefore$ the required number = 46 - 7 = 39

Ex. 8: What greatest number can be subtracted from 10,000 so that the remainder may be divisible by 32, 36, 48 and 54?

Soln: LCM of 32, 36, 48, 54 = 864

$\therefore$ the required greatest number = 10,000 - 864 = 9,136

Ex. 9: What is the least multiple of 7, which when divided by 2, 3, 4, 5 and 6, leaves the remainders 1, 2, 3, 4, 5 respectively?

Soln: LCM of 2, 3, 4, 5, 6 = 60

Other numbers divisible by 2, 3, 4, 5, 6 are 60k, where k is a positive integer.

Since 2 - 1 = 1, 3 - 2 = 1, 4 - 3 = 1, 5 - 4 = 1 and 6 - 5 = 1, the remainder in each case is less than the divisor by 1, the required number = 60k - 1 = (7 × 8k) + (4k - 1)

Now, this number is to be divisible by 7. Whatever may be the value of k the portion 7 × 8k is always divisible by 7. Hence, we must choose the least value of k which will make (4k - 1) divisible by 7. Putting k equal to 1, 2, 3, etc. in succession, we find that k should be 2.

$\therefore$ the required number = 60k - 1 = 60 × 2 - 1 = 119

EXERCISES

1. What is the greatest number that will divide 2930 and 3250 and will leave as remainders 7 and 11 respectively ?
2. What is the least number by which 825 must be multiplied in order to produce a multiple of 715 ?
3. The LCM of two numbers is 2310 and their HCF is 30. If one of the numbers is 7 × 30, find the other number.
4. Three bells commence tolling together and they toll after 0.25, 0.1 and 0.125 seconds. After what interval will they again toll together ?
5. What is the smallest sum of money which contains Rs 2.50, Rs 20, Rs 1.20 and Rs 7.50 ?

6. What is the greatest number which will divide 410, 751 and 1030 so as to leave the remainder 7 in each case ?

7. What is the HCF and LCM of $\frac{4}{5}$, $\frac{5}{6}$ and $\frac{7}{15}$?

8. Three men start together to travel the same way around a circular track of 11 km. Their speeds are 4, 5.5 and 8 km per hour respectively. When will they meet at the starting point ?

9. Find the smallest whole number which is exactly divisible by

$$1\frac{1}{2},\ 1\frac{1}{3},\ 2\frac{1}{4},\ 3\frac{1}{2} \text{ and } 4\frac{1}{5}$$

10. How many times is the HCF of 48, 36, 72 and 24 contained in their LCM ?

11. Find the least square number which is exactly divisible by 4, 5, 6, 15 and 18.

12. Find the least number which, when divided by 8, 12 and 16, leaves 3 as the remainder in each case; but when divided by 7 leaves no remainder.

13. Find the greatest number that will divide 55, 127 and 175 so as to leave the same remainder in each case.

14. Find the least multiple of 11 which, when divided by 8, 12 and 16, leaves 3 as remainder.

15. What least number should be added to 3500 to make it exactly divisible by 42, 49, 56 and 63 ?

16. Find the least number which, when divided by 72, 80 and 88, leaves the remainders 52, 60 and 68 respectively.

17. Find the greatest number of 4 digits which, when divided by 2, 3, 4, 5, 6 and 7, should leave remainder 1 in each case.

18. Find the greatest possible length which can be used to measure exactly the lengths 7m, 3m 85cm, 12m 95cm.

19. Find the least number of square tiles required to pave the ceiling of a hall 15m 17cm long and 9m 2 cm broad.

20. What is the largest number which divides 77, 147 and 252 to leave the same remainder in each case ?

21. The traffic lights at three different road crossings change after every 48 sec., 72 sec., and 108 sec. respectively. If they all change simultaneously at 8:20:00 hrs, then at what time will they again change simultaneously ?

22. The HCF and LCM of two numbers are 44 and 264 respectively. If the first number is divided by 2, the quotient is 44. What is the other number ?

23. The product of two numbers is 2160 and their HCF is 12. Find the possible pairs of numbers.

24. Find the greatest number of 4 digits and the least number of 5 digits that have 144 as their HCF.

25. Find the least number from which 12, 18, 32 or 40 may be subtracted, each an exact number of times.

26. Find the least number that, being increased by 8, is divisible by 32, 36 and 40.

27. The sum of two numbers is 528, and their HCF is 33. How many pairs of such numbers can be formed ?

28. In a school, 391 boys and 323 girls have been divided into the largest possible equal classes, so that each class of boys numbers the same as each class of girls. What is the number of classes ?

29. Is it possible to divide 1000 into two parts such that their HCF may be 15 ?

30. Show that 2205 and 4862 are prime to each other.

31. What least number must be subtracted from 1294 so that the remainder, when divided by 9, 11, 13 will leave in each case the same remainder 6 ?

32. Find the sum of the numbers between 300 and 400 such that when they are divided by 6, 9 and 12

(a) it leaves no remainder; and

(b) it leaves remainder as 4 in each case.

Solutions (Hints)

1. The greatest such number will be the HCF of (2930 - 7) and (3250 - 11), i.e. 79.

2. $825 = 3 \times 5 \times 5 \times 11$; $715 = 5 \times 11 \times 13$

Any multiple of 715 must have factors of 5, 11 and 13. So, 825 should be multiplied by the factor(s) of 715, which is (are) not present in 825.

$\therefore$ the required least number = 13

3. The required number $= \frac{2310 \times 30}{7 \times 30} = 330$

4. They will toll together after an interval of time equal to the LCM of 0.25 sec, 0.1 sec and 0.125 sec.

LCM of 0.25, 0.1 and 0.125 = (LCM of 250, 100 and 125) × 0.001

= 500 × 0.001 = 0.5 sec.

5. LCM of 2.5, 20, 1.2 and 7.5 = (LCM of 25, 200, 12 and 75) × 0.1

= 600 × 0.1 = Rs 60

6. The required number willl be the HCF of (410 - 7), (751 - 7), (1030 - 7) i.e. 31.

7. $HCF = \frac{\text{HCF of Numerators}}{\text{LCM of Denominators}} = \frac{1}{30}$

$LCM = \frac{\text{LCM of Numerators}}{\text{HCF of Denominators}} = \frac{140}{1} = 140$

8. Time taken by them to complete one revolution

$= \frac{11}{4}, \frac{11}{5.5}$ and $\frac{11}{8}$ hrs respectively $= \frac{11}{4}, \frac{2}{1}$ and $\frac{11}{8}$

LCM of $\frac{11}{4}, \frac{2}{1}$ and $\frac{11}{8} = \frac{\text{LCM of } 11, 2, 11}{\text{HCF of } 4, 1, 8} = \frac{22}{1} = 22$ hrs

$\therefore$ they will meet after 22 hrs.

9. The required smallest number = LCM of the given numbers

10. HCF of 48, 36, 72, 24 = 12; LCM of 48, 36, 72, 24 = 144

$\therefore$ LCM = 12 × HCF

11. LCM of 4, 5, 6, 15, 18 = 180, which is exactly divisible by the given numbers.

$$180 = 2 \times 2 \times 3 \times 3 \times 5 = 2^2 \times 3^2 \times 5$$

Therefore, if 180 is multiplied by 5 (180 × 5 = 900) then the number will be a perfect square as well as divisible by 4, 5, 6, 15 and 18.

12. The least number which, when divided by 8, 12 and 16, leaves 3 as remainder

= (LCM of 8, 12, 16) + 3 = 48 + 3 = 51

Other such numbers are 48 × 2 + 3 = 99, 48 × 3 + 3 = 147,....

$\therefore$ the required number which is divisible by 7 is 147.

Note: This is a hit-and-trial method. Can you get the answer by the defined method? (see Ex. 6).

13. Let x be the remainder, then the numbers (55-x), (127-x) and (175-x) are exactly divisible by the required number.

Now, we know that if two numbers are divisible by a certain number then their difference is also divisible by the number. Hence the numbers (127-x) - (55-x), (175-x) - (127-x) and (175-x) - (55-x) or, 72, 48 and 120 are divisible by the required number. HCF of 48, 72 and 120 = 24, therefore the required number = 24.

Note: Find the HCF of the positive differences of numbers. It will serve your purpose quickly.

14. LCM of 8, 12 and 16 = 48. Such numbers are (48 × 1 + 3) = 51, (48 × 2 + 3) = 99, which is divisible by 11.

$\therefore$ the required number = 9.

Note : This is a hit-and-trial method. Try to solve by the detailed method.

15. LCM of 42, 49, 56, 63 = 3528; therefore, the required least number = 3528 - 3500 = 28

16. 72 - 52 = 20, 80 - 60 = 20, 88 - 68 = 20. We see that in each case, the remainder is less than the divisor by 20. The LCM of 72, 80 and 88 = 7920, therefore, the required number = 7920 - 20 = 7900

17. The greatest number of 4 digits = 9999

LCM of 2, 3, 4, 5, 6, 7 = 420

On dividing 9999 by 420, we get 339 as remainder.

$\therefore$ the greatest number of 4 digits which is divisible by 2, 3, 4, 5, 6 and 7 = 9999 - 339 = 9660

$\therefore$ the required number = 9660 + 1 = 9661

18. The required length = HCF of 7m, 3.85m and 12.95m

= (HCF of 700, 385, 1295) × .01m = 35 × .01m = 0.35m = 35 cm

19. Side of each tile = HCF of 1517 and 902 = 41 cm.

Area of each tile = 41 × 41 cm^2

$\therefore$ the number of tiles $= \frac{1517 \times 902}{41 \times 41} = 814$

20. Solve as in Ex. 13.

The required number = HCF of (147-77), (252-147) and (252-77)

= HCF of 70, 105, 175 = 35

21. LCM of 48, 72, 108 = 432

The traffic lights will change simultaneously after 432 seconds or 7m 12 secs.

$\therefore$ they will change simultaneously at 8:27:12 hrs.

22. The first number = 2 × 44 = 88

$\therefore$ The second number $= \frac{\text{HCF} \times \text{LCM}}{88} = \frac{44 \times 264}{88} = 132$

23. HCF = 12. Then let the numbers be 12x and 12y.

Now 12x × 12y = 2160 $\therefore$ xy = 15

Possible values of x and y are (1, 15); (3, 5); (5, 3); (15, 1)

$\therefore$ the possible pairs of numbers (12, 180) and (36, 60)

24. The required numbers should be multiples of 144. We have the greatest number of 4 digits = 9999. On dividing 9999 by 144, we get 63 as the remainder.

$\therefore$ the required greatest number of 4 digits = 9999 - 63 = 9936

Again, we have the least number of 5 digits = 10000

On dividing 10,000 by 144, we get 64 as the remainder.

$\therefore$ the required least number of 5 digits = 10,000 + (144-64) = 10,08(

25. The required number = LCM of 12, 18, 32, 40 = 1440
26. LCM of 32, 36 and 40 = 1440, therefore, the required number = 1440 - 8 = 1432
27. Let the numbers be 33a and 33b.

 Now, 33a + 33b = 528

 or, 33 (a+b) = 528 $\quad \therefore$ a+b = 16

 The possible values of a and b are (1, 15); (2, 14); (3, 13); (4, 12); (5, 11); (6, 10); (7, 9); and (8, 8).

 Of these the pairs of numbers that are prime to each other are (1, 15); (3, 13); (5, 11); and (7, 9).

 $\therefore$ the possible pairs of numbers are (33, 495); (99, 429); (165, 363); (231, 297)
28. Number of classes = HCF of 391 and 323 = 17
29. *If two numbers are divisible by a certain number, then their sum is also divisible by that number.*

 According to this rule: if 15 is the HCF of two parts of 1000, then 1000 must be divisible by 15. But it is not so. Therefore, it is not possible to divide 1000 into two parts such that their HCF may be 15.
30. *If the numbers are prime to each other, then their HCF should be unity. Conversely, if their HCF is unity, the numbers are prime to each other.* In this case, the HCF is 1, so they are prime to each other.
31. LCM of 9, 11 and 13 = 1287

 Therefore, the number which, after being divided by 9, 11 and 13, leaves in each case the same remainder 6 = 1287 + 6 = 1293

 $\therefore$ the required least number = 1294 - 1293 = 1.
32. The LCM of 6, 9 and 12 = 36

 (a) Multiples of 36 which lie between 300 and 400 are 324, 360 and 396.

 $\therefore$ the required sum = 324 + 360 + 396 = 1080

 (b) Here the remainder is 4 in each case

 So, the numbers are (324 + 4 =) 328 and (360 + 4 =) 364. (The no. 396 + 4 = 400 does not lie between 300 and 400 so it is not acceptable.)

 $\therefore$ the required sum = 328 + 364 = 692.

Fractions

If any unit be divided into any number of equal parts, one or more of these parts is called a *fraction* of the unit. The fractions one-fifth, two-fifths, three-fourths are written as $\frac{1}{5}$, $\frac{2}{5}$ and $\frac{3}{4}$ respectively.

The lower number, which indicates the number of equal parts into which the unit is divided, is called **denominator.**

The upper number, which indicates the number of parts taken to form the fraction, is called the **numerator.**

The numerator and the denominator of a fraction are called its *terms*.

Note: 1. A fraction is unity when its numerator and denominator are equal.

2. A fraction is equal to zero when its numerator alone is zero. The denominator of a fraction is always assured to be non-zero.

3. A fraction is also called a rational number.

4. The value of a fraction is not altered by multiplying or dividing the numerator and the denominator by the same number.

$$\text{Ex.: } \frac{2}{5} = \frac{2 \times 5}{5 \times 5} = \frac{2 \div 4}{5 \div 4}$$

5. When the numerator and the denominator of a fraction have no common factor, the fraction is said to be in its lowest terms.

$$\text{Ex.: } \frac{15}{20} = \frac{3 \times 5}{4 \times 5}$$

The numerator and the denominator have a common factor 5, so $\frac{15}{20}$ is not in its lowest terms. If we cancel out 5 by dividing both the numerator and the denominator by 5, we get $\frac{3}{4}$, which has no common factor. Hence $\frac{3}{4}$ is the fraction $\frac{15}{20}$ in its lowest terms.

When a fraction is reduced in its lowest terms, its numerator and denominator are prime to each other i.e., they have no common factor.

6. If the numerator and the denominator are large numbers, or if their common factors cannot be guessed easily, we may find their HCF. After dividing the terms by their HCF, the fraction is reduced to its lowest term.

Ex.: $\frac{385056}{715104}$; HCF of 385056 and 715104 = 55008

$$\text{Now, } \frac{385056}{715104} = \frac{385056 \div 55008}{715104 \div 55008} = \frac{7}{13}$$

Here, $\frac{7}{13}$ is in its lowest term because the terms have no common fact

7. An integer can be expressed as a fraction with any denominat we want. For example: if we want to express 23 as a fraction who denominator is 17, the process will be as follows:

$$23 = \frac{23 \times 17}{17} = \frac{391}{17}$$

Proper fraction : *A proper fraction is one whose numerator is less tha the denominator.*

For example: $\frac{3}{4}$, $\frac{17}{19}$, $\frac{21}{42}$ are proper fractions. The value of a prop fraction is always less than 1.

Improper fraction : *A fraction whose numerator is equal to or great than the denominator is called an improper fraction.*

For example: $\frac{17}{12}$, $\frac{12}{7}$, $\frac{18}{5}$ are improper fractions.

The value of an improper fraction is always more than or equa to 1. In the above examples,

$$\frac{17}{12} = 1\frac{5}{12},\ \frac{12}{7} = 1\frac{5}{7},\ \frac{18}{5} = 3\frac{3}{5},$$

Thus, we see that an improper fraction is made up of a whol number and a proper fraction. When an improper fraction is changed t consist of a whole number and a proper fraction, it is called a **mixe number.** In the above examples $1\frac{5}{12}$, $1\frac{5}{7}$ and $3\frac{3}{5}$ are mixed number

Fractions in which the denominators are powers of 10 are calle **decimal fractions** e.g. $\frac{3}{10}, \frac{7}{10^2}, \frac{3}{100}, \frac{9}{100}$, etc.

Fractions in which the denominators are the same are calle **like fractions**

For example: $\frac{13}{17}, \frac{19}{17}, \frac{8}{17}, \frac{20}{17}, \frac{1}{12}, \frac{5}{12}, \frac{17}{12}$ etc.

In this case, the fraction having the greatest numerator is the greatest.

So $\frac{20}{17} > \frac{19}{17} > \frac{13}{17} > \frac{8}{17}$ and $\frac{1}{12} < \frac{5}{12} < \frac{17}{12}$

Fractions in which the denominators are different are called **unlike fractions.**

For example: $\frac{13}{17}, \frac{15}{8}, \frac{379}{1000}$, etc.

Solved Examples :

Ex. 1: Find the sum of a) $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{3}{4}$ b) $\frac{125}{100}$, $\frac{50}{36}$, $\frac{48}{45}$ and $1\frac{1}{2}$

Soln: a) All the fractions are in their lowest terms.

Now, LCM of 2, 3, 4 = 12

$$\text{Then, } \frac{1}{2} + \frac{2}{3} + \frac{3}{4} = \frac{(12 \div 2) \times 1 + (12 \div 3) \times 2 + (12 \div 4) \times 3}{12}$$

$$= \frac{6 + 8 + 9}{12} = \frac{23}{12} = 1\frac{11}{12}$$

Note : Students should not write the step

$$= \frac{(12 \div 2) \times 1 + (12 \div 3) \times 2 + (12 \div 4) \times 3}{12}$$

It should be done mentally.

b) First reduce the given fractions into their lowest terms.

$$\frac{125}{100} = \frac{5}{4}\ ;\ \frac{50}{36} = \frac{25}{18}\ ;\ \frac{48}{45} = \frac{16}{15}$$

Now, change the fractions into mixed numbers.

$$\frac{5}{4} = 1\frac{1}{4}\ ;\ \frac{25}{18} = 1\frac{7}{18}\ ;\ \frac{16}{15} = 1\frac{1}{15}$$

Thus, the given expression becomes : $1\frac{1}{4} + 1\frac{7}{18} + 1\frac{1}{15} + 1\frac{1}{2}$

Now, add all the whole numbers together and all the fractions together. Thus,

$$1\frac{1}{4} + 1\frac{7}{18} + 1\frac{1}{15} + 1\frac{1}{2} = (1+1+1+1) + \left(\frac{1}{4} + \frac{7}{18} + \frac{1}{15} + \frac{1}{2}\right)$$

$$= 4 + \frac{45 + 70 + 12 + 90}{180}$$

$$= 4 + \frac{217}{180} = 4 + 1\frac{37}{180} = 5\frac{37}{180}$$

Ex.2: Solve :

a) $\frac{7}{9} - \frac{11}{12} + \frac{13}{16} - \frac{1}{8}$ b) $3\frac{10}{11} + 5\frac{7}{15} - 2\frac{9}{22} - 4\frac{9}{10}$

c) $3\frac{5}{9} \times 81 \times \frac{17}{16}$ d) $10\frac{5}{6} \div 91$

e) $50\frac{4}{7} \div 14$ f) $\frac{15}{20} \times \frac{3}{4} \times \frac{4}{5}$

g) $\frac{6}{7} \div 3$ h) $\frac{6}{7} \div 4$

Soln : a) LCM of 9,12,16, 8 = 144

$$\frac{7}{9} - \frac{11}{12} + \frac{13}{16} - \frac{1}{8} = \frac{16 \times 7 - 11 \times 12 + 13 \times 9 - 18}{144}$$

$$= \frac{112 - 132 + 117 - 18}{144} = \frac{229 - 150}{144} = \frac{79}{144}$$

b) $3\frac{10}{11} + 5\frac{7}{15} - 2\frac{9}{22} - 4\frac{9}{10}$

$$= (3 + 5 - 2 - 4) + \left(\frac{10}{11} + \frac{7}{15} - \frac{9}{22} - \frac{9}{10}\right)$$

$$= 2 + \frac{300 + 154 - 135 - 297}{330}$$

$$= 2 + \frac{22}{330} = 2 + \frac{1}{15} = 2\frac{1}{15}$$

c) $3\frac{5}{9} \times 81 \times \frac{17}{16}$

Change the mixed fraction into an improper fraction. Then the expression becomes :

$$\frac{32}{9} \times 81 \times \frac{17}{16} = 306$$

d) $10\frac{5}{6} \div 91 = \frac{65}{6} \div 91 = \frac{65}{6} \times \frac{1}{91} = \frac{5}{42}$

e) $58\frac{4}{7} \div 14 = \frac{410}{7} \div 14 = \frac{410}{7} \times \frac{1}{14} = \frac{205}{49} = 4\frac{9}{49}$

Or, if the integral part of the mixed number be greater than the divisor, we proceed as follows :

$$58\frac{4}{7} \div 14 = \left(56 + 2\frac{4}{7}\right) \div 14 = 4 + \frac{18}{7} \times \frac{1}{14} = 4 + \frac{9}{49} = 4\frac{9}{49}$$

f) $\frac{15}{20} \times \frac{3}{4} \times \frac{4}{5} = \frac{9}{20}$

g) $\frac{6}{7} \div 3$; when the numerator is perfectly divisible by the divisor, divide it without changing the $\div$ sign into the $\times$ sign. Ans. $= \frac{6 \div 3}{7} = \frac{2}{7}$

h) $\frac{6}{7} \div 4$

In this case, the numerator 6 is not perfectly divisible by 4. Hence,

$$\frac{6}{7} \div 4 = \frac{6}{7} \times \frac{1}{4} = \frac{3}{14}$$

Ex. 3: Simplify :

a) $3\frac{21}{23} \div 3\frac{15}{31}$ b) $9\frac{4}{9} \div 11\frac{1}{3}$

Soln : "To divide a fraction by a fraction, multiply the fraction by the reciprocal of the divisor."

a) First change the mixed number into an improper fraction. Then multiply the fraction by the reciprocal of the divisor.

$$\frac{90}{23} \div \frac{108}{31} = \frac{90}{23} \times \frac{31}{108} = \frac{155}{138} = 1\frac{17}{138}$$

b) $9\frac{4}{9} \div 11\frac{1}{3} = \frac{85}{9} \times \frac{3}{34} = \frac{5}{6}$

Compound Fractions : *A fraction of a fraction is called a compound fraction.*

Thus, $\frac{1}{2}$ of $\frac{3}{7}$ is a compound fraction. And $\frac{1}{2}$ of $\frac{3}{7} = \frac{1}{2} \times \frac{3}{7} = \frac{3}{14}$

Combined Operations

In simplifying fractions involving various signs and brackets the following points should be remembered.

(i) The operations of multiplication and division should be performed before those of addition and subtraction.

(ii) Each of the signs $\times$ or $\div$ should be applied only to the number which immediately follows it.

Ex. 1. $\frac{4}{5} \times \frac{7}{12} \div \frac{5}{24} = \frac{4}{5} \times \frac{7}{12} \times \frac{24}{5} = \frac{56}{25}$

Ex. 2. $\frac{4}{5} \div \frac{7}{12} \times \frac{5}{24} = \frac{4}{5} \times \frac{12}{7} \times \frac{5}{24} = \frac{2}{7}$

Ex. 3. $\frac{4}{5} \div \frac{7}{12} \div \frac{5}{24} = \frac{4}{5} \times \frac{12}{7} \times \frac{24}{5} = \frac{1152}{175}$

(iii) The operations within brackets are to be carried out first.

(iv) The rule of 'BODMAS' is applied for combined operations.

Complex Fractions : *A complex fraction is one in which the numerator or denominator or both are fractions.* For example :

$$\frac{\frac{5}{7}}{8}, \frac{8}{\frac{5}{7}}, \frac{\frac{8}{9}}{\frac{7}{9}}, \frac{\frac{1}{2}+\frac{2}{3}}{\frac{3}{4}-\frac{2}{5}}$$ are complex fractions.

Ex. 1: Simplify (i) $\dfrac{\frac{4}{15}}{\frac{2}{5}}$ (ii) $\dfrac{\frac{1}{2}+\frac{2}{3}}{\frac{3}{4}-\frac{2}{9}}$

Soln : (i) $\dfrac{\frac{4}{15}}{\frac{2}{5}} = \dfrac{4}{15} \div \dfrac{2}{5} = \dfrac{4}{15} \times \dfrac{5}{2} = \dfrac{2}{3}$

(ii) $$\frac{\frac{1}{2}+\frac{2}{3}}{\frac{3}{4}-\frac{2}{9}} = \frac{\frac{3+4}{6}}{\frac{27-8}{36}} = \frac{\frac{7}{6}}{\frac{19}{36}} = \frac{7}{6} \times \frac{36}{19} = \frac{42}{19} = 2\frac{4}{19}$$

or, multiply the numerator and denominator by the LCM of their denominators 2, 3, 4, 9 namely 36.

Thus, $$\frac{\frac{1}{2}+\frac{2}{3}}{\frac{3}{4}-\frac{2}{9}} = \frac{\frac{1}{2}\times 36 + \frac{2}{3}\times 36}{\frac{3}{4}\times 36 - \frac{2}{9}\times 36} = \frac{18+24}{27-8} = \frac{42}{19} = 2\frac{4}{19}$$

Note: The second method works quickly, so we suggest that you adopt this method.

Ex. 2: Simplify: $$\frac{7}{5-\frac{8}{3}} \div \frac{3-\frac{2}{3-\frac{3}{2}}}{4-\frac{3}{2}} - \frac{5}{7} \text{ of } \left\{\frac{1}{1\frac{3}{7}} + \frac{6}{5} \text{ of } \frac{3\frac{1}{3}-2\frac{1}{2}}{\frac{47}{21}-2}\right\}$$

Soln : $$\frac{7}{\frac{7}{3}} \div \frac{3-\frac{2}{\frac{3}{2}}}{\frac{5}{2}} - \frac{5}{7} \times \left\{\frac{7}{10} + \frac{6}{5} \times \frac{\frac{10}{3}-\frac{5}{2}}{\frac{47}{21}-2}\right\}$$

$$= 3 \div \frac{3-\frac{4}{3}}{\frac{5}{2}} - \frac{5}{7} \times \left\{\frac{7}{10} + \frac{6}{5} \times \frac{5}{6} \times \frac{21}{5}\right\}$$

$$= 3 \div \left(\frac{5}{3} \times \frac{2}{5}\right) - \frac{5}{7}\left\{\frac{7}{10} + \frac{21}{5}\right\}$$

$$= 3 \div \frac{2}{3} - \frac{5}{7} \times \frac{49}{10} = \frac{9}{2} - \frac{7}{2} = \frac{2}{2} = 1$$

Ex. 3: Simplify : $$\frac{5+5\times 5}{5\times 5+5} \times \frac{\frac{1}{5}\div\frac{1}{5} \text{ of } \frac{1}{5}}{\frac{1}{5} \text{ of } \frac{1}{5}\div\frac{1}{5}} - \left(5-\frac{1}{5}\right) \times \frac{1}{\frac{2}{10}}$$

Soln : $$1 \times \frac{\frac{1}{5}\div\frac{1}{25}}{\frac{1}{25}\div\frac{1}{5}} - \left(\frac{24}{5}\right) \times \frac{10}{2} = 1 \times \frac{5}{\frac{1}{5}} - 24 = 25 - 24 = 1$$

Continued Fractions: Fractions that contain an additional fraction in the (numerator or the) denominator are called continued fraction.

Fractions of the form (i) $4 + \dfrac{1}{1+\dfrac{1}{5+\frac{2}{3}}}$ or (ii) $\dfrac{1}{1-\dfrac{2}{5+\dfrac{1}{4-\frac{2}{5}}}}$

are called continued fractions.

Rule : *To simplify a continued fraction, begin at the bottom and work upwards.*

Ex. 1: Simplify : $$\frac{1}{2+\dfrac{1}{3+\dfrac{1}{1+\frac{1}{4}}}}$$

Soln : The fraction

$$= \frac{1}{2+\dfrac{1}{3+\dfrac{1}{\frac{5}{4}}}} = \frac{1}{2+\dfrac{1}{3+\frac{4}{5}}} = \frac{1}{2+\dfrac{1}{\frac{19}{5}}}$$

$$= \frac{1}{2 + \frac{5}{19}} = \frac{1}{\frac{43}{19}} = \frac{19}{43}$$

Ex. 2: Simplify : $5 + \cfrac{1}{6 + \cfrac{1}{8 + \cfrac{1}{10}}}$

Soln: $5 + \cfrac{1}{6 + \cfrac{1}{\frac{81}{10}}} = 5 + \cfrac{1}{6 + \frac{10}{81}}$

$$= 5 + \frac{1}{\frac{496}{81}} = 5 + \frac{81}{496} = 5\frac{81}{496}$$

Note : No shortcut method has been derived for solving these questions. But you can solve these questions more quickly by
(i) more mental calculations; and (ii) skipping the steps.

Miscellaneous Solved Examples on Fractions

Ex. 1: One-quarter of one-seventh of a land is sold for Rs 30,000. What is the value of an eight thirty-fifths of land ?

Soln : One-quarter of one-seventh $= \frac{1}{4} \times \frac{1}{7} = \frac{1}{28}$

Now, $\frac{1}{28}$ of a land costs = Rs 30,000

$\therefore \frac{8}{35}$ of the land will cost $\frac{30,000 \times 28 \times 8}{35}$ = Rs 1,92,000

Ex. 2: After taking out of a purse $\frac{1}{5}$ of its contents, $\frac{1}{12}$ of the remainder was found to be Rs 7.40. What sum did the purse contain at first ?

Soln : After taking out $\frac{1}{5}$ of its contents, the purse remains with $\frac{4}{5}$ of contents. Now, $\frac{1}{12}$ of $\frac{4}{5}$ = Rs 7.40 or, $\frac{1}{15}$ = Rs 7.40 $\therefore$ 1 = Rs. 111

Ex. 3: A sum of money increased by its seventh part amounts to Rs 40. Find the sum.

Soln : $S + \frac{S}{7} = \text{Rs. } 40 \Rightarrow \frac{8S}{7} = \text{Rs } 40 \Rightarrow S = \text{Rs } 35$

Ex. 4: A train starts full of passengers. At the first station, it drops one-third of these and takes in 96 more. At the next, it drops one-half of the new total and takes in 12 more. On reaching the next station, there are found to be 248 left. With how many passengers did the train start ?

Soln : Let the train start with x passengers.

After dropping one-third and taking in 96 passengers, the train has

$x - \frac{x}{3} + 96 = \frac{2x}{3} + 96$ passengers $= \frac{2x + 288}{3}$ passengers

At the second station, the number of passengers $= \frac{2x + 288}{6} + 12$

Now, $\frac{2x + 288}{6} + 12 = 248$

or, $2x + 288 = 1416$

$\therefore x = 564$

Ex. 5: Determine the missing figures (denoted by stars) in the following equations, the fractions being given in their lowest terms.

(i) $6\frac{3}{*} \times *\frac{2}{3} = 30$ (ii) $4\frac{1}{6*} - *\frac{1}{17} = 1\frac{**}{6*}$

Soln : (i) Since (6 + a fraction) is contained (4 + a fraction) times in 30 (because 6 × 4 is just less than 30), the integral protion of the second mixed number must be 4. Then,

$$6\frac{3}{*} \times 4\frac{2}{3} = 30$$

or, $6\frac{3}{*} = \frac{30 \times 3}{14} = \frac{90}{14} = \frac{45}{7} = 6\frac{3}{7}$

Hence, the missing digits are 7 and 4.

(ii) $4\frac{1}{6*} - *\frac{1}{17} = 1\frac{**}{6*}$

The denominator in the first mixed number and denominator in the third mixed number are in sixties, which must be a multiple of 17. Thus, the equation becomes :

$$4\frac{1}{68} - *\frac{1}{17} = 1\frac{**}{68}$$

or, $(4 - *) + \left(\frac{1}{68} - \frac{1}{17}\right) = 1 + \frac{**}{68}$

Since $\frac{1}{68} < \frac{1}{17}$, $\frac{1}{68}$ will borrow 1 from 4.

Then $(3 - *) + \left(\frac{69}{68} - \frac{1}{17}\right) = 1 + \frac{**}{68}$

or, $(3-2) + \frac{65}{68} = 1 + \frac{**}{68} = 1 + \frac{65}{68} = 1\frac{65}{68}$

Hence, the equation becomes :

$$4\frac{1}{68} - 2\frac{1}{17} = 1\frac{65}{68}$$

EXERCISES

1. Which one of the following is the smallest fraction ?

$$\frac{6}{11}, \frac{13}{18}, \frac{15}{22}, \frac{19}{36}, \frac{5}{6}$$

2. What smallest fraction should be added to $3\frac{2}{3} + 6\frac{7}{12} + 4\frac{9}{36} + 5 + 7\frac{1}{12}$ to make the sum a whole number ?

3. What must be subtracted from the sum of $13\frac{7}{66}$ and $4\frac{5}{66}$ to have a remainder equal to their difference ?

4. Find the smallest fraction which, when added to $\frac{2}{5} \times \frac{15}{21} \times \frac{7}{10} \times \frac{3}{8}$ gives a whole number.

5. Find out the missing figures (denoted by stars) in the following equations, the fractions being given in their lowest terms :

i) $*\frac{3}{7} \times 2\frac{3}{*} = 14\frac{*}{14}$ ii) $7\frac{*}{3} - *\frac{5}{11} = 3\frac{*}{*3}$

iii) $8\frac{9}{**} \div *\frac{2}{27} = 7\frac{16}{17}$ iv) $2\frac{1}{2} - 3\frac{2}{3} + 1\frac{5}{6} - \frac{2}{*} = 0$

v) $\frac{1}{2\frac{3}{4}} + \frac{1}{5\frac{1}{5}} + \frac{1}{*} + \frac{1}{9\frac{8}{15}} = \frac{144}{143}$

6. A motorcycle, before overhauling, requires $\frac{5}{6}$ hour service time every 90 days, while after overhauling, it requires $\frac{5}{6}$ hour service time every 120 days. What fraction of the pre-overhauling service time is saved in the latter case?

Solution (Hints)

1. Out of the first two fractions, we see that

$6 \times 18 < 11 \times 13$ So, $\frac{6}{11}$ is smaller.

Now, from $\frac{6}{11}$ and $\frac{15}{22}$, we see that $6 \times 22 < 11 \times 15$, so $\frac{6}{11}$ is smaller.

Now, from $\frac{6}{11}$ and $\frac{19}{36}$, we see that $19 \times 11 < 6 \times 36$, so $\frac{19}{36}$ is smaller.

Now from $\frac{19}{36}$ and $\frac{5}{6}$, we see that $19 \times 6 < 5 \times 36$, so $\frac{19}{36}$ is smaller.

$\therefore$ we conclude that $\frac{19}{36}$ is the smallest.

Note : The above method does not need accurate calculations. You can decide which of the multiplications is greater by your keen observation only.

2. $3\frac{2}{3} + 6\frac{7}{12} + 4\frac{9}{36} + 5 + 7\frac{1}{12}$

Add only the fractional parts: $\frac{2}{3} + \frac{7}{12} + \frac{9}{36} + \frac{1}{12}$

$$= \frac{24 + 21 + 9 + 3}{36} = \frac{57}{36} = \frac{19}{12} = 1\frac{7}{12}$$

Thus, to make the expression a whole number we should add

$$1 - \frac{7}{12} = \frac{5}{12}$$

3. Difference $= 13\frac{7}{66} - 4\frac{5}{66} = (13 - 4) + \left(\frac{7}{66} - \frac{5}{66}\right)$

$= 9 + \frac{2}{66} = 9\frac{1}{33}$

Sum $= 13\frac{7}{66} + 4\frac{5}{66} = 17\frac{12}{66} = 17\frac{2}{11}$

$\therefore$ the required answer $= 17\frac{2}{11} - 9\frac{1}{33}$

$= 17\frac{6}{33} - 9\frac{1}{33} = 8\frac{5}{33}$

Direct formula : The required answer $= 2 \times$ smaller value

$= 2 \times 4\frac{5}{66} = 8\frac{5}{33}$

4. $\frac{2}{5}\times\frac{15}{21}\times\frac{7}{10}\times\frac{3}{8}=\frac{3}{40}$

$\therefore$ the required fraction $= 1-\frac{3}{40}=\frac{37}{40}$

5. i) $5\frac{3}{7}\times 2\frac{3}{4} = 14\frac{13}{14}$ ii) $7\frac{2}{3}-4\frac{5}{11}=3\frac{7}{33}$

iii) $8\frac{9}{17}\div 1\frac{2}{27}=7\frac{16}{17}$ iv) $2\frac{1}{2}-3\frac{2}{3}+1\frac{5}{6}-\frac{2}{3}=$

v) $* = 7\frac{1}{3}$

6. LCM of 90 and 120 = 360

So, in 360 days, the pre-overhauling service time $=\frac{5}{6}\times\frac{360}{90}=\frac{10}{3}$ hrs

and after overhauling, the service time $=\frac{5}{6}\times\frac{360}{120}=\frac{5}{2}$ hrs.

Time saved $=\frac{10}{3}-\frac{5}{2}=\frac{5}{6}$ hrs

$\therefore$ The required answer $=\frac{\frac{5}{6}}{\frac{10}{3}}=\frac{5}{6}\times\frac{3}{10}=\frac{1}{4}$

Decimal Fractions

Decimal Fraction : Fractions in which the denominators are powers of 10 are called decimal fractions.

For example : $\frac{1}{10}, \frac{7}{10}, \frac{9}{1000}$, etc.

Reading a decimal : In reading a decimal, the digits are named in order. Thus 0.457 is read as zero point (or decimal) four, five, seven.

Decimal places : The number of figures which follow the decimal point is called the number of decimal places. Thus, 1.432 has three decimal places and 7.82 has two decimal places.

Converting a decimal into a vulgar fraction

Rule : *Write down the given number without the decimal point, for the numerator, and for the denominator write 1 followed by as many zeros as many are there figures after the decimal point.*

Ex. 1: Reduce the following decimal values into the vulgar fractions.

a) 0.63 b) 0.0032 c) 3.013

Soln : a) $0.63=\frac{63}{100}$ b) $0.0032=\frac{0032}{10000}=\frac{32}{10000}$

c) $3.013 = 3\frac{013}{1000} = 3\frac{13}{1000}$

Note : Adding zeros to the extreme right of a decimal fraction does not change its value. For example: 0.9 = 0.90 = 0.9000 = 0.90000000. Why is this so ?

If numerator and denominator of a fraction contain the same number of decimal places, then we may remove the decimal sign.

Ex. 2: i) $\frac{0.53}{3.21}=\frac{053}{321}=\frac{53}{321}$ ii) $\frac{9.83051}{18.53342}=\frac{983051}{1853342}$

iii) Change $\frac{1.53}{2.4321}$ into vulgar fraction.

Soln : $\frac{1.53}{2.4321}=\frac{1.5300}{2.4321}=\frac{15300}{24321}$

Thus, to remove the decimal, we equate the number of figures in the decimal part of the numerator and the denominator (by putting zeros) and then remove the decimals.

Note : An integer may be expressed as a decimal by putting zeros in the decimal part.

Ex. : 17 = 17.0 = 17.00

Addition and Subtraction of Decimals

Rule : *Write down the numbers under one another, placing the decimal points in one column. The numbers can now be added or subtracted in the usual way.*

Ex. 3: Add together 5.032, 0.8, 150.03 and 40.

Soln :

$$\begin{array}{r} 5.032 \\ 0.8 \\ 150.03 \\ 40.00 \\ \hline 195.862 \end{array}$$

Ex. 4: Subtract 19.052 from 24.5.

Soln : Here, we write two zeros to the right of 24.5 and then subtract as in the case of integers.

$$\begin{array}{r} 24.500 \\ 19.052 \\ \hline 5.448 \end{array}$$

Multiplication of decimals

I: To multiply by 10, 100, 1000, etc.

Rule : *Move the decimal point by as many places to the right as many are there zeros in the multiplier.*

Ex. : (i) $39.052 \times 100 = 3905.2$

(ii) $42.63 \times 1000 = 42630$

II: To multiply by a whole number

Rule : *Multiply as in the case of integers, and in the product mark as many decimal places as there are in the multiplicand, prefixing zeros if necessary.*

Ex.: (i) $0.9 \times 12 = ?$

Soln: Step I: $9 \times 12 = 108$

Step II: As there is one decimal place in multiplicand, the product should also have only one decimal place. So, ans = 10.8

Ex.: (ii) $.009 \times 12 = ?$

Soln : Step I: $9 \times 12 = 108$

Step II: As there are three decimal places in the multiplicand, the product should also have three decimal places. So, ans = 0.108

Ex.: (iii) $.00009 \times 12 = ?$

Soln : Step I: $9 \times 12 = 108$

Step II: As there are five decimal places in the multiplicand, the product should also have five decimal places. So, we prefix two zeros to get the answer.

$\therefore$ Ans = 0.00108

III: To multiply a decimal by a decimal

Rule : *Multiply as in integers, and in the product mark as many decimal places as there are in the case of the multiplier and the multiplicand together, prefixing zeros, if necessary.*

Ex.: (i) $.61 \times .07 = ?$

Soln : Step I: $61 \times 7 = 427$

Step II: As there are (2+2=) 4 decimal places in the multiplier and the multiplicand together, the product should also have 4 decimal places. But there are only three digits in the product; so we prefix one zero to the product before placing the decimal. So, the answer = 0.0427

(ii) $0.2345 \times 0.24 = ?$

Soln : Step I: $2345 \times 24 = 56280$

Step II: There should be (4+2=)6 decimal places in the product. Thus, answer = 0.056280 = 0.05628

Division of Decimals

I: When the divisor is 10, 100, 1000, etc.

Rule : *To divide a decimal by 10, 100, 1000 etc., move the decimal point 1, 2, 3, etc. places to the left respectively. Thus,*

(i) $463.8 \div 10 = 46.38$

(ii) $4.309 \div 100 = 0.04309$

(iii) $0.003 \div 10000 = 0.0000003$

(iv) $234.789 \div 1000000 = 0.000234789$

(v) $5.08 \div 100000000 = 0.0000000508$

II: When the divisor is a decimal fraction

Rule : *Move the decimal points as many places to the right in both the divisor and dividend as will make the divisor a whole number, annexing zeros to the dividend, if necessary. Divide as in simple division and when you take a figure from the decimal part (if any) of the dividend so altered, set down the decimal point in the quotient.*

Ex.: (i) $\dfrac{32.5}{0.0064} = \dfrac{325000}{64} = 5078.125$

(ii) $\dfrac{0.0323}{0.00017} = \dfrac{3230}{17} = 190$

Recurring Decimals

A decimal in which a figure or set of figures is repeated continually is called a **recurring** or **periodic** or **circulating decimal**. The repeated figures or set of figures is called the **period of the decimal**.

For example : (1) $\frac{1}{3} = 0.333.....$

(2) $\frac{1}{7} = 0.\underline{142857}142857142857$ ....

(3) $\frac{13}{44} = 0.29\underline{54}5454$ ....

Recurring is expressed by putting a bar (or dots) on the set of repeating numbers. So, in the above examples :

(1) $0.333... = 0.\overline{3}$ (or, $0.\dot{3}$)

(2) $0.142857.142857 ...... = 0.\overline{142857}$ (or, $0.\dot{1}4285\dot{7}$)

(3) $0.295454.... = 0.29\overline{54}$ (or, $0.29\dot{5}\dot{4}$)

Pure Recurring Decimal : A decimal fraction in which all the figures after the decimal point are repeated, is called a pure recurring decimal. For example, $0.\overline{142857}$ is a pure recurring decimal.

Mixed Recurring Decimal : A decimal fraction in which some figures do not recur, is called a mixed recurring decimal. For example : $0.29\overline{54}$ is a mixed recurring decimal.

In example (1), the period is 3, in (2) it is 142857 and in (3) it is 54.

Note : 1. If the denominator of a vulgar fraction in its lowest terms be wholly made up of powers of 2 and 5, either alone or multiplied together, the fraction is convertible into a terminating decimal.

Ex.: (i) $\frac{12}{25} = \frac{12}{5^2} = 0.408$ (ii) $\frac{19}{50} = \frac{19}{5^2 \times 2} = 0.38$

(iii) $\frac{13}{20} = \frac{13}{2^2 \times 5} = 0.65$ (iv) $\frac{3}{8} = \frac{3}{2^3} = 0.375$

2. A pure recurring decimal is produced by a vulgar fraction in its lowest terms, whose denominator is neither divisible by 2 nor by 5.

Ex.: (i) $\frac{2}{3} = 0.\overline{6}$ (ii) $\frac{3}{11} = 0.\overline{27}$

(iii) $\frac{4}{7} = 0.\overline{571428}$ (iv) $\frac{5}{9} = 0.\overline{5}$

3. A mixed recurring decimal is produced by a vulgar fraction in its lowest terms, whose denominator contains powers of 2 or 5 in addition to other factors.

Ex.: (i) $\frac{1}{6} = \frac{1}{2 \times 3} = 0.1\overline{6}$

(ii) $\frac{1}{15} = \frac{1}{5 \times 3} = 0.0\overline{6}$

(iii) $\frac{7}{75} = \frac{7}{5^2 \times 3} = 0.09\overline{3}$

(iv) $\frac{8}{15} = \frac{8}{5 \times 3} = 0.5\overline{3}$

Question: Which of the following vulgar fractions will produce recurring decimal fractions ?

a) $\frac{12}{50}$ b) $\frac{12}{75}$ c) $\frac{3}{18}$ d) $\frac{8}{14}$ e) $\frac{1}{18}$ f) $\frac{7}{45}$ g) $\frac{1}{80}$

Soln: a) Reduce the fraction to its lowest terms, so

$$\frac{12}{50} = \frac{6}{25} = \frac{6}{(5)^2}$$

By note 1, the above vulgar fraction will not produce recurring decimal.

b) $\frac{12}{75} = \frac{4}{25} = \frac{4}{(5)^2}$

By note 1, the above vulgar fraction will not produce recurring decimal.

c) $\frac{3}{18} = \frac{1}{6} = \frac{1}{3 \times 2}$

By note 3, a mixed recurring decimal will be produced.

d) $\frac{8}{14} = \frac{4}{7}$

By note 2, a pure recurring decimal will be produced.

e) $\frac{1}{18} = \frac{1}{9 \times 2}$

By note 3, a mixed recurring decimal will be produced.

f) $\frac{7}{45} = \frac{7}{9 \times 5}$

By note 3, a mixed recurring decimal will be produced.

g) $\frac{1}{80} = \frac{1}{(2)^4 \times 5}$

By note 1, no recurring decimal will be produced.

To convert a recurring decimal fraction into a vulgar fraction

Case I: Pure Recurring Decimals

Rule : *A pure recurring decimal is equal to a vulgar fraction which has for its numerator the period of the decimal, and for its denominator the number which has for its digits as many nines as there are digits in the period.*

Ex.: Express the following recurring decimals into vulgar decimals.

a) $.\overline{5}$ b) $0.\overline{45}$ c) $0.\overline{532}$

Soln : a) We have $.\overline{5} = .555....$ ---------(1)

Multiplying both sides by 10, we get $10 \times 0.\overline{5} = 5.55...$ ---------(2)

Subtracting (1) from (2), we get

$9 \times 0.\overline{5} = 5$

$\therefore\ 0.\overline{5} = \frac{5}{9}$

Directly from the rule, we also get the same result. $0.\overline{5}$ has its period 5, so the numerator is 5. And as there is only one digit in the period, the denominator will have one nine. Thus, the vulgar fraction $= \frac{5}{9}$

b) $0.\overline{45}$

By the rule, numerator is the period (45) and the denominator is 99 because there are two digits in the period. $\therefore$ Ans $= \frac{45}{99}$

c) $.\overline{532}$

Numerator = period = 532

Denominator = as many nines as the number of digits in the denominator

$\therefore$ Ans $= \frac{532}{999}$

Try to solve Exs (b) & (c) by detailed method.

Case II: Mixed Recurring Decimals

Rule : *A mixed recurring decimal is equal to a vulgar fraction which has for its numerator the difference between the number formed by all the digits to the end of first period, and that formed by the digits which do not recur; and for its denominator the number formed by as many nines as there are recurring digits, followed by as many zeros as there are non-recurring digits.*

Ex.: Express the following as vulgar fractions.

a) $0.1\overline{8}$ b) $0.43\overline{213}$ c) $5.00\overline{983}$

Soln : a) $0.1\overline{8} = 0.18888$ --------(1)

$\therefore\ 10 \times 0.1\overline{8} = 1.8888$ -------(2)

and $100 \times 0.1\overline{8} = 18.8888$ --------(3)

Subtracting (2) from (3), we have, $90 \times 0.1\overline{8} = 18 - 1$

$\therefore\ 0.1\overline{8} = \frac{17}{90}$

Also, by the rule, numerator = 18-1 (all-digit-number — non-recurring-number) and denominator = one nine (as there is one recurring digit) followed by one zero (as there is one non-recurring digit) = 90

$\therefore$ Vulgar fraction $= \frac{18-1}{90} = \frac{17}{90}$

b) $0.43\overline{213}$; Numerator = 43213 - 43 = 43170

Denominator = 3 nines (as there are three recurring digits) followed by 2 zeros (as there are two non-recurring digits) = 99900

$\therefore$ Vulgar fraction $= \frac{43170}{99900} = \frac{4317}{9990}$

c) $5.00\overline{983} = 5\frac{00983 - 00}{99900} = 5\frac{983}{99900}$

Solve Ex. (b) and (c) by the detailed method.

Addition and Subtraction of Recurring Decimals

Addition and subtraction of recurring decimals can be understand well with the help of given examples.

Example: Add and subtract $3.\overline{76}$ and $1.4\overline{576}$.

3.7	676767	67	
1.4	576576	57	
(+) 5.2	253344	24	$\Rightarrow 5.2\overline{253344}$
(-) 2.3	100191	10	$\Rightarrow 2.3\overline{100191}$

Explanation:

Step I: We separate the expansion of recurring decimals into three parts. In the left-side part there is integral value with non-recurring decimal digits. In the above case, $1.4\overline{576}$ has 1 as integral part and 4 is as non-recurring decimal digit. So, 1.4 has been separated from 1.4576576576......

Although the first value does not have any non-recurring decimal digit (i.e. in $3.\overline{76}$, both the digits 7 and 6 are recurring), we put as many digits in the left-side part as there are non-recur-

ring digits in the second value. Thus, we have put 3.7 in left-side part.

Step II: In the middle part, the number of digits is equal to the LCM the number of recurring digits in two given values. In this ca the first value has 2 recurring digits and the second value ha recurring digits, so the middle part has 6 digits (as LCM of 2 a 3 = 6).

Step III: In the right-side part take two digits. Now add or subtract you do with simple addition and subtraction.

Step IV: Now, leave the right-side part and put a bar on the middle pa of the resultant. The left-side part will remain as it is in t resultant. You can see the same in the above example.

Verification:

$3.\overline{76} = 3\frac{76}{99} = 3 + \frac{76}{99}$ and $1.4\overline{576} = 1\frac{4576-4}{9990} = 1 + \frac{4572}{9990}$

$3.\overline{76} + 1.4\overline{576} = 3 + \frac{76}{99} + 1 + \frac{4572}{9990} = 4 + \frac{76 \times 9990 + 4572 \times}{99 \times 9990}$

$= 4 + \frac{76 \times 1110 + 4572 \times 11}{11 \times 9990} = 4 + \frac{134652}{109890} = 4 + 1 + \frac{24762}{109890}$

$= 5 + \frac{24762}{109890}$ ---------- (1)

Now, $5.2\overline{253344} = 5\frac{2253344 - 2}{9999990} = 5 + \frac{2253342}{9999990}$

$= 5 + \frac{24762 \times 91}{109890 \times 91} = 5 + \frac{24762}{109890}$ ---- (2)

From (1) and (2) it is verified that our addition is correct.

Now, for subtraction:

$3.\overline{76} - 1.4\overline{576} = 3 + \frac{76}{99} - \left(1 + \frac{4572}{9990}\right) = 2 + \frac{76 \times 9990 - 4572 \times 99}{99 \times 9990}$

$= 2 + \frac{76 \times 1110 - 4572 \times 11}{11 \times 9990} = 2 + \frac{34068}{11 \times 9990}$ --- (1)

Now, $2.3\overline{100191} = 2\frac{3100191 - 3}{9999990} = 2 + \frac{3100188}{9999990}$

$= 2 + \frac{91 \times 34068}{91 \times 11 \times 9990} = 2 + \frac{34068}{11 \times 9990}$ ----- (2)

From (1) and (2) it is verified that our subtraction is correct.

x. 1: Find $324.\overline{786} + 10.19\overline{3}$.

oln:

324.78	678	67
10.19	333	33
334.98	012	00

Thus, ans = $334.98\overline{012}$

Verification: $324.\overline{786} = 324 + \frac{786}{999}$

$10.19\overline{3} = 10 + \frac{193 - 19}{900} = 10 + \frac{174}{900}$

$324.\overline{786} + 10.19\overline{3} = 324 + 10 + \frac{786}{999} + \frac{174}{900}$

$= 334 + \frac{786 \times 900 + 174 \times 999}{999 \times 900}$

$= 334 + \frac{78600 + 174 \times 111}{99900} = 334 + \frac{97914}{99900}$

$= 334\frac{98012 - 98}{99900} = 334.98\overline{012}$

Ex. 2: Find $324.\overline{786} - 10.19\overline{3}$.

Soln:

324.78	678	67
10.19	333	33
314.59	345	34

Thus, ans = $314.59\overline{345}$

Verification: $324.\overline{786} - 10.19\overline{3} = 324 + \frac{786}{999} - \left(10 + \frac{174}{900}\right)$

$= 314 + \frac{786 \times 900 - 174 \times 999}{999 \times 900} = 314 + \frac{786 \times 100 - 174 \times 111}{99900}$

$= 314 + \frac{59286}{99900} = 314\frac{59345 - 59}{99900} = 314.59\overline{345}$

Ex. 3: Find $17.\overline{83} + 0.00\overline{7} + 310.020\overline{2}$

Soln:

17.838	38	38
0.007	77	77
310.020	22	22
327.866	38	37

Thus, ans = $327.866\overline{38}$

Ex. 4: Find $17.10\overline{86} - 7.984\overline{9}$

Soln:

$$\begin{array}{r|r|r} 17.108 & 68 & 68 \\ 7.984 & 99 & 99 \\ \hline 9.123 & 68 & 69 \end{array}$$

Thus, ans = $9.123\overline{68}$

Multiplication of Recurring Decimals

Case A: While multiplying a recurring decimal by a multiple of 10, t set of repeating digits is not altered. For example:

1. $4.\overline{03} \times 10 = 4.030303... \times 10 = 40.30303.... = 40.3\overline{03}$
2. $124.\overline{427} \times 1000 = 124.427427... \times 1000 = 124427.\overline{427}$
3. $0.006\overline{379} \times 10000 = 0.006379379.... \times 10000 = 63.79379 ... = 63.79\overline{379}$

Case B: While multiplying a recurring decimal by a number which i not a multiple of 10, first of all, the recurring decimal is change into the vulgar fraction and then the calculation is done. Fo example:

1. $7.\overline{63} \times 11 = 7\frac{63}{99} \times 11 = 7\frac{7}{11} \times 11 = \frac{84}{11} \times 11 = 84$

2. $13.34\overline{3} \times 15 = 13\frac{345-34}{900} \times 15 = 13\frac{311}{900} \times 15$

$= \frac{11700+311}{900} \times 15 = \frac{12011}{60} = \frac{12000+11}{60} = 200 + \frac{11}{60}$

$= 200 + 0.1833.... = 200 + 0.18\overline{3} = 200.18\overline{3}$

3. $27 \times 1.2\overline{2} \times 5.52\overline{62} \times 1.\overline{6} = 27 \times 1\frac{22-2}{90} \times 5\frac{5262-52}{9900} \times \frac{6}{9}$

$= 18 \times 1\frac{2}{9} \times 5\frac{521}{990} = 18 \times \frac{11}{9} \times \frac{5471}{990} = \frac{5471}{45}$

$= 121.5777... = 121.5\overline{7}$

Division of Recurring Decimals

Case A: When the divisor is a multiple of 10.

1. $0.\overline{06} \div 1000 = 0.060606... \div 1000 = 0.000060606... = 0.0000\overline{06}$
2. $16.45\overline{379} \div 10 = 1.645\overline{379}$

Case B: When the divisor is not a multiple of 10.

Ex 1: $17.2\overline{6} \div 2 = 17\frac{26-2}{90} \div 2 = 17\frac{24}{90} \times \frac{1}{2} = 17\frac{4}{15} \times \frac{1}{2}$

$= \frac{259}{30} = \frac{240+19}{30} = 8 + \frac{19}{30}$

$= 8 + \frac{57}{90} = 8 + \frac{63-6}{90} = 8 + 0.6\overline{3} = 8.6\overline{3}$

Another way of calculation is

$17.2\overline{6} \div 2 = 8 + \frac{19}{30} = 8 + \frac{6.33...}{10} = 8 + 0.633..$

$= 8 + 0.6\overline{3} = 8.6\overline{3}$

Second Method:

2) 17.266... (8.633...
16
12
12
06
6
06

Thus, ans = $8.6\overline{3}$

Ex. 2: $36.3\overline{43} \div 7 = 36.34343... \div 7$

7) 36.34343... (5.19
35
13
7
64
63
13

The process of division repeats so our quotient becomes 5.1919.... i.e. $5.\overline{19}$.

Approximation and Contraction

Rule : *Increase the last figure by 1 if the succeeding figure be 5 or greater than 5.*

For Example: (i) The approximate value of 0.3689 up to three decimal places is 0.369.

(ii) The approximate value of 0.3684 up to three decimal places is 0.368.

(iii) The approximate value of 0.3685 up to three decimal places is 0.369.

(iv) The approximate value of 0.3689 up to two decimal places is 0.37.

(v) The approximate value of 0.3468 up to one decimal place is 0.3.

Note : Decimals which are correct at one, two, three ... places are said to be correct to the nearest tenth, hundredth, thousandth, ... respectively.

Significant figures :

The following examples are given to explain the meaning of the term *significant figures.*

(a) The population of a certain place is 189000 correct to the nearest thousand. Here the assumed unit is one thousand, and the population is stated to be 189 such units correct to the nearest unit. The figure 189, which gives the number of units, is said to be significant while the three zeros, which indicate magnitude of the unit, are said to be non-significant.

(b) The distance between two places is 1400 km correct to the nearest hundred km. The figure 14 is significant and the two zeros are non-significant.

(c) The distance between two places is 1400 km correct to the nearest km. The figure 1400 is significant but there is no non-significant figure.

(d) The length of a string is 0.07 cm correct to the 2nd decimal place. This means 7 is the significant figure but the zero at the beginning is non-significant.

Note : Zeros at the beginning of a decimal are always non-significant.

Thus, significant figures are those which in any approximate result express the number of units correct to the nearest such unit.

Contracted Addition

Rule : *Set down the decimals under one another. Then add in the usual way, taking care that the last figure retained be increased by 1 if the succeeding figure be 5 or greater than 5.*

Ex : Find the sum of 320.4321, 29.042934, .0085279 and .3412 correct to 3 decimal places.

Soln :

$$\begin{array}{r} 320.4321 \\ 29.04293 \\ .00852 \\ .3412 \\ \hline 349.82475 \end{array}$$

Ans = 349.825

Remark : If you are asked to get the answer correct to three decimal places, then use not more than 5 (two more) decimal places during calculation.

Contracted Subtraction

Rule : *Write down the subtrahend under the minued in the usual way, retaining two places of decimals more than what is required.*

Ex : Find 160.342195 - 32. 0048326 correct to four decimal places.

Soln: As we are asked to get the solution correct to four decimal places, we will use not more than 6 decimal places during our calculation.

$$\begin{array}{r} 160.342195 \\ 32.004832 \\ \hline 128.337363 \end{array}$$

$\therefore$ Answer = 128.3374

EXERCISES

1. Express the following decimals as fractions in their lowest terms :-
   a) 0.0375 b) 0.00625 c) 1.008125
2. Simplify : a) 0.25 + 0.036 + 0.0075 b) 34.07 + 0.007 + 0.07
   c) 30.9 + 3.09 + 0.309 + 0.039 d) 35 - 7.892 + 0.005 - 10.345
   e) 0.6 + 0.66 + 0.066 - 6.606 + 66.06
3. (Remove decimals)
   a) 0.35×10^6 b) 0.275×10^{-7} c) $0.0034 \times 10{,}000$
   d) 0.132×500 e) 5.302×513
4. Divide : a) 28.9 by 17 b) 0.457263 by 18 c) 64 by 800
   d) 64 by 0.008 e) 2.375 by 0.0005 f) 0.1 by 0.0005
   g) 0.1 by 5000
5. Find the quotient to three places of decimals :-
   a) $0.5 \div 0.71$ b) $4.321 \div 0.77$ c) $5.002 \div 0.00078$
6. Simplify the followings:
   a) $12 \div 0.09$ of 0.3×2 b) $\frac{0.0025 \times 1.4}{0.0175}$
   c) $\frac{9.5 \times 0.085}{0.0017 \times 0.19}$ d) $\frac{3}{11}$ of 0.176
7. What should come in place of question mark (?) ?
   a) $3 \times 0.3 \times 0.03 \times 0.003 \times 300 = ?$
   b) $0.25 \div 0.0025 \times 0.025$ of $2.5 = ?$
   c) $0.00033 \div 0.11$ of $30 \times 100 = ?$
   d) $0.8 \times ? = 0.00004$

e) $\frac{3420}{19} = \frac{?}{0.01} \times 7$

f) $\frac{17.28 \div ?}{3.6} \times 0.2 = 400$

g) $\sqrt{\frac{0.324 \times 0.081 \times 4.624}{1.5625 \times 0.0289 \times 72.9 \times 64}}$

h) $\frac{20 + 8 \times 0.05}{40 - ?} = 16$

i) ?% of 10.8 = 32.4

j) $3.79 \times 31 + 3.79 \times 37 + 3.79 \times 32 = ?$

k) $321 \times 11.54 - 203 \times 11.54 - 105 \times 11.54 = ?$

l) $(1.27)^3 + 3(1.23)^2 \times 1.27 + 3(1.27)^2 \times 1.23 + (1.23)^3 = ?$

m) $(2.3)^3 - 3 \times (2.3)^2 \times (0.3) + 3(2.3)(0.09) - (0.3)^3 = ?$

8. State in each case whether the equivalent decimal is terminating or non-terminating:

a) $\frac{1}{6}$ b) $\frac{8}{625}$ c) $\frac{17}{90}$ d) $\frac{104}{111}$ e) $\frac{33}{165}$

9. Express the following recurring decimals in their vulgar fractions :

a) $0.\bar{3}$ b) $0.\overline{037}$ c) $0.\overline{09}$ d) $2.4\overline{32}$ e) $10.03\bar{6}$

10. If $\frac{1}{36.18} = 0.0276$, then what is the value of $\frac{1}{0.0003618}$?

11. If $13324 \div 145 = 91.9$, then what is the value of $133.24 \div 9.19$?

12. If $\sqrt{5} = 2.24$, then what is the value of $\frac{3\sqrt{5}}{2\sqrt{5} - 0.48}$?

13. If $\sqrt{15} = 3.88$, then what is the value of $\sqrt{\frac{5}{3}}$?

14. If $\sqrt{2916} = 54$, then what is the value of
$\sqrt{29.16} + \sqrt{0.2916} + \sqrt{0.002916} + \sqrt{0.00002916}$?

15. What decimal of an hour is a second ?

16. If $1.5x = 0.05y$, then what is the value of $\frac{y - x}{y + x}$?

17. $(0.\bar{6} + 0.\bar{7} + 0.\bar{8} + 0.\bar{3}) \times 9000 = ?$

18. $0.3\overline{467} + (0.\overline{45} \times 0.\bar{3}) \times 11 = ?$

Answers

1. a) $0.0375 = \frac{375}{10,000} = \frac{3}{80}$

b) $0.00625 = \frac{625}{1,00,000} = \frac{1}{160}$

c) $1.008125 = 1\frac{8125}{10,00,000} = 1\frac{13}{1600}$

2. a) 0.2935 b) 34.147 c) 34.338 d) 16.768 e) 60.78

3. a) 35×10^4 b) 275×10^{-10} c) 34 d) 66 e) 2719926×10^{-3}

4. a) 1.7 b) 0.0254035 c) 0.08 d) 8000
e) 4750 f) 200 g) 0.00002

5. a) 0.704 b) 5.612 c) 6412.821

6. a) 888.88 b) 0.2 c) 2500 d) 0.048

7. a) 0.0243 b) 6.25 c) 0.01 d) 0.00005
e) 0.257 f) 0.0024

g) $\sqrt{\frac{0.324 \times 0.081 \times 4.624}{1.5625 \times 0.0289 \times 72.9 \times 64}}$

$= \sqrt{\frac{324 \times 81 \times 4624 \times 10^{-9}}{15625 \times 289 \times 729 \times 64 \times 10^{-9}}}$

$= \frac{18 \times 9 \times 68}{125 \times 17 \times 27 \times 8} = 0.024$

h) 38.725 i) 300

j) Given expression $= 3.79(31 + 37 + 32) = 3.79(100) = 379$

k) Given expression $= 11.54(321 - 203 - 105) = 11.54 \times 13 = 150.02$

l) Given expression $= (1.27 + 1.23)^3 = (2.5)^3 = 15.625$

m) Given expression $= (2.3 - 0.3)^3 = 2^3 = 8$

8. If the denominator of a vulgar fraction in its lowest terms be wholly made up of powers of 2 and 5,either alone or multiplied together, the fraction is convertible into a terminating decimal. Following the same rule :

a) $\frac{1}{6}$ is non-terminating since its denominator has a factor (3) other than 2 and 5.

b) $\frac{8}{625} = \frac{8}{(5)^4}$ is terminating since its denominator is made up of

powers of 5 only.

c) $\frac{17}{90} = \frac{17}{2 \times 3 \times 3 \times 5}$ is non-terminating since its denominator has factors other than 2 and 5.

d) $\frac{104}{111} = \frac{104}{3 \times 37}$ is non-terminating.

e) $\frac{33}{165} = \frac{1}{5}$ is terminating.

9. a) $0.\overline{3} = \frac{3}{9} = \frac{1}{3}$ b) $0.\overline{037} = \frac{37}{999}$

c) $0.\overline{09} = \frac{9}{99} = \frac{1}{11}$ d) $2.4\overline{32} = 2\frac{432-4}{990} = 2\frac{428}{990} = 2\frac{214}{495}$

e) $10.03\overline{6} = 10\frac{36-3}{900} = 10\frac{33}{900} = 10\frac{11}{300}$

10. $\frac{1}{0.0003618} = \frac{1}{36.18 \times (10)^{-5}} = \frac{(10)^5}{36.18} = (0.0276) \times 10^5 = 2760$

This question should not be solved by detailed method. If you observe the following points you will be able to answer within seconds:

i) To get $\frac{1}{0.0003618}$ from $\frac{1}{36.18}$ the decimal in denominator is moved 5 positions left.

ii) So, to get the answer from 0.0276 the decimal should be moved 5 places right.

Note : Without going into details, we can get the answer. You should know that $\frac{1}{0.0003618}$ is larger than $\frac{1}{36.18}$. Therefore the required answer should also be larger than 0.0276. So, the required larger value should be obtained by moving the decimal to the right.

11. We have, $13324 \div 145 = 91.9$ or, $13324 \div 91.9 = 145$.

Then we have to find $133.24 \div 9.19 = ?$

To get the answer divide the whole number of dividend by the whole number of divisor and observe the number of digits in the whole number of quotient. In this case when 133 is divided by 9 the quotient (whole number) will have two digits. So, our answer should have a 2-digit whole number.

$\therefore$ required answer = 14.5

12. $\frac{3\sqrt{5}}{2\sqrt{5} - 0.48} = \frac{6.72}{4} = 1.68$

13. $\sqrt{\frac{5}{3}} = \sqrt{\frac{5 \times 3}{3 \times 3}} = \frac{\sqrt{15}}{3} = \frac{3.88}{3} = 1.2933$

14. If $\sqrt{2916} = 54$, then the required expression $= 5.4 + 0.54 + 0.054 + 0.0054 = 5.9994$

15. $\frac{1}{60 \times 60} = 0.00028$ (approx)

16. $1.5x = 0.05y$

or, $\frac{x}{y} = \frac{0.05}{1.5}$

By the rule of componendo-dividendo, we have,

$\frac{y - x}{y + x} = \frac{1.5 - 0.05}{1.5 + 0.05} = 0.935$

17. The given expression $= \left(\frac{6}{9} + \frac{7}{9} + \frac{8}{9} + \frac{3}{9}\right) \times 9000$

$= \frac{24}{9} \times 9000 = 24{,}000$

18. The given expression $= \left[\frac{3467 - 3}{9990}\right] + \left[\frac{45}{99} \times \frac{5}{9}\right] \times 11$

$= \frac{3464}{9990} + \frac{225}{81}$

$= \frac{93642}{9990 \times 3} = \frac{31214}{9990} = \frac{15607}{4995}$

Elementary Algebra

Algebraic Expressions: A number, including literal numbers, along with the signs of fundamental operations is called an algebraic expression. They may be *monomials*, having only one term as $+6x$, $-3y$ etc. They could also be *binomials*, i.e.having two terms as p^2+2r, $-x^2-2x$ etc. They may even be *polynomials*, i.e. having more than two terms.

Addition and Subtraction of Polynomials: The sum or difference of coefficients of like terms is performed.

For example, let the two polynomials be $3x^2+2x-3$, $5x^3-2x^2+x$. The sum of these two expressions is $5x^3+(3x^2-2x^2)+(2x+x)-3 = 5x^3+x^2+3x-3$. The difference between these two is the subtraction of smaller expression from the greater expression i.e. $5x^3-2x^2+x-(3x^2+2x-3) = 5x^3-2x^2+x-3x^2-2x+3 = 5x^3+(-2x^2-3x^2)+(x-2x)+3 = 5x^3-5x^2-x+3$.

Ex. 1: Find the sum of

$$-15a^2+3ab-6b^2,\ a^2-5ab+11b^2,\ -7a^2-18ab-13b^2 \text{ and } 26a^2-16ab-7b^2$$

Soln:

$$\left(-15a^2+3ab-6b^2\right)+\left(a^2-5ab+11b^2\right)+\left(-7a^2-18ab-13b^2\right)$$
$$+\left(26a^2-16ab-7b^2\right)$$
$$=\left(-15a^2+a^2-7a^2+26a^2\right)+(3ab-5ab-18ab-16ab)$$
$$+\left(-6b^2+11b^2-13b^2-7b^2\right)$$
$$=5a^2-36ab-15b^2$$

Ex. 2: If $P = a^4+a^3+a^2-6$

$Q = a^2-2a^3-2+3a$ and

$R = 8-3a-2a^2+a^3$

Find the value of $P+Q+R$

Soln: $P+Q+R =$

$$\left(a^4+a^3+a^2-6\right)+\left(a^2-2a^3-2+3a\right)+\left(8-3a-2a^2+a^3\right)$$
$$=a^4+\left(a^3-2a^3+a^3\right)+\left(a^2+a^2-2a^2\right)+(3a-3a)+(-6-2+8)$$
$$=a^4+0+0+0+0=a^4$$

Remainder Theorem: This theorem represents the relationship between the divisor of the first degree in the form $(x-a)$ and the remainder $r(x)$.

If an integral function of x is divided by x - a, until the remainder does not contain x, then the remainder is the same as the original expression with 'a' put in place of 'x'. In other words, if f(x) is divided by x - a, the remainder is f(a); e.g. when $f(x) = x^3 - 2x^2 + 3x - 4$ is divided by x - 2, the remainder is f(2).

$f(2) = 2^3 - 2(2)^2 + 3(2) - 4 = 2$

Note: (1) If f(x) is divided by x + a, the remainder is f(–a)

(2) If f(x) is divided by ax + b, the remainder is $f\left(-\frac{b}{a}\right)$.

(3) If f(x) is divided by ax - b, the remainder is $f\left(\frac{b}{a}\right)$.

(4) The method of finding the remainder is as follows:

I: Put the divisor equal to zero. Like, if the divisor is ax + b then

$ax + b = 0 \Rightarrow x = -\frac{b}{a}$

II: The remainder will be a function of the value of x, i.e., $f\left(-\frac{b}{a}\right)$.

Similarly, you can find the other remainders.

Ex. 1: Without using the division process, find the remainder when $x^3 + 4x^2 + 6x - 2$ is divided by (x + 5).

Soln: Step I: Put divisor equal to zero and find the value of x.

$x + 5 = 0 \Rightarrow x = -5.$

Step II: The remainder will be f(-5).

$f(-5) = (-5)^3 + 4(-5)^2 + 6(-5) - 2$
$= -125 + 100 - 30 - 2 = -57.$

Ex. 2: Find the remainder when $27x^3 - 9x^2 + 3x - 8$ is divided by 3x + 2.

Soln: Step I: $3x + 2 = 0 \Rightarrow x = -\frac{2}{3}$.

Step II: Remainder is $f\left(-\frac{2}{3}\right)$.

$$f\left(-\frac{2}{3}\right) = 27\left(-\frac{2}{3}\right)^3 - 9\left(-\frac{2}{3}\right)^2 + 3\left(-\frac{2}{3}\right) - 8$$

$= -8 - 4 - 2 - 8 = -22.$

Ex. 3: If the expression $Px^3 + 3x^2 - 3$ and $2x^3 - 5x + P$ when divided by x - 4 leave the same remainder, find the value of P.

Soln: The remainder are:

$R_1 = f(4) = P(4)^3 + 3(4)^2 - 3 = 64P + 45$

$R_2 = f(4) = 2(4)^3 - 5(4) + P = P + 108$

Since, $R_1 = R_2$, we have

$64P + 45 = P + 108$

or, $63P = 63$

$\therefore P = 1$

Ex. 4: Find the values of p and q when $px^3 + x^2 - 2x - q$ is exactly divisible by (x - 1) and (x + 1).

Soln: When the expression is exactly divisible by any divisor, the remainder will be zero.

Now, the remainder, when the divisor is x - 1, is

$f(1) = p + 1 - 2 - q - 0$

$\Rightarrow p - q = 1$ ------- (1)

and the remainder, when the divisor is x + 1, is

$f(-1) = p(-1)^3 + (-1)^2 - 2(-1) - q = 0$

$\Rightarrow -p + 1 + 2 - q = 0$

$\Rightarrow p + q = 3$ --------- (2)

Solving (1) & (2), we have,

$p = 2,\ q = 1$

Factorization of Polynomials: The factor theorem based on the remainder theorem is useful in the factorization of polynomials.

Factor Theorem: Let f (x) be a polynomial and a be a real number. Then the following two results hold:

(i) If f(a) = 0 then (x-a) is a factor of f (x).

(ii) If (x-a) is a factor of f (x) then f (a) = 0.

Ex. 1: Let $f(x) = x^3 - 12x^2 + 44x - 48$

Find out whether (x-2) and (x-3) are factors of f (x).

Soln : (a) To check whether (x-2) is a factor of f (x) we find f (a) i.e., f(2). If this becomes zero then (x-2) is a factor of f(x) according to the factor theorem (i) of division algorithm.

$f(2) = 2^3 - 12 \times 2^2 + 44 \times 2 - 48 = 0$

Hence, by the factor theorem, (x - 2) is a factor of f (x).

(b) To check whether (x-3) is a factor of f (x), we repeat the above process with f (3).

$f(3) = 3^3 - 12 \times 3^2 + 44 \times 3 - 48 = 3$

$\Rightarrow f(a) = f(3) \neq 0$. Hence (x-3) is not a factor of f (x).

Ex. 2: Find out whether $(3x - 1)$ is a factor of $27x^3 - 9x^2 - 6x + 2$ by the above rule.

Soln: We have, $3x - 1 = 0 \Rightarrow x = \frac{1}{3}$

If $(3x - 1)$ is a factor of f(x) then $f\left(\frac{1}{3}\right)$ should be equal to zero.

Here, $f\left(\frac{1}{3}\right) = 27\left(\frac{1}{3}\right)^3 - 9\left(\frac{1}{3}\right)^2 - 6\left(\frac{1}{3}\right) + 2$
$= 1 - 1 - 2 + 2 = 0$

$\therefore$ $(3x - 1)$ is a factor of the above expression.

Note: (1) We can see how the factor theorem has been derived from the remainder theorem: "When remainder is zero after dividing an expression."

(2) If $f(a) = 0$, then $x - a$ is a factor of f(x)

(3) If $f(-a) = 0$, then $x + a$ is a factor of f(x)

(4) If $f(x) = 0$, when $x = a$ and $x = b$ then f(x) is exactly divisible by $(x - a)(x - b)$ ie, $(x - a)$ and $(x - b)$ both are the factors of f(x).

(5) If an integral function of two or more variables is equal to zero when two of these variables are supposed to be equal, then the function is exactly divisible by the difference of these variables.

e.g.
$(b - c)$, $(c - a)$ and $(a - b)$ are the factors of
$a(b - c)^3 + b(c - a)^3 + c(a - b)^3$

Proof: Put $b = c$ in the given expression.

$a(c - c)^3 + c(c - a)^3 + c(a - c)^3 = 0 + c(c - a)^3 - c(c - a)^3 = 0$

$\therefore$ $(b - c)$ is a factor of the given expression. Similarly, it can be shown that $c - a$ and $a - b$ are the factors of the given expression.

Conditions of Divisibility

1. $x^n + a^n$ is exactly divisible by $(x + a)$ only when n is odd.
   e.g. $a^5 + b^5$ is exactly divisible by $a + b$.
2. $x^n + a^n$ is not exactly divisible by $(x + a)$ when n is even.
   e.g. $a^8 + b^8$ is not exactly divisible by $a + b$.
3. $x^n + a^n$ is never divisible by $(x - a)$.
   e.g. $a^7 + b^7$ or $a^{10} + b^{10}$ is not divisible by $a - b$.
4. $x^n - a^n$ is exactly divisible by $x + a$ when n is even.
   e.g. $x^6 - a^6$ is exactly divisible by $x + a$

5. $x^n - a^n$ is exactly divisible by $x - a$ (whether n is odd or even).
   e.g. $x^9 - a^9$ and $x^{10} - a^{10}$ are exactly divisible by $x - a$.

Proof: All the above statements can be proved. We are going to prove only statements (1) and (5). You should try to prove the remaining three yourself.

(1) $x^n + a^n$ is exactly divisible by $x + a$.
Put $x + 4 = 0 \Rightarrow x = -a$
Then $f(-a) = (-a)^n + a^n = 0$
This is possible only when n is odd.

(5) $x^n - a^n$ is exactly divisible by $x - a$.
Put $x - a = 0 \Rightarrow x = a$
Then $f(a) = a^n - a^n = 0$
This is true in all cases.

Ex. 3: The expression $5^{2n} - 2^{3n}$ has a factor
1) 3 2) 7 3) 10 4) 17 5) None of these

Soln: $5^{2n} - 2^{3n} = (5^2)^n - (2^3)^n = (25)^n - (8)^n$
As we don't know about n, by the condition (5), $(25 - 8)$ is a factor, i.e. 17 is a factor of $5^{2n} - 2^{3n}$.

Ex. 4: The last digit in the expansion of $(41)^n - 1$ when n is any +ve integer is
1) 2 2) 1 3) 0 4) -1 5) None of these

Soln: The last digit in the $(41)^n$ for any value of n is 1. Thus, the last digit in $(41)^n - 1$ is 0.

or,

$(41 - 1)$ is a factor or exact divisor of $(41)^n - (1)^n$ for any +ve integer of n. Now, when $41 - 1 = 40$ is a factor, the last digit in the expansion should be 0.

Ex. 5: Find the last two digits of the expansion of $2^{12n} - 6^{4n}$ when n is any positive integer.

Soln: $(2^6)^{2n} - (6^2)^{2n} = (64)^{2n} - (36)^{2n}$
Since 2n is an even integer, the above expression must be divisible by $(64 + 36)$ {By condition (4)}. Hence, $64 + 36 = 100$ is the factor of the above expression. Hence, the last two digits of its expansion must be 00.

Ex. 6: For all integral values of n, the expression $7^{2n} - 3^{3n}$ is a multiple

1) 22 2) 12 3) 10 4) 11 5) 24

Soln: $7^{2n} - 3^{3n} = (7^2)^n - (3^3)^n = (49)^n - (27)^n$

By the condition (5), 49 - 27 = 22 is a factor of the expressi $7^{2n} - 3^{3n}$.

Ex. 7: What should be subtracted from $27x^3 - 9x^2 - 6x - 5$ to make exactly divisible by (3x - 1)?

Soln: We will find the remainder by the rule of remainder.

We have, $3x - 1 = 0 \Rightarrow x = \frac{1}{3}$

Thus, the remainder is $f\left(\frac{1}{3}\right) = 27\left(\frac{1}{3}\right)^3 - 9\left(\frac{1}{3}\right)^2 - 6\left(\frac{1}{3}\right) - 5$

$= 1 - 1 - 2 - 5 = -7$

If we reduce the given expression by the remainder (-7), expression will be exactly divisible by the given divisor. Hen our required value is -7.

Theorem for Zero of a Polynomial

Let $f(x) = a_0x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots\ldots + a_n$ be a polynon with integral co-efficients. If an integer k is a zero polynon then k is a factor of a_n.

Solved Example : Find all integral zeros of the polynomial,

$f(y) = y^3 - 2y^2 + y + 4.$

Solution : Suppose (k) is an integral zero of the polynomial f(y). T by the above theorem, k is a factor of a_n i.e., 4. Hence, possi values of k are 1, -1, 2, -2, 4 and -4.

Now, test each of them to see whether it is zero of the polynon or not.

(i) $f(1) = 1^3 - 2 \times 1^2 + 1 + 4 = 4$. Since $f(1) \neq 0$, so 1 is not a zero of f

(ii) $f(-1) = (-1)^3 - (2)(-1)^2 + (-1) + 4 = 0$. Since $f(-1) = 0$, there -1 is the zero of f(y).

(iii) Similarly, 2, -2, 4 and -4 are not the zeros of f(y). Thus, the integral zero of f(y) is -1. If there are other zeros of the f(y), they not integers.

Quadratic Equations : *An equation in which the highest power of variable is 2 is called a quadratic equation.* For example, equation of the type $ax^2 + bx + c = 0$ denotes a quadratic equat

The product of multiplication of two linear polynomials also gives a quadratic polynomial. Let the two linear polynomials be (lx + m) and (px + q), where $l \neq 0$, $p \neq 0$. Then the product of these polynomials is the quadratic polynomial, $lpx^2 + (lq + mp)x + mq$. This is written in the standard form as $ax^2 + bx + c$. Then $a = lp$, $b = (lq + mp)$ and $c = mq$.

Factorization of Quadratic Equations : The quadratic polynomial $ax^2 + bx + c$ can be factorized only if there exist two numbers r and s such that

(i) $r = lq$, $s = mp$

(ii) $r + s = b$ = co-efficient of $x = lq + mp$

(iii) $r \times s = ac = l.p.m.q$ = co-efficient of x^2 × constant

Solved Example : Factorize $2x^2 + 11x + 5$.

Solution : (i) Here a = 2, b = 11 and c = 5

(ii) Find two numbers r and s such that r + s = b = 11 and $r \times s = a \times c = 2 \times 5 = 10$

So the numbers are 10 and 1.

(iii) Now, break up the middle term 11x of the given polynomial as 10x + 1x

$\therefore\ 2x^2 + 11x + 5 = 2x^2 + 10x + 1x + 5$

$= 2x(x+5) + 1(x+5) = (2x+1)(x+5)$

Conditions for Factorization of a Quadratic Equation: All quadratic expressions cannot be factorized. To test whether it can be factorized or not follow the points given below :

(1) if $b^2 - 4ac > 0$, then the quadratic equation can be factorized.

(2) If $b^2 - 4ac < 0$, then the quadratic equation cannot be factorized.

Some important formulae that are used in basic operations and in finding the factors of an expression are summarized below:

1. $a^2 - b^2 = (a+b)(a-b)$
2. $(a + b)^2 = a^2 + 2ab + b^2$
3. $(a - b)^2 = a^2 - 2ab + b^2$
4. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
5. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
6. $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$
7. $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$
8. $(a + b)^2 = (a - b)^2 + 4ab$

9. $(a-b)^2 = (a+b)^2 - 4ab$

10. $a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$
$= \frac{1}{2}(a+b+c)[(a-b)^2+(b-c)^2+(c-a)^2]$

11. $(a+b+c)^3 = a^3+b^3+c^3+3(b+c)(c+a)(a+b)$

12. $a^2+b^2 = (a+b)^2 - 2ab$

13. $a^2+b^2 = (a-b)^2 + 2ab$

14. If $a+b+c = 0$ then the value of $a^3+b^3+c^3$ is $3abc$.

15. $x^2 + x(a+b) + ab = (x+a)(x+b)$

16. $a^2(b+c)+b^2(c+a)+c^2(a+b)+3abc = (a+b+c)(ab+bc+ca)$

17. $ab(a+b)+bc(c+b)+ca(c+a)+2abc = (a+b)(b+c)(c+a)$

18. $a^2(b-c)+b^2(c-a)+c^2(a-b) = -(a-b)(b-c)(c-a)$

19. $(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$

Theory of Indices: The expression of a^m means the product of m factors, each equal to 'a'. When m is a positive integer, m is called the exponent or index or the power of 'a'. Any quantity raised to the power zero is always equal to 1. For example,

$a^0 = 1$ (where $a \neq 0$)

$1^0 = 1, (-1)^0 = 1, \left(\frac{1}{2}\right)^0 = 1, (3)^0 = 1, (1000)^0 = 1$, etc.

Whenever any index of the power is taken from the numerator to denominator or from the denominator to numerator, the sign of the power changes. For example,

$a^m = \frac{1}{a^{-m}}, \frac{1}{a^m} = a^{-m}, \frac{a^m}{a^n} = a^m a^{-n} = a^{m-n}$

Law of Indices: While solving the problems of exponents, the following laws are useful:

(1) $x^m \times x^n = x^{m+n}$

(2) $\left(x^m\right)^n = x^{mn}$

(3) $x^{m^n} = x^{(m^n)}$

(4) $(xy)^m = x^m y^m$

(5) $\frac{x^m}{x^n} = x^{m-n}$ if $m > n$ and $\frac{x^m}{x^n} = \frac{1}{x^{n-m}}$ if $n > m$

Note: The difference between (2) & (3) can be seen in the following examples:

(2): $\left(3^3\right)^2 = 3^{3\times 2} = 729$

(3): $3^{3^2} = 3^9 = 3^{6+3}$
$= 3^6 \times 3^3 = 729 \times 27 = 19683$

In all the above laws, m and n are positive integers. In algebra, the root problems are solved by laws of indices, for example $\sqrt{a}$ means $a^{1/2}$, $\sqrt[4]{a}$ means $a^{1/4}$, $a^{3/4}$ means $\sqrt[4]{a^3}$.

H.C.F.: The highest common factor of two or more algebraic expressions can be determined by the following methods:

1. **By the factor method**
2. **By the division method**

In case of the factor method, at first, the factors of the given expressions are found separately. Then the maximum number of common factors are taken out and multiplied, the product of which becomes the H.C.F.

For example, if we are asked to find the H.C.F. of $8(x^2-5x+6)$ and $12(x^2-9)$, we shall first find the factors of the given expressions separately.

Thus, $8(x^2-5x+6) = 4\times 2(x-3)(x-2)$

and $12(x^2-9) = 4\times 3(x-3)(x+3)$

Then the maximum number of common elements of the two will give the HCF of the expressions.

Therefore, HCF $= 4(x-3) = 4x-12$

L.C.M.: The minimum multiplied quantities out of two or more algebraic expressions is called LCM. If only two expressions are given, then at first we find their HCF by the factor method or by the division method and apply the following formula to find their LCM.

$$\text{LCM of two expressions} = \frac{\text{Product of the two expressions}}{\text{Their HCF}}$$

Thus, LCM of $8(x^2-5x+6)$ and $12(x^2-9)$ will be equal to

$\frac{8(x^2-5x+6)\times 12(x^2-9)}{4(x-3)}$ (by the formula)

$= 24 (x^2 - 5x + 6)(x + 3)$
$= 24 (x^3 - 5x^2 + 6x + 3x^2 - 15x + 18)$
$= 24 (x^3 - 2x^2 - 9x + 18)$

Solving Quadratic Equations: A quadratic equation when solved wi always give two values of the variable. These values of th variable are called the **roots of the equation.**

Any quadratic equation can either be solved by the factor metho or by a formula.

(1) **By the Factor Method:** When we solve a quadratic equation by th factor method, we first find the factors of the given equatio making the right-hand side equal to zero and then by equating th factors to zero, we get the values of the variable. For example, solving the quadratic equation $x^2 - 5x + 6 = 0$.

We first find the factors of $x^2 - 5x + 6$ which are $(x - 3)$ and $(x - 2$
Then we say that $(x - 3)(x - 2) = 0$
After that, we take each bracket putting it equal to zero in turn an find the values of x.
i.e. when $x - 3 = 0$ gives $x = 3$
and when $x - 2 = 0$ gives $x = 2$
Therefore, solution is $x = 3$ or $x = 2$.

(2) **By Formula:** When we want to solve a quadratic equation $ax^2 + bx + c = 0$, we apply the following formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Taking + and - signs separately, we get two values of x. Th quantity $b^2 - 4ac$ is called the **discriminant**. The two values of x obtaine from a quadratic equation are called the **roots of the equation**. Thes roots are denoted by α and β. It is seen that the sum of the roots of quadratic equation $ax^2 + bx + c = 0$, is equal to $\frac{-b}{a}$, i.e., $\alpha + \beta = \frac{-}{a}$

and product of the root is equal to $\frac{c}{a}$ i.e., $\alpha\beta = \frac{c}{a}$.

For a quadratic equation $ax^2 + bx + c = 0$.
(i) The roots will be equal if $b^2 = 4ac$.
(ii) The roots will be unequal and real if $b^2 > 4ac$.
(iii) The roots will be unequal and unreal if $b^2 < 4ac$.

Formulating a quadratic equation from given Roots: Whenever we are given the roots of a quadratic equation x^2 - x (sum of the roots) + product of the roots = 0.

For example, if the roots are given as 2 and 3, then the quadratic equation will be as follows:

$x^2 - (3 + 2)x + 3 \times 2 = 0$
$x^2 - 5x + 6 = 0$

Note:

(i) A quadratic equation $ax^2 + bx + c = 0$ will have reciprocal roots, if $a = c$.

(ii) When a quadratic equation $ax^2 + bx + c = 0$ has one root equal to zero, then $c = 0$.

(iii) When the roots of the quadratic equation $ax^2 + bx = c$ are negative reciprocals of each other, then $c = -a$.

(iv) When both the roots are equal to zero, $b = 0$ and $c = 0$.

(v) When one root is infinite, then $a = 0$ and when both the roots are infinite, then $a = 0$ and $b = 0$.

(vi) When the roots are equal in magnitude but opposite in sign, then $b = 0$.

(vii) If two quadratic equations $ax^2 + bx + c = 0$ and $a_1x^2 + b_1x + c_1 = 0$ and have a common root (i.e., one root common), then $(bc_1 - b_1c)(ab_1 - a_1b) = (ca_1 - c_1a)^2$.

(viii) If they have both the roots common, then $\frac{a}{a_1} = \frac{b}{b_1} = \frac{c}{c_1}$.

(ix) The square root of any negative number is an imaginary number (e.g. $\sqrt{-2}, \sqrt{-4}, \sqrt{-25}, \sqrt{-0.04}, \sqrt{-20.2}, \sqrt{-3^4}, \sqrt{-\frac{13}{17}}$, etc.), we do not have to deal with the problems regarding imaginary numbers. So a simple introduction is sufficient. $i^2 = -1$, so, $i = \sqrt{-1}$

So, $\sqrt{-4} = \sqrt{(-1) \times 4} = \sqrt{-1} \times \sqrt{4} = 2i$, which is an imaginary number.

Solved examples

Ex. 1: Find the discriminant of the following quadratic equations. Tell the nature of the roots of the equations. Verify them.
(i) $x^2 - 4x + 3 = 0$
(ii) $3x^2 + 4x - 3 = 0$
(iii) $-5x^2 + 7x = 0$
(iv) $2x^2 - 6x + 7 = 0$

Soln: (i) In $x^2 - 4x + 3 = 0$, $a = 1$, $b = -4$, $c = 3$ $\therefore$ $D = b^2 - 4ac = (-4$
$- 4 \times 1 \times 3 = 16 - 12 = 4 > 0$ and also a perfect square. So, the ro
will be distinct rational numbers.

Now, the roots are $\frac{-b \pm \sqrt{D}}{2a} = \frac{-(-4) \pm \sqrt{4}}{2 \times 1} = \frac{4 \pm 2}{2} = 2 \pm 1 = 3$ and

(These are distinct rational numbers.)

(ii) In $3x^2 + 4x - 3 = 0$, $a = 3$, $b = 4$, $c = -3$ $\therefore$ $D = b^2 - 4ac = (4$
$- 4 \times 3 \times (-3) = 16 + 36 = 52 > 0$ and is not a perfect square. S
the roots will be distinct irrational numbers.

Now, the roots are $\frac{-b \pm \sqrt{D}}{2a} = \frac{-4 \pm \sqrt{52}}{2 \times 3} = \frac{-4 \pm 2\sqrt{13}}{2 \times 3}$
$\frac{-2 \pm \sqrt{13}}{3} = \frac{-2 + \sqrt{13}}{3}$ and $\frac{-2 - \sqrt{13}}{3}$

(iii) Here, $a = -5$, $b = 7$, $c = 0$.
$\therefore$ $D = b^2 - 4ac$ becomes $b^2 = 7^2 = 49 > 0$ and also a perfect squar
So, the roots will be distinct rational numbers.

Now, the roots are $\frac{-b \pm \sqrt{D}}{2a} = \frac{-7 \pm \sqrt{7^2}}{2 \times (-5)} = \frac{-7 \pm 7}{-10}$

$= \frac{0}{-10}$ and $\frac{-14}{-10}$ i.e., 0 and 1.4.

As $c = 0$, we can simply solve this equation as
$-5x^2 + 7x = 0$ or, $x(-5x + 7) = 0$

$\Rightarrow$ either $x = 0$ or $-5x + 7 = 0$, i.e., $5x = 7$ i.e. $x = \frac{7}{5} = 1.4$

Finally, we get $x = 0$ and 1.4.

(iv) Here $a = 2$, $b = -6$, $c = 7$ $\therefore$ $D = b^2 - 4ac = (-6)^2 - 4 \times 2 \times$
$= 36 - 56 = -20 < 0$; so the roots will be imaginary.

Now, the roots are $\frac{-b \pm \sqrt{D}}{2a} = \frac{-(-6) \pm \sqrt{-20}}{2 \times 2}$

$= \frac{6 \pm 2\sqrt{5}i}{2 \times 2} = \frac{3 \pm \sqrt{5}i}{2} = \frac{3}{2} + \frac{\sqrt{5}}{2}i$ and $\frac{3}{2} - \frac{\sqrt{5}}{2}i$

Ex. 2: Form a quadratic equation whose roots are
(i) 3 and -5 (ii) 2 and 7, and verify them.

Soln: (i) The quadratic equation will be
x^2 - sum of the roots $\times x$ + products of the roots = 0
or, $x^2 - \{3 + (-5)\} \times x + 3 \times (-5) = 0$
or, $x^2 - (-2)x + (-15) = 0$ or, $x^2 + 2x - 15 = 0$

Verification: $D = 4 + 60 = 64$

$\therefore$ The roots are $\frac{-2 \pm \sqrt{64}}{2 \times 1} = \frac{-2 \pm 8}{2} = -1 \pm 4$ i.e. 3 and (-)5.

(ii) $x^2 - (2 + 7) \times x + 2 \times 7 = 0$ or, $x^2 - 9x + 14 = 0$
Verification: $D = 81 - 56 = 25$

$\therefore$ The roots are $\frac{-(-9) \pm \sqrt{25}}{2 \times 1} = \frac{9 \pm 5}{2} = \frac{14}{2}$ and $\frac{4}{2} = 7$ and 2.

Ex. 3: (i) If one of the roots of the equation $x^2 - 19x + 88 = 0$ be 8, find the other root.
(ii) If one of the roots of the equation $4x^2 - 27x + 18 = 0$ be 6, find the other root.

Soln: (i) We have the product of the roots $= \frac{c}{a} = \frac{88}{1} = 88$

$\therefore$ The other root $= \frac{88}{\text{one root i.e.}8} = 11$

Or, the sum of the roots $= -\frac{b}{a} = \frac{(-19)}{1} = 19$

$\therefore$ the other root = 19 - 8 = 11

(ii) The required other root = sum of the roots $\left(= -\frac{b}{a} = \frac{27}{4}\right)$ - one of the roots (= 6) i.e. $\frac{27}{4} - 6 = \frac{3}{4}$

Ex. 4: Find the roots of the equations
(i) $2x^2 + 3x - 5 = 0$ (ii) $x^2 - 8x + 7 = 0$

Note: Whenever we get $a + b + c = 0$, one of the roots will always be 1.

Soln: (i) Here $a + b + c = 2 + 3 + (-5) = 0$, so, 1 is one of the roots of the equation $2x^2 + 3x - 5 = 0$

$\therefore$ The other root = sum of the roots - one of the roots $= -\frac{3}{2} - 1$
$= -\frac{5}{2}$

So, the required roots are 1 and $-\frac{5}{2}$

Ex. 5: A motorcycle travels 20 km an hour faster than a cycle our a journey of 600 kms. The cycle takes 15 hours more than the motorcycle. Find their speed.

Soln: Let, the speed of the cycle = x kmph
then that of the motorcycle = x + 20.

Time taken by the cycle $= \frac{\text{distance}}{\text{speed}} = \frac{600}{x}$

and the time taken by the motorcycle $= \frac{600}{x+20}$ = 15 hours less than the time taken by the cycle.

i.e. $\frac{600}{x+20} = \frac{600}{x} - 15$

or, $\frac{40}{x+20} = \frac{40}{x} - 1 = \frac{40-x}{x}$

or, $40x = 40(x+20) - x(x+20)$

or, $40x = 40x + 800 - x^2 - 20x$

or, $x^2 + 20x - 800 = 0$

$\therefore\ x = \frac{-20 \pm \sqrt{(20)^2 - 4 \times 1 \times (-800)}}{2 \times 1} = \frac{-20 \pm \sqrt{400 + 3200}}{2}$

$= \frac{-20 \pm 60}{2} = -10 \pm 30 = 20$ or -40

As x, the speed of the cycle, cannot be negative, so x = -40 is not acceptable.

$\therefore$ x, the speed of the cycle = 20 kmph

and the speed of the motorcycle = x + 20 = 40 kmph

Ex. 6: Solve the equation $3^{2x+1} - 3^x = 3^{x+3} - 3^2$

Soln: $3^{2x} \times 3 - 3^x = 3^3 \times 3^3 - 3^2$

or, $3(3^x)^2 - 3^x = 27 \times 3^x - 9$

or, $3m^2 - m = 27m - 9$ Where $m = 3^x$

or, $3m^2 - 28m + 9 = 0$

$\therefore\ m = \frac{28 \pm \sqrt{(28)^2 - 4 \times 3 \times 9}}{2 \times 3} = \frac{28 \pm \sqrt{676}}{2 \times 3}$

$= \frac{28 \pm 26}{2 \times 3} = \frac{14 \pm 13}{3} = 9, \frac{1}{3}$

When m = 9, then $3^x = 9 = 3^2\ \therefore\ x = 2$

and when $m = \frac{1}{3}$ then $3^x = \frac{1}{3} = 3^{-1}\ \therefore\ x = -1$

Ex. 7: For what value of m, can the equation $-9x^2 + 12x - m = 0$ be a perfect square of a linear expression?

Soln: A linear expression is of the form ax + b = 0; (a ≠ 0), a and b are constant. A quadratic equation whose roots are α and β is given by $(x - \alpha)(x - \beta) = 0$

If α = β then the equation becomes $(x - \alpha)^2 = 0$. For both the roots to be equal, we have D = 0.

So, the given equation $-9x^2 + 12x - m = 0$ can be a perfect square of a linear expression if

$D = 0$ or, $b^2 - 4ac = (12)^2 - 4 \times (-9)(-m) = 0$

or, $144 - 36m = 0\ \therefore\ m = \frac{144}{36} = 4$

Verification: If we put m = 4 in the equation $-9x^2 + 12x - m = 0$ the equation becomes $-9x^2 + 12x - 4 = 0$

or, $9x^2 - 12x + 4 = 0$ or, $(3x - 2)^2 = 0$

Clearly, 3x - 2 is a linear expression of the form ax + b. Here a = 3 and b = -2.

Quadratic Expression

An expression of the form $ax^2 + bx + c$, (a ≠ 0) where a, b, c are real numbers is called a quadratic expression in x. The corresponding equation of the expression $ax^2 + bx + c$ is $ax^2 + bx + c = 0$

Before discussing the sign scheme for the quadratic expression, let us study the concept of real number line.

-ve side +ve side

$-\infty$ ←——————————→ ∞

..... -3 -2 -1 0 1 2 3 ......

smaller no. greater no.

(left) ——————————→ (right)

It is the real number line. As we move right, the value becomes greater.

So, 2 < 3, -3 < -2, (on the real number line as -2 is in the right side of -3 so -2 is greater than -3)

Also -2 < 0, -2 < 1, -1.5 < -0.5, -1.999 > -2, and so on.

Sign scheme for the quadratic expression

Note: 1. The sign sheme for the quadratic expression is always meant for the real values of x. We cannot compare any two imaginary numbers. So to say that ri > 2i or, 4i < wi is absolutely incorrect.

Let α and β be the roots of the corresponding quadratic equation (i.e. $ax^2 + bx + c = 0$) of the quadratic expression $ax^2 + bx + c$ (= y, suppose) Then we have for all the real values of x:

Case I: If α and β are real and equal or both imaginary i.e., D ≤ 0 then y i.e., $ax^2 + bx + c$ will have the same sign as that of a, the coefficient of x^2. That is, if D ≤ 0 and a is +ve, y will be always +ve, and if a is -ve y will be always -ve.

Case II: If α and β are real and unequal i.e., if $D > 0$ th
sign scheme for y i.e. $ax^2 + bx + c$ is as follow:

opposite to that of a

$-\infty$ ———————— α ———————— β ———————— ∞ $(\alpha < \beta)$

same as that of a α β same as that of a

Ex. 1: Find the sign scheme for the quadratic expression $x^2 - 4x + 7$.

Soln: The corresponding equation is $x^2 - 4x + 7 = 0$

$D = (-4)^2 - 4 \times 1 \times 7 = 16 - 28 - 12 < 0$ i.e. roots are imaginary

Here, $a = 1 > 0$. Therefore, for all the real values of x, the give
expression $x^2 - 4x + 7$ is always positive.

Note: 1. For all the real values of x (x may be 0.2, 32.07, $\frac{3}{8}$, -4, - 32.07
$2 - \sqrt{5}$, etc) $x^2 - 4x + 7 > 0$.

You can put x = any real number and verify that $x^2 - 4x + 7 > 0$

Let us suppose $x = -4.2$ then $x^2 - 4x + 7 = (4.2)^2 - 4 \times (-4.2) + 7$
$= 17.64 + 16.8 + 7 > 0$

When $x = 0.07$ then $x^2 - 4x + 7 = (0.07)^2 - 4 \times 0.07 + 7$
$= 0.07(0.07 - 4) + 7$
$= 0.07 \times (3.93) + 7 = 7 - 0.2751 > 0$

When $x = 2 - \sqrt{5}$ then $x^2 - 4x + 7 = (2 - \sqrt{5})^2 - 4(2 - \sqrt{5}) + 7$
$= 4 + 5 - 4\sqrt{5} - 8 + 4\sqrt{5} + 7 = 1 + 7 > 0$

2. Here $D = -12 < 0$ i.e., the roots are imaginary.

The roots are $\frac{-b \pm \sqrt{D}}{2a}$ i.e. $\frac{-(-4) \pm \sqrt{-12}}{2 \times 1} = \frac{4 \pm 2\sqrt{3}i}{2} = 2 \pm \sqrt{3}i$

So, if we put $x = 2 + \sqrt{3}i$ or $2 - \sqrt{3}i$, the given expression will become zero. The expression cannot be +ve or -ve.

When $x = 2 + \sqrt{3}i$, $x^2 - 4x + 7$
$= (2 + \sqrt{3}i)^2 - 4(2 + \sqrt{3}\ i) + 7$
$= 4 + 3i^2 + 4\sqrt{3}i - 8 - 4\sqrt{3}i + 7$
$= 4 + 3(\times -1) - 8 \times 7$ (we have $i^2 = -1$)
$= 4 - 3 - 1 = 0$

Similarly, when $x = 2 - \sqrt{3}i$, $x^2 - 4x + 7 = 0$.

3. The sign scheme is not valid for imaginary values of x. If we put $x = 4i$ in the given expression, we get
$x^2 - 4x + 7 = (4i)^2 - 4(4i) + 7$
$= 16i^2 - 16i + 7$
$= 16 \times (-1) - 16i + 7$

$= -16 - 16i + 7 = -8 - 16i$ which is imaginary and it cannot be compared with any real or imaginary number. So the purpose of the sign scheme becomes meaningless except for the roots (2 $\pm \sqrt{3}i$ which are imaginary).

Ex. 2: For what values of x, $9x^2 + 42x + 49 > 0$?

Soln: The corresponding equation is $9x^2 + 42x + 49 = 0$

$D = (42)^2 - 4 \times 9 \times 49 = 1764 - 1764 = 0$

Here, the coefficient of $x^2 = a = 9 > 0$

So, for all real values of x, $9x^2 + 4x + 49 > 0$ i.e. the given expression is always positive.

Ex. 3: Find the sign scheme for $-x^2 + 3x + 28$.

Soln: The corresponding equation is $-x^2 + 3x + 28 = 0$

$D = (3)^2 - 4 \times (-1) \times 28 = 9 + 112 = 121 > 0$

The roots of the equation are $\frac{-3 \pm \sqrt{121}}{2 \times (-1)} = \frac{-3 \pm 11}{-2} = \frac{8}{-2}, \frac{-14}{-2}$
$= -4, 7$

Here, the coefficient of x^2 is $-1 < 0$.

So, the sign scheme for the given expression $-x^2 + 3x + 28$ is as follows:

(+ve)

$-\infty$ ———————— -4 ———————— 7 ———————— ∞

(-ve) -4 7 (-ve)

That is, Case (i): if $-4 < x < 7$, $-x^2 + 3x + 28 > 0$

Case (ii): if $x < -4$ or $x > 7$, $-x^2 + 3x + 28 < 0$

Case (iii): if $x = -4$ or 7, $-x^2 + 3x + 28 = 0$

Note: You can verify the above Cases (i) and (ii) by putting such values of x as may be easier for calculation.

If we put $x = 0$, $-4 < 0 < 7$ we see,
$-x^2 + 3x + 28 = 0 + 0 + 28 = 28$ 0.

If we put $x = -5 < -4$, $-x^2 + 3x + 28 = -25 - 15 + 28 = -12 < 0$

If we put $x = 10 > 7$, $-x^2 + 3x + 28 = -100 + 30 + 28 = -42 < 0$

Ex. 4: For what range of the values of x, is $2x^2 - 4x - 7$ non-positive?

Soln: The corresponding equation is $2x^2 - 4x - 7 = 0$

$D = (-4)^2 - 4 \times 2 \times (-7) = 16 + 56 = 72 > 0$

Here, the coefficient of x^2 is $2 > 0$

The roots are $\frac{-(-4) \pm \sqrt{72}}{2 \times 2} = \frac{4 \pm 6\sqrt{2}}{2 \times 2} = \frac{2 \pm 3\sqrt{2}}{2}$

So, the sign scheme for the given expression $2x^2 - 4x + 7$ is as follows:

(-ve)

$-\infty$ ——+——————+—— ∞

(-ve) $\frac{2-3\sqrt{2}}{2}$ $\frac{2+3\sqrt{2}}{2}$ (-ve)

for $x = \frac{2 \pm 3\sqrt{2}}{2}$, the given expression will be zero

and for $\frac{2-3\sqrt{2}}{2} < x < \frac{2+3\sqrt{2}}{2}$ the given expression will be negative. For other values, the expression will be +ve.

Thus, for $\frac{2-3\sqrt{2}}{2} \le x \le \frac{2+3\sqrt{2}}{2}$ the given expression is not positive i.e., the expression is either zero or negative.

Note: If you want to check the sign scheme, you simply take an approximate value of $\sqrt{2}$ and then proceed. For example, we take $\sqrt{2} = 1.4$ (approximately)

Now, the roots are $\frac{2 \pm 3 \times 1.4}{2} = 1 \pm 3 \times 0.7$

$= 1 \pm 2.1 = 3.1, -1.1$

As the roots have been found by approximation, so while checking for the sign scheme, you should not take such values of x which are nearer to the roots.

Putting $x = 1$ $\left(\because \frac{2-3\sqrt{2}}{2} < 1 < \frac{2+3\sqrt{2}}{2}\right)$

$2x^2 - 4x - 7 = 2 - 4 - 7 = -9 < 0.$

Putting $x = 5$, $2x^2 - 4x - 7 = 2 \times 25 - 20 - 7 = 23 > 0$

Putting $x = -2$, $2x^2 - 4x - 7 = 2 \times 4 + 8 - 7 = 9 > 0$

Thus, we find that the sign scheme is correct.

Ex. 5: The inequality of $b^2 + 8b \ge 9b + 14$ is correct for

(i) $b \ge 5, b \le -5$ (ii) $b \ge 5, b \le -4$ (iii) $b \ge 6, b \le -6$

(iv) $b \ge 4, b \le -4$ (v) $b \ge 6, b \le 4$

Soln: The given equality is $b^2 + 8b \ge 9b + 14$

i.e. $b^2 + 8b - 9b - 14 \ge 0$ or, $b^2 - b - 14 \ge 0.$

For the equation $b^2 - b - 14 = 0$ we have

$D = (-1)^2 - 4 \times 1 \times (-14) = 1 + 56 = 57 > 0$

Here, the coefficient of $b^2 = 1 > 0$ and

the roots are $\frac{-(-1) \pm \sqrt{57}}{2 \times 1} = \frac{1 \pm 7.6}{2}$ [$\sqrt{57} = 7.6$ (approximately)]

$= 4.3$ and -3.3

So, the sign scheme for the expression $b^2 - b - 14$ is as follows:

(+ve)

$-\infty$ ——+——————+—— ∞

(-ve) -3.3 4.3 (-ve)

Thus for $b \le -3.3$ or, $b \ge 4.3$, $b^2 - b - 14 \ge 0$

To decide the correct option, we draw the points of the option on the real number line of the sign scheme

(+ve) (-ve) (+ve)

$-\infty$ ——+——————+—— ∞

..... -7 -6 -5 -4 -3.3 -3 4 4.3 5 6 7 .....

Now, we tally each of the options one by one.

Clearly, option (i) is correct. Option (ii) is also correct.

Between these two options, we observe that option (ii) is more suitable than option (i). Though option (iii) is correct, it is not as close as option (ii).

Options (iv) and (v) are incorrect.

Hence (ii) is the answer.

Ex. 6: If $x + y$ is constant, prove that xy is maximum when $x = y$

Soln: $xy = \left(\frac{x+y}{2}\right)^2 - \left(\frac{x-y}{2}\right)^2$,

We know that, the square of any real no. ≥ 0.

So, $\left(\frac{x+y}{2}\right)^2 \ge 0$ and $\left(\frac{x-y}{2}\right)^2 \ge 0$

As $x + y$ is constant, xy is the maximum when $\left(\frac{x-y}{2}\right)^2 = 0$

or, $\frac{x-y}{2} = 0$ or, $x - y = 0$ or, $x = y$

Ex. 7: Which of the following values of P satisfy the inequality $= P(P-3) < 4P - 12$?

1) $P > 14$ or $P < 13$ 2) $24 \le P < 71$ 3) $P > 13; P < 51$

4) $3 < P < 4$ 5) $P = 4, P = +3$

Soln: $P(P-3) < 4(P-3)$; $P(P-3) - 4(P-3) < 0$

$(P-3)(P-4) < 0$

This means that when $(P-3) > 0$ then $(P-4) < 0$ ... (i)

or, when (P - 3) < 0 then (P - 4) > 0 .... (ii)
From (i) P > 3 and P < 4 $\therefore$ 3 < P < 4
From (ii) P < 3 and P > 4
Which is not in choices. Hence from (i) our answer is (4).

Directions (Ex. 8-12): In each question one or more equation(s) is/are provided. On the basis of these you have to find out the relation between p and q.

Give answer (1) if p = q,
Give answer (2) if p > q,
Give answer (3) if q > p,
Give answer (4) if $p \geq q$, and
Give answer (5) if q > p.

8. **I.** $2p + \frac{5}{2} = p + 3$ **II.** $q - \frac{5}{2} = 1$

9. **I.** $\frac{p}{2} - \frac{p}{3} = 1$ **II.** $q^2 + 36 = 12q$

10. **I.** $\frac{p}{5} - \frac{2}{7} = 0$ **II.** $q^2 - 2q - 1$

11. **I.** $p^2 + 3 = 12$ **II.** $3q - 5 = 1 + q$

12. **I.** $7p^2 - 8p + 1 = 0$ **II.** $\frac{5q}{2} - \frac{q}{4} = \frac{1}{8}$

Soln:

8. 3; **I.** $\rightarrow 2p - p = 3 - \frac{5}{2} \Rightarrow p = \frac{1}{2};$

II. $\rightarrow q = 1 + \frac{5}{2} = 3.5$ Therefore q > p

9. 1; **I.** $\rightarrow \frac{p}{2} - \frac{p}{3} = 1 \Rightarrow \frac{p}{6} = 1$ $\therefore p = 6$

II. $\rightarrow q^2 - 12q + 36 = 0 \Rightarrow (q - 6)^2 = 0$

$\therefore q = 6$ Therefore p = q

10. 2; **I.** $\rightarrow \frac{p}{5} = \frac{2}{7} \Rightarrow p = \frac{10}{7}$

II. $\rightarrow q^2 - 2q + 1 = 0 \Rightarrow (q - 1)^2 = 0$ $\therefore q = 1$

Therefore p > q

11. 5; **I.** $\rightarrow p^2 = 9$ $\therefore p = \pm 3$

II. $\rightarrow 3q - q = 1 + 5$ $\therefore q = 3$ Therefore $q \geq p$

12. 2; **I** $\rightarrow p = \frac{8 \pm \sqrt{64 - 28}}{14} = \frac{8 \pm 6}{14} = \frac{2}{14}, \frac{14}{14} = \frac{1}{7}, 1$

II $\rightarrow \frac{7q}{4} = \frac{1}{8}$ or, $q = \frac{1}{14}$

$\therefore q < p$

Ex. 14: The inequality $3n^2 - 18n + 24 > 0$ gets satisfied for which of the following values of n?

1) n < 2 & n > 4 2) 2 < n < 4 3) n > 2
4) n > 4 5) None of these

Soln: The equation for the given inequality is

$3n^2 - 18n + 24 = 0$

$\Rightarrow 3(n - 2)(n - 4) = 0$

$\therefore n = 2, 4$

The real number line of the sign scheme is

$-\infty$ $+\infty$
+ve 2 -ve 4 +ve

The two values of n give the two end points of the inequality. On the above axis we have three ranges in which value of n can move.
(i) n < 2 (ii) 2 < n < 4 (iii) n > 4

Put a value which is less than 2. Suppose n = 0

Then $3n^2 - 18n + 24 = 0 - 0 + 24 = 24 > 0$ which satisfies the inequality. Hence n < 2 is valud value.

Again put a value which is between 2 and 4.

Suppose n = 3. Then $3n^2 - 18n + 24 = 27 - 54 + 24 = -3 < 0$ which does not satisfies the inequality. Hence 2 < n < 4 is not a valud value.

Again put a value which is greater than 4. Suppose n = 5. The $3n^2 - 18n + 24 = 75 - 90 + 24 = +9 > 0$ which satisfies the inequality. Hence n > 4 is valid value.

Note: The inequality (3n - 6) (n - 4) > 0

or, 3(n - 2) (n - 4) > 0 is true when (n - 2) and (n - 4) both are either +ve or -ve.

When both are +ve, we have

n - 2 > 0 and n - 4 > 0

or, n > 2 and n > 4 $\Rightarrow$ n > 4

When both are -ve

(n - 2) < 0 and (n - 4) < 0

$\Rightarrow$ n < 2 and n < 4 $\Rightarrow$ n < 2

Thus required value is n < 2 and n > 4

Ex. 15: Which of the following values of x satisfies the inequality $2x(x - 2) < x + 12$?

1) $-\frac{3}{2} < x < 4$ 2) $-3 < 2x < 4$ 3) $x > 4, x < -\frac{3}{2}$

4) $x > 4, x < \frac{3}{2}$ 5) None of these

Soln: 1; $2x(x-2) < x + 12$

or, $2x^2 - 4x - x - 12 < 0$

or, $2x^2 - 5x - 12 < 0$

or, $2x^2 - 8x + 3x - 12 < 0$

or, $2x(x-4) + 3(x-4) < 0$

or, $(x-4)(2x+3) < 0$

There are two cases for the above inequality.

Case I $x - 4 < 0$ and $2x + 3 > 0$

$\Rightarrow x < 4$ and $x > -\frac{3}{2}$

or, $-\frac{3}{2} < x < 4$

Case II $x - 4 > 0$ and $2x + 3 < 0$

or, $x > 4$ and $x < -\frac{3}{2}$

Which is not possible. Although it is given in choice (3) but both inequalities ($x > 0$ and $x < -3/2$) are not possible at a time. So, from case I only our answer is (1).

Note: If you have no idea about quadratic equation, you can verify the inequality by putting the suitable value of x from each choice. Whichever satisfies the inequality should be our answer.

Directions (Ex. 16-20): In each question one or more equation(s) is (are) provided. On the basis of these you have to find out relation between p and q.

Give answer (1) if $p = q$

Give answer (2) if $p > q$

Give answer (3) if $q > p$

Give answer (4) if $p \geq q$ and

Give answer (5) if $q \geq p$

16. (i) $\frac{5}{28} \times \frac{9}{8}p = \frac{15}{14} \times \frac{13}{16}q$

17. (i) $p - 7 = 0$ (ii) $3q^2 - 10q + 7 = 0$

18. (i) $4p^2 = 16$ (ii) $q^2 - 10q + 25 = 0$

19. (i) $4p^2 - 5p + 1 = 0$ (ii) $q^2 - 2q + 1 = 0$

20. (i) $q^2 - 11q + 30 = 0$ (ii) $2p^2 - 7p + 6 = 0$

Solutions:

16. 2; $\frac{5}{28} \times \frac{9}{8}p = \frac{15}{14} \times \frac{13}{16}q$ or, $\frac{p}{q} = \frac{15}{14} \times \frac{13}{16} \times \frac{8}{9} \times \frac{28}{5} = \frac{13}{3}$ .... (*)

$\Rightarrow p > q \quad \therefore$ Answer = (2)

Note: (*) shows that if p = 13 then q is 3.

17. 2; (i) $p - 7 = 0$ (ii) $3q^2 - 10q + 7 = 0$

(i) $\Rightarrow p = 7$

(ii) $\Rightarrow 3q^2 - 3q - 7q + 7 = 0 \Rightarrow 3q(q-1) - 7(q-1) = 0$

$\Rightarrow (3q-7)(q-1) = 0 \Rightarrow q = \frac{7}{3}$ or 1

$\therefore p > q \quad \therefore$ Answer = (2)

Note: In a quadratic equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\therefore q = \frac{-(-10) \pm \sqrt{(-10)^2 - 4 \times 3 \times 7}}{2 \times 3} = \frac{10 \pm 4}{6} = 1, \frac{7}{3}$$

18. 3; (i) $4p^2 = 16$ (ii) $q^2 - 10q + 25 = 0$

(i) $\Rightarrow p = \pm 2$ (ii) $\Rightarrow q = \frac{10 \pm \sqrt{100 - 4 \times 1 \times 25}}{2} = 5$

We see that $q > p$

19. 5; (i) $4p^2 - 5p + 1 = 0$ (ii) $q^2 - 2q + 1 = 0$

(i) $\Rightarrow p = \frac{5 \pm \sqrt{25 - 16}}{8} = \frac{5 \pm 3}{8} = \frac{1}{4}, 1$

(ii) $\Rightarrow q = \frac{2 \pm \sqrt{4 - 4}}{2} = 1$ We see that $p \leq q$ or $q \geq p$

20. 3; (i) $q = \frac{11 \pm \sqrt{121 - 120}}{2} = \frac{11 \pm 1}{2} = 5, 6$

(ii) $p = \frac{7 \pm \sqrt{49 - 48}}{4} = \frac{7 \pm 1}{4} = \frac{6}{4}, 2$

We see that $p < q$ or $q > p$

Conditions for the Maximum and the Minimum Values of the Quadratic Expression $ax^2 + bx + c$ (= y, suppose)

For the minimum value of y, the condition is $a > 0$. (if $a < 0$, y has no minimum value). The minimum value of $y = \frac{-D}{4a}$ which is possible when $x = -\frac{b}{2a}$

For the maximum value of y, the condition is $a < 0$ (if $a > 0$,

has no maximum value). The maximum value of $y = \frac{-D}{4a}$ which

possible when $x = -\frac{b}{2a}$

Ex. 1: If x be real, find the maximum value of $-2x^2 + x + 3$ and also fin the corresponding value of x.

Soln: The corresponding equation is $-2x^2 + x + 3 = 0$

$D = (1)^2 - 4 \times (-2) \times 3 = 1 + 24 = 25$

$\therefore$ The required maximum value of the given expression $= \frac{-D}{4a}$

$= \frac{-25}{4 \times (-2)} = \frac{25}{8} = 3.125$ and the corresponding value of x

$-\frac{b}{2a} = -\frac{1}{2 \times (-2)} = \frac{1}{4} = 0.25$

Explanation: $-2x^2 + x + 3 = y$ ..... (i)

or, $-2x^2 + x + (3 - y) = 0$

$D = (1)^2 - 4 \times (-2) \times (3 - y)$

$= 1 + 8(3 - y) = 1 + 24 - 8y = 25 - 8y$

Given that x is real, so $D \geq 0$ or, $25 - 8y \geq 0$, or, $25 \geq 8y$

$\therefore\ y \leq \frac{25}{8}$ i.e. the maximum value of y is $\frac{25}{8}$

Substituting $y = \frac{25}{8}$ in (i), we get $-2x^2 + x + 3 = \frac{25}{8}$

or, $-16x^2 + 8x + 24 = 25$

or, $16x^2 - 8x + 1 = 0$

or, $(4x - 1)^2 = 0$, or, $4x - 1 = 0$

$\therefore\ x = \frac{1}{4} = 0.25$

Ex. 2: If x be real, find the minimum value of $2x^2 - 5x - 3$ and also find the corresponding value of x.

Soln: The corresponding equation is $2x^2 - 5x - 3 = 0$

$D = (-5)^2 - 4 \times 2 \times (-3) = 25 + 24 = 49$

$\therefore$ The required minimum value of the given expression

$= -\frac{D}{4a} = -\frac{49}{4 \times 2} = -\frac{49}{8} = -6.125$ and the corresponding value of x

$= -\frac{b}{2a} = -\frac{(-5)}{2 \times 2} = \frac{5}{4} = 1.25$

Explanation: $2x^2 - 5x - 3 = y$ ..... (i)

or, $2x^2 - 5x - (y + 3) = 0$

$D = (-5)^2 - 4 \times 2 \times \{-(y + 3)\} = 25 + 8(y + 3) = 49 + 8y$

Given that x is real $\therefore$ $D \geq 0$ or, $49 + 8y \geq 0$ or, $49 \geq (-8)y$

or, $y \leq \frac{49}{(-8)} = -6.125$

i.e. the minimum value of y is -6.125 and substituting $y = -\frac{49}{8}$ in (i), we get

$2x^2 - 5x - 3 = \frac{49}{-8}$

or, $16x^2 - 40x - 24 + 49 = 0$

or, $16x^2 - 40x + 25 = 0$

or, $(4x - 5)^2 = 0$

or, $4x - 5 = 0$ $\therefore$ $x = \frac{5}{4} = 1.25$

Ex. 3: A certain number of tennis balls were purchased for Rs 450. Five more balls could have been purchased for the same amount if each ball was cheaper by Rs 15. Find the number of balls purchased.

1) 15 2) 20 3) 10 4) 25 5) None of these

Soln: Suppose he purchased x balls.

Then comparing the prices in two conditions, we get an equation

$\frac{450}{x} = \frac{450}{x+5} + 15$ .... (*)

or, $\frac{30}{x} = \frac{30}{x+5} + 1$ or, $\frac{30}{x} = \frac{30 + x + 5}{x + 5}$

or, $30(x + 5) = x(35 + x)$

or, $30x + 150 = 35x + x^2$

or, $x^2 + 5x - 150 = 0$

or, $x^2 + 15x - 10x - 150 = 0$

or, $x(x + 15) - 10(x + 15) = 0$

or, $(x - 10)(x + 15) = 0$

$\therefore$ x = 10, -15

Neglecting the -ve value we find the no. of balls = 10.

Note: (1) In equation (*)

Cost price of a ball $= \frac{450}{x}$

When we get 5 balls more for the same amount the cost price of a ball $= \frac{450}{x+5}$

We are given that price of a ball in second case is cheaper by Rs 15. So, $\frac{450}{x} = \frac{450}{x+5} + 15$

Quicker Approach: Equation (*) is our first step of the solution. From this very first step we see that we are going to be traped in a quadratic equation. Naturally, it will take more time to solve it by solving the quadratic equation. So, we suggest you to stop your further calculation and look at the choices given.

Choice (1): Put x = 15. It can't be our answer because at the right-hand side we get $\frac{450}{15+5}$; which is not a complete number.

Choice (2): Put x = 20. It also gives absurd values.

Choice (3): Put x = 10. This satisfies the equation and give meaningful values. So, our answer is (3).

Ex. 4: A businessman knows that the price of commodity will increase by Rs 5 per packet. He bought some packets of this commodity for Rs 4,500. If he bought this packet on new price then he gets 10 packets less. What is the number of packets bought by him?

1) 90 2) 100 3) 50 4) 125 5) None of these

Soln: Suppose he bought x packets.

Then cost price of a packet $= \frac{4500}{x}$

When he gets 10 packet less then cost price of a packet $= \frac{4500}{x-10}$

Now, we have, $\frac{4500}{x} + 5 = \frac{4500}{x-10}$ ..... (*)

The above equation is a quadratic equation so we should stop our further calculation (to save time). Put the value of x from choices given in the question. The value of x and x - 10 should be such that they divide 4500 exactly. So, our correct answer should be 100.

Note: (*) should be our first and last step in examination hall.

Linear Equations

Equations with One Variable: A statement of equality that contains an unknown quantity or variable is called an equation. The graph of such

an equation is a straight line, whose abscissae x or ordinates y satisfy the given equation.

Root or Solution: Any value of the variable that makes the statement of equation true is called a root of the equation.

Solved Example: Ravi's father is four times as old as Ravi. Four years ago, his father was six times as old as he was then. Find their present ages.

Solution:

(i) Suppose the present age of Ravi is 'x' years, and the present age of his father is 'y' years. Ravi's father is four times older than Ravi. This statement forms an equation, i.e., 4x = y. Converted to general form of ax + by + c = 0. It looks like 4x - y + 0 = 0.

(ii) Now, four years ago, their ages were x - 4 and y - 4 years respectively. The statement that his father was six times older than Ravi four years ago, forms another equation i.e., 6(x - 4) = y - 4. Converted to general form of ax + by + c = 0, it looks like

$$6x - y - 20 = 0.$$

(iii) The two simultaneous equations are:

(1) $4x - y + 0 = 0$

(2) $6x - y - 20 = 0$

(iv) Test for common solution $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ i.e. $\frac{4}{6} \neq \frac{-1}{-1}$

Therefore, there exists an ordered pair which can satisfy both the equations.

(v) Solving for x by elimination, by subtracting (2) from (1), we get $-2x + 20 = 0$ (by transposition).

$\Rightarrow x = 10$; putting the value of x = 10 in (1)

$4x - y + 0 = 0 \Rightarrow 40 - y = 0 \quad \therefore y = 40$

The present ages of Ravi and his father are 10 and 40 years respectively.

Miscellaneous Examples

Ex 1: If $x + \frac{1}{x} = 2$ the value of $x^2 + \frac{1}{x^2} = ?$

Soln: $x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2(x)\left(\frac{1}{x}\right) = 4 - 2 = 2$

Ex 2: If $x + \frac{1}{x} = 3$, the value of $x^8 + \frac{1}{x^8} = ?$

Soln: $x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 \cdot x \cdot \frac{1}{x} = 9 - 2 = 7$

$x^4 + \frac{1}{x^4} = \left(x^2 + \frac{1}{x^2}\right)^2 - 2 \cdot x^2 \cdot \frac{1}{x^2} = 49 - 2 = 47$

$x^8 + \frac{1}{x^8} = \left(x^4 + \frac{1}{x^4}\right)^2 - 2 \cdot x^4 \cdot \frac{1}{x^4} = (47)^2 - 2 = 2207$

Ex. 3: If $x + y = 3$, $xy = 2$; find the value of $x^3 - y^3$

Soln: $x^3 - y^3 = (x - y)\ (x^2 + y^2 + xy)$

Now, $x - y = \sqrt{(x+y)^2 - 4xy} = \sqrt{9 - 8} = 1$

$x^2 + y^2 + xy = (x + y)^2 - xy = 9 - 2 = 7$

$\therefore x^3 - y^3 = 1 \times 7 = 7$

Ex. 4: If $x - \frac{1}{x} = \sqrt{21}$, the value of $\left(x^2 + \frac{1}{x^2}\right)\left(x + \frac{1}{x}\right)$ is ___

Soln: $x^2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2 + 2 \cdot x \cdot \frac{1}{x} = 21 + 2 = 23$

$\left(x + \frac{1}{x}\right)^2 = \left(x - \frac{1}{x}\right)^2 + 4 \cdot x \cdot \frac{1}{x} = 21 + 4 = 25$

$\therefore\ x + \frac{1}{x} = \sqrt{25} = 5 \quad \therefore\ \left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2}\right) = 5 \times 23 = 115$

Ex. 5: $\frac{27^{3n+1} \cdot 81^{-n}}{9^{n+5} \cdot 3^{3n-7}}$.

Soln: $27 = 3^3 \Rightarrow 27^{3n+1} = 3^{3(3n+1)} = 3^{9n+3}$.

$81 = 3^4 \Rightarrow (81)^{-n} = 3^{-4n}$

$9 = 3^2 \Rightarrow 9^{n+5} = 3^{2(n+5)} = 3^{2n+10}$

$\therefore$ the given expression becomes $\frac{3^{9n+3} \cdot 3^{-4n}}{3^{2n+10} \cdot 3^{3n-7}}$

$\Rightarrow 3^{(9n+3-4n)-[(2n+10)+(3n-7)]} \Rightarrow 3^0 = 1$

Ex. 6: If $x = 12$, $y = 4$; find the value of $(x + y)^{x/y}$.

Soln: $(x + y)^{x/y} = (16)^3 = 4096$.

Ex. 7: If $a + b + c = 0$, find the value of $a^3 + b^3 + c^3$.

Soln: $a^3 + b^3 + c^3 = (a + b + c)^3 - 3(b + c)(c + a)(a + b)$

$= 0 - 3(b + c)(c + a)(a + b) = -3(-a)(-b)(-c)$

$\Rightarrow a^3 + b^3 + c^3 = 3abc$ **(Remember)**

Ex. 8: If $x = \frac{\sqrt{3}+1}{\sqrt{3}-1}$, $y = \frac{\sqrt{3}-1}{\sqrt{3}+1}$ find the value of $x^2 + y^2$.

Soln: $x^2 + y^2 = \left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)^2 + \left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)^2$

Now, $\frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{(\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{4-2\sqrt{3}}{3-1} = 2 - \sqrt{3}$

Similarly, $\frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{(\sqrt{3}+1)^2}{(\sqrt{3}-1)(\sqrt{3}+1)} = 2 + \sqrt{3}$.

$\therefore\ x^2 + y^2 = (2 - \sqrt{3})^2 + (2 + \sqrt{3})^2 = 2\left[(2)^2 + (\sqrt{3})^2\right] = 14$

$\left[\text{As } (a - b)^2 + (a + b)^2 = 2(a^2 + b^2)\right]$

Ex. 9 : If y varies as x and y = 7 when x = 2, find y when x = 5.

Soln: If y varies as x then $\frac{y}{x}$ = constant

$\therefore\ \frac{7}{2} = \frac{y}{5} \Rightarrow y = \frac{35}{2}$

Ex. 10: If $x + 2$ exactly divides $x^3 + 6x^2 + 11x + 6k$; find the value of k.

Soln: If $x + 2$ divides the given expression, then the given expression must be equal to zero when x equals -2, ie., $f(-2) = 0$

$\Rightarrow (-2)^3 + 6(-2)^2 + 11(-2) + 6k = 0$

$\Rightarrow -8 + 24 - 22 + 6k = 0 \Rightarrow 6k = 6 \Rightarrow k = 1$.

Ex. 11 : What must be added to $\frac{x}{y}$ to make it $\frac{y}{x}$?

Soln: The required value $= \frac{y}{x} - \frac{x}{y} = \frac{y^2 - x^2}{xy}$.

Ex. 12 : What is the square root of $(x^2 + 4x + 4)\ (x^2 + 6x + 9)$?

Soln: $(x^2 + 4x + 4)(x^2 + 6x + 9) = (x + 2)^2\ (x + 3)^2$

$\therefore$ the square root is : $(x + 2)(x + 3)$

Ex. 13 : If $3^{x-y} = 27$ and $3^{x+y} = 243$, find the value of x.

Soln: $3^3 = 27 = 3^{x-y}$

$\Rightarrow x - y = 3$ -------- (1)

$3^5 = 243 = 3^{x+y}$

$\Rightarrow 5 = x + y$ --------- (2)

Adding (1) and (2), we get

$(x + y) + (x - y) = 5 + 3$

$\Rightarrow 2x = 8 \Rightarrow x = 4.$

Ex. 14 : If $x = 9$, $y = \sqrt{17}$, the value of $(x^2 - y^2)^{-1/3}$ is equal to ______

Soln: $(x^2 - y^2)^{-1/3} = (81 - 17)^{-1/3} = \frac{1}{\sqrt[3]{81-17}} = \frac{1}{\sqrt[3]{64}} = \frac{1}{4}.$

Ex. 15 : Find the value of

$$\left[x^{b+c}\right]^{b-c}\left[x^{c+a}\right]^{c-a}\left[x^{a+b}\right]^{a-b}$$

Soln: We have : $x^{(b+c)(b-c)}\,x^{(c+a)(c-a)}\,x^{(a+b)(a-b)}$

$= x^{(b^2-c^2)}\,x^{(c^2-a^2)}\,x^{(a^2-b^2)}$

$= x^{b^2-c^2+c^2-a^2+a^2-b^2} = x^0 = 1.$

Ex. 16: The roots of $2kx^2 + 5kx + 2 = 0$ are equal if k is equal to ______

Soln: The roots will be equal if in $ax^2 + bx + c = 0$, $b^2 = 4ac$.

Hence, here the roots are equal if:

$(5k)^2 - 4\ (2k)\ 2 = 0$

$\Rightarrow 25k^2 - 16k = 0$

$\Rightarrow k(25k - 16) = 0$

i.e., if $k = 0$ or if $k = \frac{16}{25}$

Ex. 17: What is the condition that one of the roots of the equation is double the other in $ax^2 + bx + c = 0$?

Soln: Let one root be equal to α

Then other root will be 2α

Now, we know that the

sum of roots $= \frac{-b}{a}$ and the product of roots $= \frac{c}{a}$ **(Note)**

$\Rightarrow \alpha + 2\alpha = 3\alpha = \frac{-b}{a}$ and $\alpha(2\alpha) = 2\alpha^2 = \frac{c}{a}$ -------- (1)

$\alpha = \left(\frac{-b}{3a}\right) \Rightarrow \alpha^2 = \frac{b^2}{9a^2} \Rightarrow 2\alpha^2 = \frac{2b^2}{9a^2}$ ------ (2)

Hence, from (1) and (2) the condition is: $\frac{2b^2}{9a^2} = \frac{c}{a} \Rightarrow 2b^2 = 9ac$

Ex. 18: The sum of the roots of the equation $\frac{1}{x+a} + \frac{1}{x+b} = \frac{1}{c}$ is zero. Find the product of the roots.

Soln: Multiplying both sides of the equation by $c(x + a)(x + b)$ we obtain:

$c(x + b) + c(x + a) = (x + a)(x + b)$

On rearrangement, we obtain:

$x^2 + x(a + b - 2c) + ab - bc - ca = 0.$

Now, the sum of roots $= -(a + b - 2c) = 0 \Rightarrow a + b = 2c.$

$\therefore$ Product of roots $= ab - bc - ca$

$= ab - c(b + a) = ab - \left(\frac{a+b}{2}\right)(a + b)$

$= \frac{-1}{2}(a^2 + b^2)$, on simplification.

Ex. 19: Find the roots of the equation, $\sqrt{3y+1} = \sqrt{y-1}$

Soln: We have, on squaring.

$3y + 1 = y - 1 \Rightarrow 2y = -2 \Rightarrow y = -1.$

But $y = -1$ means $\sqrt{y-1} = \sqrt{-2}$ which is not a real number.

Hence, no real root exists.

Ex. 20: If α, β are the roots of the equation

$x^2 + 3ax + c = 0$ and if $\alpha^2 + \beta^2 = 5$, find the value of a.

Soln : Since α, β satisfy the given equation, we must have:

$\alpha^2 + 3a\alpha + c = 0$

$\beta^2 + 3a\beta + c = 0$

On adding, we get

$(\alpha^2 + \beta^{2)} + 3a(\alpha + \beta) + 2c = 0$

$\Rightarrow$ but $\alpha + \beta$ = sum of roots $= -\left(\frac{3a}{1}\right) = -3a.$

$\therefore 5 + 3a(-3a) + 2c = 0$

$\Rightarrow 9a^2 = 5 + 2c \Rightarrow a = \sqrt{\frac{5+2c}{9}}$

Ex. 21: If α, β are the roots of the equation

$x^2 + 3ax + 2a^2 = 0$ and if $\alpha^2 + \beta^2 = 5$, find the value of a.

Soln: In the previous example, we obtained:

$9a^2 = 5 + 2c$

Now, put, $c = 2a^2$

We get, $9a^2 = 5 + 4a^2 \Rightarrow 5a^2 = 5 \Rightarrow a^2 = 1 \Rightarrow a = \pm 1$

Ex. 22: The area of a rectangle is the same as that of a circle of radiu $\sqrt{\frac{35}{11}}$ cm. If the length of the rectangle exceeds its breadth b cm; find the dimensions of the rectangle.

Soln: Area of the circle $= \frac{22}{7} \times \sqrt{\frac{35}{11}} \times \sqrt{\frac{35}{11}}$

$= \frac{22}{7} \times \frac{35}{11} = 10$ sq. units.

Let the breadth be x.
Then the length is x + 3.
Area of rectangle = Area of circle

$\Rightarrow x(x+3) = 10 \Rightarrow x^2 + 3x - 10 = 0.$

$\therefore x = \frac{-3 \pm \sqrt{3^2 - 4 \times 1 \times (-10)}}{2}$

$= \frac{-3 \pm \sqrt{49}}{2} = \frac{-3 \pm 7}{2} = -5, 2.$

But breadth cannot be negative. Hence, we discard x = -5 an accept x = 2.

$\therefore$ Breadth = 2 cm, Length = 5 cm.

Ex. 23: The surface area of a pipe, open at both ends, is equal to 628 sq. m. The difference between its radius and its length is 15 m the length being the larger. If the pipe was closed at one end, wha amount of water can it hold? (Use $\pi = 3.14$).

Soln: Let radius = x. Then length = x + 15.

Now, we have surface area $= 2\pi rh = 628$

$\Rightarrow 6.28\, x(x+15) = 628 \Rightarrow x(x+15) = 100$

$\Rightarrow x^2 + 15x = 100$

$\Rightarrow x^2 + 15x - 100 = 0$

Solving the quadratic equation using the formula,

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ we obtain,

x = 5, -20. But radius can't be negative.
Hence, we accept x = 5.
Now, the volume of the pipe is

$\Rightarrow \pi r^2 h = 3.14 \times (5)^2 \times (5 + 15)$

$= 3.14 \times 25 \times 20 = 1570$ cub. m.

Ex. 24: Show that

$(a + b - 2c)^3 + (b + c - 2a)^3 + (c + a - 2b)^3$
$= 3(a + b - 2c)(b + c - 2a)(c + a - 2b)$

Soln: Let $x = a + b - 2c$

$y = b + c - 2a$

$z = c + a - 2b$

Now, we see that $x + y + z = 0$

Thus, $x^3 + y^3 + z^3 = 3xyz$ (From Ex. 7)

Therefore, $(a + b - 2c)^3 + (b + c - 2a)^3 + (c + a - 2b)^3$
$= 3(a + b - 2c)(b + c - 2a)(c + a - 2b)$

Ex. 25: Find the value of $x^3 + y^3 + z^3 - 3xyz$ when $x + y + z = 16$ and $xy + yz + zx = 78$

Soln: $x + y + z = 16$

or, $(x + y + z)^2 = 256$

or, $x^2 + y^2 + z^2 + 2(xy + yz + xz) = 256$

or, $x^2 + y^2 + z^2 + 2 \times 78 = 256$

or, $x^2 + y^2 + z^2 = 100$

Now, we have,

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)\{x^2 + y^2 + z^2 - (xy + yz + zx)\}$

$= 16\{100 - 78\} = 352$

Ex. 26: Find the value of $a^2 + b^2 + c^2 - 2ab + 2ac - 2bc$, if $a = x+y$, $b = x - y$ and $c = 2x - 1$

Soln: $a^2 + b^2 + c^2 - 2ab + 2ac - 2bc$

$= a^2 + (-b)^2 + c^2 + 2(a)(-b) + 2ac + 2(-b)c$

$= \{a + (-b) + c\}^2 = (a-b+c)^2$

$= (x + y - x + y + 2x - 1)^2 = \{2x - 2y - 1\}^2$

Ex. 27: Find the combined product of

$(a^4 + b^4)(a^2 + b^2)(a + b)(a - b)$

Soln: Given expression $= (a^4 + b^4)(a^2 + b^2)(a^2 - b^2)$

$= (a^4 + b^4)(a^4 - b^4) = a^8 - b^8$

Ex. 28: Find the value of $x^3 - \frac{1}{x^3}$ when $x - \frac{1}{x} = a$

Soln: $x - \frac{1}{x} = a$ or, $\left(x - \frac{1}{x}\right)^3 = a^3$

Since, $(a - b)^3 = a^3 - b^3 - 3ab(a-b)$, we have

$$\left(x-\frac{1}{x}\right)^3 = x^3-\left(\frac{1}{x}\right)^3-3x\frac{1}{x}\left(x-\frac{1}{x}\right) = a^3$$

or, $x^3-\frac{1}{x^3}-3a = a^3$

$\therefore\ x^3-\frac{1}{x^3} = a^3+3a$

Ex. 29: Find the value of $\frac{3^{n+4}-6\times 3^{n+1}}{3^{n+2}}$

Soln: $\frac{3^{n+4}-6\times 3^{n+1}}{3^{n+2}} = \frac{3^{n+4}-2\times 3\times 3^{n+1}}{3^{n+2}}$

$= \frac{3^{n+4}-2\times 3^{n+2}}{3^{n+2}} = \frac{3^{n+2}\left(3^2-2\right)}{3^{n+2}}$

$= 3^2-2 = 7$

Ex. 30: If $x = 2^{1/3}-2^{-1/3}$, find the value of $2x^3+6x$

Soln: Let $2^{1/3} = a$, then $2^{-1/3} = \frac{1}{2^{1/3}} = \frac{1}{a}$

Now, we have $x = a-\frac{1}{a}$

So, $2x^3+6x = 2\left\{\left(a-\frac{1}{a}\right)^3+3\left(a-\frac{1}{a}\right)\right\}$

$= 2\left\{a^3-\frac{1}{a^3}-3\times a\times\frac{1}{a}\times\left(a-\frac{1}{a}\right)+3\left(a-\frac{1}{a}\right)\right\}$

$2\left\{a^3-\frac{1}{a^3}\right\} = 2\left\{2-\frac{1}{2}\right\} = 2\left(\frac{3}{2}\right) = 3$

Ex. 31: Find the value of $\frac{(a-b)^2}{(b-c)(c-a)}+\frac{(b-c)^2}{(a-b)(c-a)}+\frac{(a-c)^2}{(a-b)(b-c)}$

Soln: Given expression $= \frac{(a-b)^3+(b-c)^3+(a-c)^3}{(a-b)(b-c)(c-a)}$

$= \frac{3(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)} = 3$

Note: We see that, $a-b+b-c+c-a = 0$

Therefore, by the well-known conditional identity, when $x+y+z = 0$ then $x^3+y^3+z^3 = 3xyz$ we have

$(a-b)^3+(b-c)^3+(c-a)^3 = 3(a-b)(b-c)(c-a)$

Ex. 32: If $a+b+c = 0$, then find the value of

$$\frac{1}{b^2+c^2-a^2}+\frac{1}{c^2+a^2-b^2}+\frac{1}{a^2+b^2-c^2}$$

Soln: $a+b+c = 0$

or, $a+b = -c$

or, $(a+b)^2 = (-c)^2$

or, $a^2+2ab+b^2 = c^2$

or, $a^2+b^2-c^2 = -2ab$

Similarly, $a^2+c^2-b^2 = -2ac$ and $b^2+c^2-a^2 = -2bc$

Hence, the given expression $= \frac{1}{-2bc}+\frac{1}{-2ac}+\frac{1}{-2ab}$

$= \frac{(a+b+c)}{-2a^2b^2c^2} = 0$, since $a+b+c = 0$

Ex. 33: If $x+y+z = 0$, then find the value of $\frac{(x+y)(y+z)(z+x)}{xyz}$

Soln: $x+y+z = 0$

$x+y = -z;\ y+z = -x;\ z+x = -y$

$\therefore$ The given expression $= \frac{(-z)(-x)(-y)}{xyz} = -1$

Ex. 34: If $a+b+c = 0$ then find the value of $\frac{a^4+b^4+c^4}{b^2c^2+c^2a^2+a^2b^2}$

Soln: We have, $a+b+c = 0$

or, $(a+b+c)^2 = 0$

or, $a^2+b^2+c^2 = -2(ab+ac+bc)$

Squaring both sides; $\left(a^2+b^2+c^2\right)^2 = 4(ab+bc+ca)^2$

or, $a^4+b^4+c^4+2a^2b^2+2b^2c^2+2a^2c^2$

$= 4\left[a^2b^2+b^2c^2+a^2c^2+2abc(a+b+c)\right]$

or, $a^4+b^4+c^4 = 2\left(a^2b^2+b^2c^2+a^2c^2\right)\quad(\because a+b+c = 0)$

$$\therefore \frac{a^4+b^4+c^4}{a^2b^2+b^2c^2+a^2c^2}=2$$

Ex. 35: If $x+y=2z$, then find the value of $\frac{x}{x-z}+\frac{z}{y-z}$

Soln : We have, $x+y=2z$

or, $x-z=z-y$

Thus, the given expression $=\frac{x}{z-y}-\frac{z}{z-y}=\frac{x-z}{z-y}=\frac{z-y}{z-y}=1$

Ex. 36 : If $x+\frac{1}{y}=1$ and $y+\frac{1}{z}=1$ then find the value of $z+\frac{1}{x}$.

Soln : $x+\frac{1}{y}=1$

$\Rightarrow x=1-\frac{1}{y}=\frac{y-1}{y}$

$\therefore \frac{1}{x}=\frac{y}{y-1}$ --- (1)

Again, $y+\frac{1}{z}=1 \Rightarrow \frac{1}{z}=1-y$

$\therefore z=\frac{1}{1-y}$ ----- (2)

From (1) & (2), the given expression =

$z+\frac{1}{x}=\frac{1}{1-y}+\frac{y}{y-1}=\frac{1-y}{1-y}=1$

Ex. 37 : If $a=b=c$, then find the value of $\frac{(a+b+c)^2}{a^2+b^2+c^2}$

Soln: The given expression $=\frac{(3a)^2}{3a^2}=\frac{9a^2}{3a^2}=3$

EXERCISES

1. The values of a and b for which $3x^3-ax^2-74x+b$ is a multiple of $x^2+2x-24$ are
   1) $a=-5, b=24$ 2) $a=5, b=24$ 3) $a=13, b=16$ 4) $a=-13, b=16$ 5) None of these
2. The remainder when $4x^6-5x^3-3$ is divided by x^3-2 is
   1) 0 2) 1 3) 2 4) 3 5) None of these

3. What should be added to $8x^3-12x^2+4x-5$ to make it exactly divisible by $2x+1$?
   1) 11 2) 5 3) 6 4) -11 5) None of these
4. Find the values of a and b when $f(x)=2x^3+ax^2-11x+b$ is exactly divisible by $(x-2)(x+3)$.
   1) $a=3, b=6$ 2) $a=3, b=-6$ 3) $a=-3, b=6$ 4) $a=-3, b=-6$ 5) None of these
5. If $f(x)=4x^3-2x^2+5x-8$ is divided by $x-2$, what will be the remainder?
   1) 25 2) 42 3) 16 4) 26 5) None of these
6. x^n-y^n is exactly divisible by
   1) $x-y$ 2) $x+y$ 3) both $x-y$ and $x+y$ 4) Neither $x-y$ nor $x+y$ 5) None of these
7. $\frac{1}{2}(a+b+c)\{(a-b)^2+(b-c)^2+(c-a)^2\}=?$
   1) $a^3+b^3+c^3+3abc$ 2) $a^3+b^3+c^3-3abc$ 3) $a^3+b^3+c^3+3abc(a+b+c)$ 4) $3abc$ 5) 3
8. Find the value of $a^3+b^3+c^3-3abc$ when $a+b+c=9$ and $a^2+b^2+c^2=29$
   1) 9 2) 3 3) 27 4) 81 5) None of these
9. Find the value of $x^3+y^3+z^3-3xyz$ when $x=89, y=87, z=84$
   1) 260 2) 19 3) 4940 4) 4490 5) None of these
10. If $x=a(b-c)$, $y=b(c-a)$ and $z=c(a-b)$, then $\left(\frac{x}{a}\right)^3+\left(\frac{y}{b}\right)^3+\left(\frac{z}{c}\right)^3=?$
    1) $\frac{3xyz}{abc}$ 2) $\frac{xyz}{abc}$ 3) 3xyzabc 4) 3 5) None of these
11. When $x+\frac{1}{x}=3$, find the value of $x^2+\frac{1}{x^2}$
    1) 3 2) 6 3) 9 4) 27 5) None of these
12. When $x-\frac{1}{x}=5$, find the value of $x^2+\frac{1}{x^2}$
    1) 9 2) 27 3) 81 4) 7 5) 15
13. Find the value of $a^2+b^2+c^2-2ab-2ac+2bc$ when $a=17, b=15$ and $c=13$.
    1) 111 2) 121 3) 225 4) 361 5) None of these

14. When $x + \frac{1}{x} = 3$ find the value of $x^3 + \frac{1}{x^3}$

1) 9 2) 27 3) 18 4) 21 5) None of these

15. When a = -5, b = -6 and c = 10 find the value of

$$\frac{a^3 + b^3 + c^3 - 3abc}{(ab + bc + ca - a^2 - b^2 - c^2)}$$

1) -1 2) 1 3) 2 4) -2 5) 3

16. If a = 1, find the value of

$$15a^3 - (3a^3 - 1) - (4a^4 + a^3 - 3) + (a^3 - 1)$$

1) 11 2) 1 3) 10 4) 17 5) None of these

17. If $a + b + c = 10$ and $ab + bc + ac = 31$, find the value of $a^2 + b^2 + c^2$

1) 69 2) 162 3) 131 4) 38 5) None of these

18. Find the value of $(a + 1)(1 - a)(1 - a + a^2)(1 + a + a^2)(1 + a^6)$

1) $1 - a^{12}$ 2) $1 + a^{12}$
3) $1 - a^{36}$ 4) $1 + a^{36}$
5) None of these

19. If $x + y = 1$, find the value of $x^3 + y^3 + 3xy$.

1) 1 2) 0 3) 2 4) 6 5) None of these

20. Find the value of $\frac{2^n \times 6^{m+1} \times 10^{m-n} \times 15^{m+n-2}}{4^m \times 3^{2m+n} \times 25^{m-1}}$

1) 2 2) $\frac{2}{3}$ 3) $\frac{1}{3}$ 4) 3 5) None of these

21. What is the value of the expression $\frac{(a-b)^3 + (b-c)^3 + (c-a)^3}{(a-b)(b-c)(c-a)}$?

1) 1 2) 0 3) 2 4) 3 5) None of these

22. If $a + b + c = 0$, find the value of $\frac{a^2 + b^2 + c^2}{a^2 - bc}$

1) 0 2) 1 3) 2 4) 3 5) None of these

23. If $a + b + c = 0$, find the value of $\frac{a^2 + b^2 + c^2}{c^2 - ab}$

1) 0 2) 1 3) 2 4) 3 5) None of these

24. If $a + b + c = 0$, then find the value of $\frac{(a^2 + b^2 + c^2)^2}{a^2 b^2 + b^2 c^2 + c^2 a^2}$

1) 1 2) 2 3) 3 4) 4 5) None of these

25. If $x^2 = y + z$, $y^2 = z + x$, $z^2 = x + y$, then the value of

$$\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1} \text{ is}$$

1) 1 2) -1 3) 2 4) 4 5) None of these

ANSWERS

1. 1; $x^2 + 2x - 24 = (x + 6)(x - 4)$
Since (x + 6) (x - 4) is a multiple of the given expression, (x + 6) and (x - 4) should divide the expression exactly. That is, if we divide the given expression by (x + 6) and (x - 4), there should be no remainder. Thus, by the remainder theorem,
$f(-6) = 3(-6)^3 - a(-6)^2 - 74(-6) + b = 0$
and $f(4) = 3(4)^3 - a(4)^2 - 74(4) + b = 0$
Solving these two equations, we get the values of a and b.

2. 4; Put $x^3 = y$ then the divisor is y - 2 and the given expression is $4y^2 - 5y - 3$. By the remainder theorem, the remainder is
$f(2) = 4(2)^2 - 5(2) - 3 = 3$

3. 1; By the remainder theorem, the remainder is
$$f\left(-\frac{1}{2}\right) = 8\left(-\frac{1}{2}\right)^3 - 12\left(-\frac{1}{2}\right)^2 + 4\left(-\frac{1}{2}\right) - 5$$
$$= -1 - 3 - 2 - 5 = -11$$
For the expression to be exactly divisible, the remainder should be zero. Hence, 11 should be added.

4. 2; $f(2) = 2(2)^3 + a(2)^2 - 11 \times 2 + b = 0$
or, $16 + 4a - 22 + b = 0$
or, $4a + b = 6$ ----- (1)
$f(-3) = 2(-3)^3 + a(-3)^2 - 11(-3) + b = 0$
or, $-54 + 9a + 33 + b = 0$
or, $9a + b = 21$ ----- (2)
Solving (1) and (2), we get the values of a and b.

5. 4; $f(2) = 4(2)^3 - 2(2)^2 + 5(2) - 8$
$= 32 - 8 + 10 - 8 = 26$.

6. 1; $x^n - y^n$ is exactly divisible by x - y for any +ve integer (odd or even).

7. 2; $\frac{1}{2}(a + b + c)\{2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ac\}$
$= (a + b + c)\{a^2 + b^2 + c^2 - ab - bc - ac\} = a^3 + b^3 + c^3 - 3abc$

8. 3; $a + b + c = 9$

or, $(a + b + c)^2 = 81$

or, $a^2 + b^2 + c^2 + 2(ab + bc + ac) = 81$

or, $29 + 2(ab + bc + ac) = 81$

or, $(ab + bc + ac) = \frac{81 - 29}{2} = 26$

Now, $a^3 + b^3 + c^3 - 3abc = (a + b + c)\{a^2 + b^2 + c^2 - ab - bc - ac\}$

$= 9\{29 - 26\} = 27$

9. 3; Use;

$$x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)\{(x - y)^2 + (y - z)^2 + (x - z)^2\}$$

10. 1; $\frac{x}{a} = b - c$, $\frac{y}{b} = c - a$; $\frac{z}{c} = a - b$

Now, we have, $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0$

Now, use the well-known conditional identity

If $l + m + n = 0$ then $l^3 + m^3 + n^3 = 3lmn$

11. 5; $\left(x + \frac{1}{x}\right)^2 = 3^2 = 9$

or, $x^2 + \frac{1}{x^2} + 2x\frac{1}{x} = 9$ $\quad\therefore x^2 + \frac{1}{x^2} = 9 - 2 = 7$

12. 2; $\left(x - \frac{1}{x}\right)^2 = 25$ $\quad\therefore x^2 + \frac{1}{x^2} = 25 + 2 = 27$

13. 2; The given expression $= (a - b - c)^2$

$= (17 - 15 - 13)^2 = (-11)^2 = 121$

14. 3; $\left(x + \frac{1}{x}\right)^3 = 27$

or, $x^3 + \frac{2}{x^3} + 3 \times x \times \frac{1}{x}\left(x + \frac{1}{x}\right) = 27$

or, $x^3 + \frac{1}{x^3} + 3 \times 3 = 27$ $\quad\therefore x^3 + \frac{1}{x^3} = 18$

15. 2; The given expression $= -(a + b + c)$ (find it)

$= -(-5 - 6 + 10) = -(-1) = 1.$

16. 1; $15a^2 - 3a^3 - a^3 + a^3 - 4a^4 + 4 + 3 - 1$

$= 12a^3 - 4a^4 + 3 = 12 - 4 + 3 = 11$

17. 4; $a + b + c = 10$

or, $(a + b + c)^2 = 100$

or, $a^2 + b^2 + c^2 + 2(ab + bc + ac) = 100$

$\therefore a^2 + b^2 + c^2 = 100 - 2(31) = 38$

18. 1; The given expression =

$\{(a + 1)(1 - a + a^2)\}\{(1 - a)(1 + a + a^2)\}\{1 + a^6\}$

$= (1 + a^3)(1 - a^3)(1 + a^6) = (1 - a^6)(1 + a^6) = 1 - a^{12}$

Note : We have used $(a - b)(a + b) = a^2 - b^2$

19. 1; $x + y = 1$

or, $(x + y)^3 = 1$ or, $x^3 + y^3 + 3xy(x + y) = 1$ or, $x^3 + y^3 + 3xy = 1$

20. 2; Given expression =

$$\frac{2^n \times 2^{m+1} \times 3^{m+1} \times 2^{m-n} \times 5^{m-n} \times 3^{m-n-2} \times 5^{m+n-2}}{2^{2m} \times 3^{2m+n} \times 5^{2m-2}}$$

$$= \frac{2^{2m+1} \times 3^{2m+n-1} \times 5^{2m-2}}{2^{2m} \times 3^{2m+n} \times 5^{2m-2}} = \frac{2}{3}$$

21. 4; Use: when $x + y + z = 0$, $\quad x^3 + y^3 + z^3 = 3xyz$

or, $\frac{x^3 + y^3 + z^3}{xyz} = 3$

In this case, $(a - b) + (b - c) + (c - a) = 0$

22. 3; $a + b + c = 0$

or, $(a + b + c)^2 = 0$

or, $a^2 + b^2 + c^2 + 2(ab + bc + ac) = 0$

or, $a^2 + b^2 + c^2 = -2\{bc + a(b + c)\}$

$= -2\{bc + a(-a)\} = -2(bc - a^2) = 2(a^2 - bc)$

$\therefore \frac{a^2 + b^2 + c^2}{a^2 - bc} = 2$

23. 3; The same as question (22).

24. 4; $(a + b + c)^2 = 0$

or, $a^2 + b^2 + c^2 = -2(ab + bc + ac)$

or, $\left(a^2 + b^2 + c^2\right)^2 = 4(ab + bc + ac)^2$

$= 4\{a^2b^2 + b^2c^2 + a^2c^2 + 2abc(a + b + c)\}$

$= 4\{a^2b^2 + b^2c^2 + a^2c^2\}$ (Since $a + b + c = 0$)

$\therefore \dfrac{\left(a^2+b^2+c^2\right)^2}{a^2b^2+b^2c^2+a^2c^2}=4$

25. 1; $x^2=y+z$, $y^2=z+x$ and $z^2=x+y$

or, $x+x^2=x+y+z$, $y+y^2=x+y+z$ and $z+z^2=x+y+z$

or, $x+x^2 = y+y^2 = z+z^2=x+y+z=k$ (say)

or, $x(1+x)=y(1+y)=z(1+z)=k$

$\therefore \dfrac{1}{1+x}=\dfrac{x}{k}, \dfrac{1}{1+y}=\dfrac{y}{k}$ and $\dfrac{1}{1+z}=\dfrac{z}{k}$

$\therefore$ the given expression $=\dfrac{x}{k}+\dfrac{y}{k}+\dfrac{z}{k}=\dfrac{x+y+z}{k}=\dfrac{k}{k}=1$

Surds

The roots of those quantities which cannot be exactly obtained are called ***surds****; e.g.* $\sqrt{2}$*,* $4\sqrt{8}$*, etc.*

Mixed Surds : A rational factor and a surd multiplied together produce a mixed surd; e.g. $2\sqrt{3}$, $4\sqrt{5}$, etc.

Order of Surds : $a^{1/m}$ is called a surd of the mth order.

Changing the surds into that of the same order :

Ex.: Express $3^{1/4}$, $2^{1/3}$ and $5^{1/6}$ as surds of the same order and arrange them in the ascending order of magnitude.

Soln : The LCM of 4, 3 and 6 (the root indices) is 12. We then reduce them to the 12th order.

$$3^{1/4} = 3^{3/12} = 3^{3^{1/12}} = 27^{1/12}$$

$$2^{1/3} = 2^{4/12} = 2^{4^{1/12}} = 16^{1/12}$$

$$5^{1/6} = 5^{2/12} = 5^{2^{1/12}} = 25^{1/12}$$

Hence, in order of magnitude, they should be

$$16^{1/12} < 25^{1/12} < 27^{1/12} \text{ or, } 2^{1/3} < 5^{1/6} < 3^{1/4}$$

Addition and Subtraction of Surds : Similar surds like $2\sqrt{5}$, $5\sqrt{5}$, $12\sqrt{5}$ can be added but dissimilar surds like $5\sqrt{3}$, $3\sqrt{2}$, $4\sqrt{7}$ cannot be added.

Ex.: Simplify $\sqrt{75} + \sqrt{48}$

Soln : $\sqrt{75} + \sqrt{48} = \sqrt{25 \times 3} + \sqrt{16 \times 3} = 5\sqrt{3} + 4\sqrt{3} = 9\sqrt{3}$

Multiplication of Surds:

Ex.: Find the product of $4^{1/3}$, $6^{1/6}$ and $\sqrt{5}$.

Soln : $4^{1/3} \times 6^{1/6} \times 5^{1/2}$

$$= 4^{2/6} \times 6^{1/6} \times 5^{3/6} = \left[4^2 \times 6 \times 5^3\right]^{1/6} = 12000^{1/6}$$

Division of Surds :

Ex.: Divide $12 \times 4^{1/3}$ by $3\sqrt{2}$.

Soln : $\dfrac{12 \times 4^{1/3}}{3\sqrt{2}} = \dfrac{4 \times 4^{1/3}}{2^{1/2}} = \dfrac{4 \times 4^{2/6}}{2^{3/6}} = 4 \times \left[\dfrac{4^2}{2^3}\right]^{1/6} = 4\left(\dfrac{16}{8}\right)^{1/6} = 4 \times 2^{1/6}$

In solving the examples under this chapter, the following simple results will be used:

1) $\sqrt{a} \times \sqrt{a} = a$
2) $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$
3) $\sqrt{a^2 \times b} = a\sqrt{b}$
4) $(\sqrt{a} + \sqrt{b})^2 = a + b + 2\sqrt{ab}$
5) $(\sqrt{a} - \sqrt{b})^2 = a + b - 2\sqrt{ab}$
6) $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$

The following roots are also useful; so they should be remembered.

$\sqrt{2} = 1.41421$; $\sqrt{3} = 1.73205$; $\sqrt{5} = 2.23607$
$\sqrt{6} = 2.44949$; $\sqrt{7} = 2.64575$; $\sqrt{8} = 2.82842$

Ex. 1: Find the value of $\sqrt{300}$.

Soln: $\sqrt{300} = \sqrt{10 \times 10 \times 3} = 10\sqrt{3} = 10 \times 1.732 = 17.32$

Ex. 2: Evaluate the following:

a) $\sqrt{75} + \sqrt{147}$ b) $\sqrt{80} + 3\sqrt{245} - \sqrt{125}$

Soln: a) $\sqrt{75} + \sqrt{147} = \sqrt{5 \times 5 \times 3} + \sqrt{7 \times 7 \times 3}$
$= 5\sqrt{3} + 7\sqrt{3} = 12\sqrt{3} = 20.7846$

b) $\sqrt{80} + 3\sqrt{245} - \sqrt{125}$
$= \sqrt{4 \times 4 \times 5} + 3\sqrt{7 \times 7 \times 5} - \sqrt{5 \times 5 \times 5}$
$= 4\sqrt{5} + 21\sqrt{5} - 5\sqrt{5} = 20\sqrt{5} = 44.7214$

Ex. 3: Evaluate the following:

a) $\sqrt{2} \times \sqrt{3}$ b) $\sqrt{6} \times \sqrt{150}$ c) $\sqrt{242} \div \sqrt{72}$

Soln: a) $\sqrt{2} \times \sqrt{3} = \sqrt{6} = 2.44949 \approx 2.4495$

b) $\sqrt{6} \times \sqrt{150} = \sqrt{6 \times 150} = \sqrt{900} = 30$

c) $\sqrt{242} \div \sqrt{72} = \dfrac{\sqrt{121 \times 2}}{\sqrt{36 \times 2}} = \dfrac{11\sqrt{2}}{6\sqrt{2}} = \dfrac{11}{6} = 1\dfrac{5}{6}$.

Ex. 4: Find the values of

a) $\dfrac{1}{\sqrt{2}}$ b) $\dfrac{1}{\sqrt{3}}$ c) $\dfrac{1}{\sqrt{5}}$ d) $\dfrac{1}{\sqrt{6}}$ e) $\dfrac{1}{\sqrt{7}}$

Soln: a) $\dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \dfrac{\sqrt{2}}{2} = 0.7071$

b) $\dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \dfrac{\sqrt{3}}{3} = 0.5773$

c) $\dfrac{1}{\sqrt{5}} = \dfrac{\sqrt{5}}{5} = 0.4472$

d) $\dfrac{1}{\sqrt{6}} = \dfrac{\sqrt{6}}{6} = 0.4082$

e) $\dfrac{1}{\sqrt{7}} = \dfrac{\sqrt{7}}{7} = 0.3779$

Ex. 5: Evaluate the following:

a) $\dfrac{1}{\sqrt{3} - 1}$ b) $\dfrac{14}{3 + \sqrt{2}}$ c) $\dfrac{\sqrt{2} - 1}{\sqrt{2} + 1}$

d) $\dfrac{\sqrt{5} + 1}{\sqrt{5} - 1}$ e) $\dfrac{2 + \sqrt{3}}{2 - \sqrt{3}}$ f) $\dfrac{4 + \sqrt{2}}{\sqrt{2} + 1}$

g) $\dfrac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}}$

Soln: a) $\dfrac{1}{\sqrt{3} - 1} = \dfrac{\sqrt{3} + 1}{(\sqrt{3} - 1) \times (\sqrt{3} + 1)} = \dfrac{\sqrt{3} + 1}{2} = 1.3660$

b) $\dfrac{14}{3 + \sqrt{2}} = \dfrac{14 \times (3 - \sqrt{2})}{(3 + \sqrt{2})(3 - \sqrt{2})} = \dfrac{14(3 - \sqrt{2})}{9 - 2} = 2(3 - \sqrt{2}) = 3.1716$

c) $\dfrac{\sqrt{2} - 1}{\sqrt{2} + 1} = \dfrac{(\sqrt{2} - 1)(\sqrt{2} - 1)}{(\sqrt{2} + 1)(\sqrt{2} - 1)} = \dfrac{(\sqrt{2} - 1)^2}{2 - 1} = (\sqrt{2} - 1)^2 = 0.1716$

d) $\dfrac{\sqrt{5} + 1}{\sqrt{5} - 1} = \dfrac{(\sqrt{5} + 1)^2}{(\sqrt{5} - 1)(\sqrt{5} + 1)} = \dfrac{(\sqrt{5} + 1)^2}{4} = 2.6180$

e) $\dfrac{2 + \sqrt{3}}{2 - \sqrt{3}} = \dfrac{(2 + \sqrt{3})^2}{(2 - \sqrt{3})(2 + \sqrt{3})} = \dfrac{(2 + \sqrt{3})^2}{1} = 13.9282$

f) $\dfrac{4 + \sqrt{2}}{\sqrt{2} + 1} = \dfrac{(4 + \sqrt{2})(\sqrt{2} - 1)}{(\sqrt{2} + 1)(\sqrt{2} - 1)}$
$= 4\sqrt{2} + 2 - 4 - \sqrt{2} = 3\sqrt{2} - 2 = 2.2426$

g) $\dfrac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}} = \dfrac{(\sqrt{3} + \sqrt{2})^2}{(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})} = \dfrac{(\sqrt{3} + \sqrt{2})^2}{1} = 9.8989$

Note : In each of the above examples, we made the denominator a whole number.

Suggested Quicker Method (Direct Formula) for

(i) $\dfrac{\sqrt{a} - \sqrt{b}}{\sqrt{a} + \sqrt{b}}$ and (ii) $\dfrac{\sqrt{a} + \sqrt{b}}{\sqrt{a} - \sqrt{b}}$

(i) $\dfrac{\sqrt{a} - \sqrt{b}}{\sqrt{a} + \sqrt{b}} = \dfrac{(\sqrt{a} - \sqrt{b})^2}{(\sqrt{a})^2 - (\sqrt{b})^2} = \dfrac{a + b - 2\sqrt{ab}}{a - b} = \dfrac{a + b}{a - b} - \dfrac{2\sqrt{ab}}{a - b}$

(ii) $\dfrac{\sqrt{a} + \sqrt{b}}{\sqrt{a} - \sqrt{b}} = \dfrac{(\sqrt{a} + \sqrt{b})^2}{(\sqrt{a})^2 + (\sqrt{b})^2} = \dfrac{a + b + 2\sqrt{ab}}{a - b} = \dfrac{a + b}{a - b} + \dfrac{2\sqrt{ab}}{a - b}$

If we combine both the results, we have

$$\frac{\sqrt{a} \pm \sqrt{b}}{\sqrt{a} + \sqrt{b}} = \frac{a+b}{a-b} \pm \frac{2\sqrt{ab}}{a-b} \text{ ----- } (*)$$

The above formula has the first term $\left(\frac{a+b}{a-b}\right)$ and the second term $\frac{2\sqrt{ab}}{a-b}$. Both the terms are the same for both the cases. The two terms are added when the numerator of the surd has a '+' sign and subtracted when the numerator of the surd has a '-' sign. For example:

$$\frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}} = \frac{7+5}{7-5} - \frac{2\sqrt{7\times 5}}{7-5} = 6 - \sqrt{35}$$

$$\frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}} = \frac{7+5}{7-5} + \frac{2\sqrt{7\times 5}}{7-5} = 6 + \sqrt{35}$$

Note: You are suggested to remember the direct result (*). It will save you several precious seconds in your examination.

Ex. 6: Find the value of the following expressions correct to 3 decimal places:

1) $\frac{1}{\sqrt{7}}$ 2) $\frac{1}{\sqrt{11}}$ 3) $\frac{1}{\sqrt{3}-1}$ 4) $\frac{1}{\sqrt{7}-1}$ 5) $\frac{\sqrt{2}-1}{\sqrt{2}+1}$

6) $\frac{\sqrt{5}+1}{\sqrt{5}-1}$ 7) $\frac{2+\sqrt{3}}{2-\sqrt{3}}$ 8) $\sqrt{\frac{\sqrt{5}+1}{\sqrt{5}-1}}$ 9) $\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}$

Solution:

1) $\frac{1}{\sqrt{7}} = \frac{\sqrt{7}}{7} = 0.378$

2) $\frac{1}{\sqrt{11}} = \frac{\sqrt{11}}{11} = 0.302$

3) $\frac{1}{\sqrt{3}-1} = \frac{\sqrt{3}+1}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{\sqrt{3}+1}{3-1} = \frac{\sqrt{3}+1}{2} = 1.366$

4) $\frac{1}{\sqrt{7}-1} = \frac{\sqrt{7}+1}{7-1} = \frac{\sqrt{7}+1}{6} = 0.608$

5) $\frac{\sqrt{2}-1}{\sqrt{2}+1} = \frac{(\sqrt{2}-1)^2}{2-1} = 0.172$

6) $\frac{\sqrt{5}+1}{\sqrt{5}-1} = \frac{(\sqrt{5}+1)^2}{5-1} = 2.618$

7) $\frac{2+\sqrt{3}}{2-\sqrt{3}} = \frac{(2+\sqrt{3})^2}{4-3} = 13.928$

8) $\sqrt{\frac{(\sqrt{5}+1)^2}{5-1}} = \frac{\sqrt{5}+1}{2} = 1.618$

9) $\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}} = \sqrt{\frac{(2+\sqrt{3})^2}{4-3}} = 2+\sqrt{3} = 3.732$

Ex. 7: Arrange the following in an ascending order.

$\sqrt{7}-\sqrt{5}$, $\sqrt{5}-\sqrt{3}$, $\sqrt{9}-\sqrt{7}$, $\sqrt{11}-\sqrt{9}$

Soln : $\sqrt{7}-\sqrt{5} = \frac{(\sqrt{7}-\sqrt{5})\times(\sqrt{7}+\sqrt{5})}{\sqrt{7}+\sqrt{5}} = \frac{7-5}{\sqrt{7}+\sqrt{5}} = \frac{2}{\sqrt{7}+\sqrt{5}}$

$\sqrt{5}-\sqrt{3} = \frac{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})}{\sqrt{5}+\sqrt{3}} = \frac{5-3}{\sqrt{5}+\sqrt{3}} = \frac{2}{\sqrt{5}+\sqrt{3}}$

Similarly, $\sqrt{9}-\sqrt{7} = \frac{2}{\sqrt{9}+\sqrt{7}}$ and $\sqrt{11}-\sqrt{9} = \frac{2}{\sqrt{11}+\sqrt{9}}$

We know that when the denominator is greater, the value of a fraction is lower. This way, we may say that

$$\frac{2}{\sqrt{11}+\sqrt{9}} < \frac{2}{\sqrt{9}+\sqrt{7}} < \frac{2}{\sqrt{7}+\sqrt{5}} < \frac{2}{\sqrt{5}+\sqrt{3}}$$

or, $\sqrt{11}-\sqrt{9} < \sqrt{9}-\sqrt{7} < \sqrt{7}-\sqrt{5} < \sqrt{5}-\sqrt{3}$

Note: The above example gives an important result. It should be remembered.

Number System

Quantitative Aptitude deals mainly with the different topics in Arithmetic, which is the science which deals with the relations of numbers to one another. It includes all the methods that are applicable to numbers.

Numbers are expressed by means of figures - 1,2,3,4,5,6,7,8,9 and 0 — called digits. Out of these, 0 is called *insignificant* digit whereas the others are called *significant* digits.

Numerals: A group of figures, representing a number, is called a numeral. Numbers are divided into the following types:

Natural Number: Numbers which we use for counting the objects are known as natural numbers. They are denoted by 'N'.

$$N = \{1, 2, 3, 4, 5, ...\}$$

Whole Number: When we include 'zero' in the natural numbers, it is known as whole numbers. They are denoted by 'W'.

$$W = \{0, 1, 2, 3, 4, 5, ...\}$$

Prime Number: A number other than 1 is called a prime number if it is divisible only by 1 and itself.

To test whether a given number is prime number or not

If you want to test whether any number is a prime number or not, take an integer larger than the approximate square root of that number. Let it be 'x'. Test the divisibility of the given number by every a prime number less than 'x'. If it is not divisible by any of them, then it is prime number; otherwise it is a composite number (other than prime).

Ex. 1: Is 349 a prime number ?

Soln. The square root of 349 is approximately 19. The prime numbers less than 19 are 2, 3, 5, 7, 11, 13, 17.
Clearly, 349 is not divisible by any of them. Therefore, 349 is a prime number.

Ex. 2: Is 881 a prime number?

Soln: The approximate sq. root of 881 is 30.
Prime numbers less than 30 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29.
881 is not divisible by any of the above numbers, so it is a prime number.

Ex. 3: Is 979 a prime number?

Soln: The approximate sq. root of 979 is 32.

Prime numbers less than 32 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31.
We observe that 979 is divisible by 11, so it is not a prime number.

Ex. 4: Are 857, 509, 757, 1003, 1009, 919, 913, 647, 649, 657, 659 prime numbers? (Solve it.)

Composite Numbers: A number, other than 1, which is not a prime number is called a composite number.

e.g., 4, 6, 8, 9, 12, 14.

Even Number: The number which is divisible by 2 is known as a even number.

e.g., 2, 4, 8, 12, 24, 28....

It is also of the form **2n** {where n = whole number}

Odd Number: The number which is not divisible by 2 is known as odd number.

e.g., 3, 9, 11, 17, 19, ...

Consecutive Numbers: A series of numbers in which each is greater than that which precedes it by 1, is called consecutive numbers.

e.g., 6, 7, 8 or, 13, 14, 15, 16 or, 101, 102, 103, 104

Integers: The set of numbers which consists of whole numbers and negative numbers is known as integers. It is denoted by I.

e.g., I = {-4, -3, -2, -1, 0, 1, 2, 3, 4, 5, ...}

Rational Numbers: When the numbers are written in fractions, they are known as rational numbers. They are denoted by Q.

e.g., $\frac{1}{2}$, $\frac{3}{4}$, $\frac{8}{9}$, $\frac{13}{15}$ are rational numbers.

Or, the numbers which can be written in the form $\frac{a}{b}$ {where a and b are integers and $b \neq 0$} are called rational numbers.

Irrational Numbers: The numbers which cannot be written in the form of p/q are known as irrational numbers (where p and q are integers and $q \neq 0$).

For example : $\sqrt{3} = 1.732....$, $\sqrt{2} = 1.414....$

But recurring decimals like $\frac{8}{3} = 2.666$ or $2.\bar{6}$ can be written in the $\frac{p}{q}$ form, so they are rational numbers.

Real Numbers: Real numbers include both rational as well as irrational numbers.

Rule of Simplification

(i) In simplifying an expression, first of all vinculum or bar must be removed. For example: we know that -8-10 = -18
but, $-\overline{8-10} = -(-2) = 2$

(ii) After removing the bar, the brackets must be removed, strictly in the order ( ), { } and [ ].

(iii) After removing the brackets, we must use the following operations strictly in the order given below. (a) of (b) division (c) multiplication (d) addition and (e) subtraction

Note: The rule is also known as the rule of 'VBODMAS' where V, B, O, D, M, A and S stand for Vinculum, Bracket, Of, Division, Multiplication, Addition and Subtraction respectively.

Ex.: Simplify: $1 \div \frac{3}{7} \text{ of } (6 + 8 \times \overline{3-2}) + \left[\frac{1}{5} \div \frac{7}{25} - \left\{\frac{3}{7} + \frac{8}{14}\right\}\right]$

Soln: $1 \div \frac{3}{7} \text{ of } (6 + 8 \times 1) + \left[\frac{1}{5} \div \frac{7}{25} - \frac{14}{14}\right]$

$= 1 \div \frac{3}{7} \text{ of } (6 + 8) + \left[\frac{1}{5} \times \frac{25}{7} - 1\right]$

$= 1 \div \frac{3}{7} \text{ of } 14 + \left[\frac{5}{7} - 1\right]$

$= 1 \div 6 + \left[-\frac{2}{7}\right]$

$= \frac{1}{6} - \frac{2}{7} = \frac{7-12}{42}$

$= -\frac{5}{42}$

General Rules for Solving Problems in Arithmetic

1) $(a + b)(a - b) = a^2 - b^2$

or, $\frac{a^2 - b^2}{a + b} = a - b$

or, $\frac{a^2 - b^2}{a - b} = a + b$

2) $(a + b)^2 = a^2 + 2ab + b^2$

3) $(a-b)^2 = a^2 - 2ab + b^2$

4) $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
$= a^3 + b^3 + 3ab(a+b)$

5) $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
$= a^3 - b^3 - 3ab(a-b)$

6) $\frac{a^3+b^3}{a^2-ab+b^2} = a+b$
or, $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

7) $\frac{a^3-b^3}{a^2+ab+b^2} = a-b$
or, $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

8) $\frac{a^3+b^3+c^3-3abc}{a^2+b^2+c^2-ab-bc-ca} = (a+b+c)$

9) $a^x \times a^y = a^{x+y}$

10) $a^x \div a^y = a^{x-y}$

11) $(a^x)^y = a^{xy}$

12) $a^x = b^x \Rightarrow$ either $a = b$ or, $x = 0$

13) $a^x = a^y \Rightarrow$ either $x = y$ or, $a = 0,1$

14) $a^x = 1$, then x is 0 for all values of a (except 0).

Ascending or Descending Orders in Rational Numbers

Under this chapter, we shall first learn to compare two fractions.

Rule 1: When the numerator and the denominator of the fractions increase by a constant value, the last fraction is the biggest.

Ex. 1: Which one of the following fractions is the greatest?

$$\frac{3}{4}, \frac{4}{5} \text{ and } \frac{5}{6}$$

Soln: We see that the numerators as well as denominators of the above fractions increase by 1, so the last fraction, i.e., $\frac{5}{6}$, is the greatest fraction.

Ex. 2: Which one of the f' 'iowing fractions is the greatest?

$$\frac{2}{5}\ \frac{4}{7} \text{ and } \frac{6}{9}$$

Soln: We see that the numerators as well as the denominators of the above fractions increase by 2, so the last fraction, ie, $\frac{6}{9}$, is the greatest fraction.

Ex. 3: Which one of the following fractions is the greatest?

$$\frac{1}{8}, \frac{4}{9}, \frac{7}{10}$$

Soln: We see that the, numerator increases by 3 (a constant value) and the denominator also increases by a constant value (1), so the last fraction, ie $\frac{7}{10}$, is the greatest fraction.

Thus, a generalised form can be seen as

> In the group of fractions
> $\frac{x}{y}, \frac{x+a}{y+b}, \frac{x+2a}{y+2b}, \frac{x+3a}{y+3b}, \ldots\ldots, \frac{x+na}{y+nb}$
> $\frac{x+na}{y+nb}$ has the highest value.
> where i) $a = b$ or, ii) $a > b$

But what happens when $a < b$? See in the following examples

Ex. 4: Which one of the following is the greatest?

$$\frac{1}{8}, \frac{2}{12}, \frac{3}{16}, \frac{4}{20}$$

Soln: In the above example, we see that the numerators and the denominators of the fractions increase by constant values (the numerators by 1 and denominators by 4). If we change the above fractions into decimal values, we see that the fractions are in an increasing order and hence, the last fraction, i.e., $\frac{4}{20}$ is the greatest.

Ex. 5: Which one of the following fractions is the greatest?

$$\frac{2}{7}, \frac{4}{15} \text{ and } \frac{6}{23}$$

Soln: In the above example also, we see that the numerators and denominators increase by 2 and 8 respectively. But the last fraction, i.e., $\frac{6}{23}$ is the least.

Note: In Ex. 4; $\frac{\text{Increase in Num.}}{\text{Increase in Den.}} = \frac{1}{4}$ is greater than the first fraction $\frac{1}{8}$.

In Ex. 5; $\frac{\text{Increase in Num.}}{\text{Increase in Den.}} = \frac{2}{8}$ is less than the first fraction $\frac{2}{7}$

In such a case when a < b we have the following relations:

1. If $\frac{\text{Increase in Num.}}{\text{Increase in Den.}}$ > First fraction, the last value is the greatest.
2. If $\frac{\text{Increase in Num.}}{\text{Increase in Den.}}$ < First fraction, the last value is the least.
3. If $\frac{\text{Increase in Num.}}{\text{Increase in Den.}}$ = First fraction, all the values are equal.

Rule 2: The fraction whose numerator after cross-mutliplication gives the greater value is greater.

Ex. 6: Which is greater : $\frac{5}{8}$ or $\frac{9}{14}$?

Soln: Students generally solve this question by changing the fraction into decimal values or by equating the denominators. But we suggest you a better method for getting the answer more quickly.

Step I. Cross-multiply the two given fractions.

$$\frac{5}{8} \times \frac{9}{14}$$

we have, $5\times14 = 70$ and $8\times9 = 72$

Step II: As 72 is greater than 70 and the numerator involved with the greater value is 9, the fraction $\frac{9}{14}$ is the greater of the two.

Ex. 7: Which is greater : $\frac{4}{15}$ or $\frac{6}{23}$?

Soln: Step I : $4 \times 23 > 15 \times 6$

Step II: As the greater value has the numerator 4 involved with it, $\frac{4}{15}$ is greater.

Ex. 8: Which is greater: $\frac{13}{15}$ or $\frac{20}{23}$?

Soln: **Step I:** $13 \times 23 < 15 \times 20$

Step II: $\frac{20}{23}$ is greater.

You can see how quickly this method works. After good practice, you won't need to calculate before answering the question.

The arrangement of fractions into the ascending or descending order becomes easier now. Choose two fractions at a time. See which one is greater. This way you may get a quick arrangement of fractions.

Note: Sometimes, when the values are smaller (i.e., less than 10), the conventional method, i.e., changing the values into decimals or equating the denominators after getting LCM, will prove more convenient for some of you.

Ex. 9: Arrange the following in ascending order.

$$\frac{3}{7}, \frac{4}{5}, \frac{7}{9}, \frac{1}{2} \text{ and } \frac{3}{5}$$

Soln: Method I

The LCM of 7, 5, 9, 2, 5 is 630.

Now, to equate the denominators, we divide the LCM by the denominators and multiply the quotient by the respective numerators.

Like, for $\frac{3}{7}$, $630 \div 7 = 90$ so, multiply 3 by 90.

Thus, the fractions change to

$\frac{270}{630}, \frac{504}{630}, \frac{490}{630}, \frac{315}{630}$ and $\frac{378}{630}$.

The fraction which has larger numerator is naturally larger. So,

$$\frac{504}{630} > \frac{490}{630} > \frac{378}{630} > \frac{315}{630} > \frac{270}{630}$$

$$\text{or } \frac{4}{5} > \frac{7}{9} > \frac{3}{5} > \frac{1}{2} > \frac{3}{7}$$

Method II

Change the fractions into decimals like

$\frac{3}{7} = 0.428$, $\frac{4}{5} = 0.8$, $\frac{7}{9} = 0.777$, $\frac{1}{2} = 0.5$, $\frac{3}{5} = 0.6$

Clearly, $\frac{4}{5} > \frac{7}{9} > \frac{3}{5} > \frac{1}{2} > \frac{3}{7}$

Method III

Rule of CM (cross-multiplication)

Step I: Take the first two fractions. Find the greater one by the rule of CM.

$$\frac{3}{7} \times \frac{4}{5}$$

$3 \times 5 < 7 \times 4$

$\therefore \frac{4}{5} > \frac{3}{7}$

Step II : Take the third fraction. Apply CM with the third fraction and the larger value obtained in Step I.

$$\frac{4}{5} \times \frac{7}{9}$$

$$4 \times 9 > 5 \times 7$$

$$\therefore \frac{4}{5} > \frac{7}{9}$$

Now we see that $\frac{7}{9}$ can lie after $\frac{3}{7}$ or between $\frac{4}{5}$ and $\frac{3}{7}$. Therefore, we apply CM with $\frac{3}{7}$ and $\frac{7}{9}$ and see that $\frac{7}{9} > \frac{3}{7}$.

$$\therefore \frac{4}{5} > \frac{7}{9} > \frac{3}{7}$$

Step III: Take the next fraction. Apply CM with $\frac{3}{7}$ and $\frac{1}{2}$ and see that $\frac{1}{2} > \frac{3}{7}$. Next, we apply CM with $\frac{7}{9}$ and $\frac{1}{2}$ and see that $\frac{7}{9} > \frac{1}{2}$. Therefore,

$$\frac{4}{5} > \frac{7}{9} > \frac{3}{5} > \frac{1}{2} > \frac{3}{7}$$

Step IV: With similar applications, we get the final result as :

$$\frac{4}{5} > \frac{7}{9} > \frac{3}{5} > \frac{1}{2} > \frac{3}{7}$$

Note : This rule has some disadvantages also. But if you act fast, it gives faster results. Don't reject this method at once. This can prove to be the better method for you.

Some Rules on Counting Numbers

I. Sum of all the first n natural numbers $= \frac{n(n+1)}{2}$

For example: $1+2+3+....+105 = \frac{105\ (105+1)}{2} = 5565$

II. Sum of first n odd numbers $= n^2$

For example: $1+3+5+7 = 4^2 = 16$ (as there are four odd numbers).

For example: 1+3+5+.....+20th odd number (ie, $20\times2-1 = 39$) $= 20^2 = 400$

III. Sum of first n even numbers $= n(n+1)$

For example: 2+4+6+8+.......+100 (or 50th even number) $= 50 \times (50+1) = 2550$

IV. Sum of squares of first n natural numbers $= \frac{n(n+1)(2n+1)}{6}$

For example: $1^2 + 2^2 + 3^2 + ... + 10^2 = \frac{10(10+1)(2\times10+1)}{6}$

$$= \frac{10 \times 11 \times 21}{6} = 385$$

V. Sum of cubes of first n natural numbers $= \left[\frac{n(n+1)}{2}\right]^2$

For example: $1^3 + 2^3 + ... + 6^3 = \left[\frac{6\times(6+1)}{2}\right]^2 = (21)^2 = 441$

Note: 1. In the first n counting numbers, there are $\frac{n}{2}$ odd and $\frac{n}{2}$ even numbers provided n, the number of numbers, is even. If n, the number of numbers, is odd, then there are $\frac{1}{2}(n+1)$ odd numbers and $\frac{1}{2}(n-1)$ even numbers.

For example, from 1 to 50, there are $\frac{50}{2} = 25$ odd numbers and $\frac{50}{2} = 25$ even numbers. And from 1 to 51, there are $\frac{51+1}{2} = 26$ odd numbers and $\frac{51-1}{2} = 25$ even numbers.

2. The difference between the squares of two consecutive numbers is always an odd number.

Ex. 1: 16 and 25 are squares of 4 and 5 respectively (two consecutive numbers). 25 - 16 = 9 an odd number.

Ex. 2: Difference between $(26)^2$ and $(25)^2 = 51$ (an odd number)

Reasoning: Derived from the above rule II.

3. The difference between the squares of two consecutive numbers is the sum of the two consecutive numbers.

Ex. 1: $5^2 - 4^2 = 5 + 4 = 9$

Ex. 2: $(26)^2 - (25)^2 = 26 + 25 = 51$

Reasoning: $a^2 - b^2 = (a-b)(a+b) = (a+b) \quad \because a-b = 1$

Solved Examples

(1) What is the total of all the even numbers from 1 to 400?

Soln: From 1 to 400, there are 400 numbers.

So, there are $\frac{400}{2} = 200$ even numbers.

Hence, sum = 200 (200+1) = 40200 [From Rule III]

(2) What is the total of all the even numbers from 1 to 361?

Soln: From 1 to 361, there are 361 numbers; so there are

$$\frac{361-1}{2} = \frac{360}{2} = 180 \text{ even numbers.}$$

Thus, sum = 180 (180 + 1) = 32580

(3) What is the total of all the odd numbers from 1 to 180?

Soln: There are $\frac{180}{2} = 90$ odd numbers between the given range. So, the sum = $(90)^2 = 8100$

(4) What is the total of all the odd numbers from 1 to 51?

Soln: There are $\frac{51+1}{2} = 26$ odd numbers between the given range. So, the sum = $(26)^2 = 676$

(5) Find the sum of all the odd numbers from 20 to 101.

Soln: The required sum = Sum of all the odd numbers from 1 to 101 – sum of all the odd numbers from 1 to 20

= Sum of first 51 odd numbers - sum of first 10 odd numbers

= $(51)^2 - (10)^2$

= 2601 - 100 = 2501

Power and Index

If a number 'p' is multiplied by itself n times, the product is called nth power of 'p' and is written as p^n. In p^n, p is called the **base** and n is called the **index** of the power.

Some solved examples

(1) What is the number in the unit place in $(729)^{59}$?

Soln: When 729 is multiplied twice, the number in the unit place is 1. In other words, if 729 is multiplied an even number of times, the number in the unit place will be 1. Thus, the number in the unit place in $(729)^{58}$ is 1.

$\therefore\ (729)^{59} = (729)^{58} \times (729) = (.....1) \times (729) = 9$ in the unit place.

(2) Find the number in the unit place in

$(623)^{36}$, $(623)^{38}$ and $(623)^{39}$.

Soln: When 623 is multiplied twice, the number in the unit place is 9. When it is multiplied 4 times, the number in the unit place is 1. Thus, we say that if 623 is multiplied 4n number of times, the number in the unit place will be 1. So,

$(623)^{36} = (623)^{4\times 9} = 1$ in the unit place

$(623)^{38} = (623)^{4\times 9} \times (623)^2 = (...1) \times (....9) = 9$ in the unit place

$(623)^{39} = (623)^{4\times 9} \times (623)^3 = (.....1) \times (......7) = 7$ in the unit place

Note: When you solve this type of questions (<u>for odd numbers</u>) try to get the last digit 1, as has been done in the above two examples.

(3) Find the number in the unit place in $(122)^{20}$, $(122)^{22}$ and $(122)^{23}$.

Soln: $(....2)\times(....2) = ...4$

$(....2)\times(....2)\times(....2) = ...8$

$(....2)\times(....2)\times(....2)\times(2....) = ...6$

We know that $(....6)\times(....6) = (....6)$

Thus, when (122) is multiplied 4n times, the last digit is 6. Therefore,

$(122)^{20} = (122)^{4\times 5} = (....6) = 6$ in the unit place

$(122)^{22} = (122)^{4\times 5} \times (122)^2 = (....6)\times(....4) = 4$ in the unit place.

$(122)^{23} = (122)^{4\times 5} \times (122)^3 = (....6)\times(....8) = 8$ in the unit place.

(4) Find the number in the unit place in $(98)^{40}$, $(98)^{42}$ and $(98)^{43}$

Soln: $(98)^4 = (...6)$

$\therefore\ (98)^{4n} = (....6)$

Thus, $(98)^{40} = (98)^{4\times 10} = (....6) = 6$ in the unit place

$(98)^{42} = (98)^{4\times 10} \times (98)^2 = (....6)\times(....4) = 4$ in the unit place.

$(98)^{43} = (98)^{4\times 10} \times (98)^3 = (....6)\times(....2) = 2$ in the unit place

Note: When there is an <u>even number</u> in the unit place of base, try to get 6 in the unit place, as has been done in the above two questions.

This chapter should be concluded with some general rules derived for this type of questions.

Rule I: For odd numbers

When there is an odd digit in the unit place (except 5), multiply the number by itself until you get 1 in the unit place.

$(....1)^n = (....1)$

$(....3)^{4n} = (....1)$

$(....7)^{4n} = (....1)$
where n=1,2,3,....

Rule 2: For even numbers

When there is an even digit in the unit place, multiply the number by itself until you get 6 in the unit place.

$(....2)^{4n} = (....6)$
$(....4)^{2n} = (....6)$
$(....6)^{n} = (....6)$
$(....8)^{4n} = (....6)$, where n =1, 2, 3, ...

Note: If there is 1, 5 or 6 in the unit place of the given number, then after any times of its multiplication, it will have the same digit in the unit place, i.e.,

$(....1)^{n} = (....1)$
$(..5)^{n} = (....5)$
$(...6)^{n} = (....6)$

(5) What is the number in the unit place when 781, 325, 497 and 243 are multiplied together?

Soln: Multiply all the numbers in the unit place, i.e., $1\times5\times7\times3$; the result is a number in which 5 is in the unit place.

Miscellaneous Examples

In a division sum, we have four quantities — Dividend, Divisor, Quotient and Remainder. These are connected by the relation

Dividend = (Divisor × Quotient) + Remainder

When the division is exact, the remainder is zero (0).
In this case, the above relation becomes

Dividend = Divisor × Quotient

Ex. 1: The quotient arising from the division of 24446 by a certain number is 79 and the remainder is 35; what is the divisor?

Soln: Divisor × Quotient = Dividend - Remainder
$\therefore$ 79 × Divisor = 24446 - 35 = 24411
$\therefore$ Divisor = 24411 ÷ 79 = 309.

Ex. 2: What least number must be added to 8961 to make it exactly divisible by 84?

Soln: On dividing 8961 by 84, we get 57 as the remainder.
$\therefore$ the number to be added = 84 - 57 = 27

Ex. 3: What least number must be subtracted from 8961 to make it exactly divisible by 84?

Soln: On dividing 8961 by 84, we get 57 as the remainder. Therefore, the number to be subtracted is 57.

Note: In Ex 2, we see that the given number needs 27 to make it exactly divisible by 84. But in Ex 3, the given number exceeds by 57.

Ex. 4: Find the least number of 5 digits which is exactly divisible by 89.

Soln: The least number of 5 digits is 10,000.
On dividing 10,000 by 89 we get 32 as remainder.
$\therefore$ if we add (89 - 32) or 57 to 10,000, the sum will be divisible by 89.
$\therefore$ the required number = 10,000 + 57 = 10,057.

Ex. 5: Find the greatest number of 5 digits which is exactly divisible by 137.

Soln: The greatest number of five digits is 99,999. On dividing 99,999 by 137, we get 126 as remainder.
$\therefore$ the required number = 99,999 - 126 = 99873

Note : Do you find the difference between Ex. 4 and Ex. 5 ?

Ex. 6: Find the nearest integer to 1834 which is exactly divisible by 12.

Soln: On dividing 1834 by 12, we get 10 as the remainder.
Since the remainder 10 is more than the half of the divisor 12, the nearest integer will be found by adding (12-10) =2
Thus, the required number = 1834+(12-10) =1836

Ex. 7: Find the nearest integer to 1829 which is exactly divisible by 12.

Soln: On dividing 1829 by 12, we get 5 as the remainder. Since the remainder 5 is less than half of the divisor 12, the nearest integer will be found by subtracting 5 from 1829.
$\therefore$ the required number = 1829 - 5 = 1824.

Note : Do you realize the difference between Ex. 6 and Ex. 7?

Ex. 8: A number when divided by 899 gives a remainder 63. What remainder will be obtained by dividing the same number by 29?

Soln: Number = 899 × Quotient + 63 = 29×31 × Quotient + 2×29+5
Therefore, the remainder obtained by dividing the number by 29 is clearly 5.

Ex. 9: A number when divided by 899 gives a remainder 62. What remainder will be obtained by dividing the same number by 31?

Soln: Number = 899 × Quotient + 62 = 31×29 × Quotient + 31×2+0
Therefore, the remainder obtained by dividing the number by 31 is clearly 0.

Note: From Ex. (8) and (9) it is clear that the first divisor must be a multiple of the second divisor. But what happens when the first

divisor is not a multiple of second divisor? See in the following examples.

Ex. 10: A number when divided by 12 leaves a remainder 7. What remainder will be obtained by dividing the same number by 7?

Soln: We see that in the above example, the first divisor 12 is not a multiple of the second divisor 7. Now, we take the two numbers 139 and 151, which when divided by 12, leave 7 as the remainder. But when we divide the above two numbers by 7, we get the respective remainders as 6 and 4. Thus, we conclude that the question is wrong.

Ex. 11. A boy was set to multiply 432051 by 56827, but reading one of the figures in the question erroneously, he obtained 21959856177 as his answer. Which figure did he mistake?

Soln: On dividing we find that 56827 does not exactly divide 21959856177. Hence the error was in reading 56827. But on dividing the number by 432051, we get 50827. Hence the boy read the figure 6 of the multiplier as 0.

Ex. 12: A boy multiplied 423 by a certain number and obtained 65589 as his answer. If both the fives are wrong, but the other figures are right, find the correct answer.

Soln: Step I: In the product, the figures 9,8 and 6 are correct. To get 9 in the unit place, we must multiply by 3 units. So,

$$\begin{array}{r} 423 \\ **3 \\ \hline 1269 \\ ---- \\ ---- \\ \hline 6**89 \end{array}$$

Step II: To get 8 in the ten's place, we must have 2 under 6 in the first line. Hence, we must multiply by 4 tens. So,

$$\begin{array}{r} 423 \\ *43 \\ \hline 1269 \\ 1692 \\ ---- \\ \hline 6**89 \end{array}$$

Step III: To get 6 in the product we must have 4 under 1 in the second line. Hence, we must multiply by 1 hundred. So,

$$\begin{array}{r} 423 \\ 143 \\ \hline 1269 \\ 1692 \\ 423 \\ \hline 60489 \end{array}$$

Thus, the correct answer is 60489.

Ex. 13: How many prime numbers exist in $6^7 \times 35^3 \times 11^{10}$?

Soln: $6^7 \times 35^3 \times 11^{10} = (2 \times 3)^7 \times (5 \times 7)^3 \times 11^{10}$

$= 2^7 \times 3^7 \times 5^3 \times 7^3 \times 11^{10}$

Thus, there are 7+7+3+3+10 =30 prime numbers.

Ex. 14: On dividing a number by 5, 7 and 8 successively the remainders are respectively 2, 3 and 4. What will be the remainders if the order of division is reversed?

Soln:

$$\begin{array}{r|l|l} 5 & *** & \\ \hline 7 & ** & 2 \\ \hline 8 & * & 3 \\ \hline & 1 & 4 \end{array}$$

We have $* = 8 \times 1 + 4 = 12$

$** = 7 \times 12 + 3 = 87$

$*** = 5 \times 87 + 2 = 437$

Thus, the number **may be** 437.

Now, when order of division is reversed,

$$\begin{array}{r|l|l} 8 & 437 & \\ \hline 7 & 54 & 5 \\ \hline 5 & 7 & 5 \\ \hline & 1 & 2 \end{array}$$

Hence, the required remainders will be 5, 5 and 2.

Note: We have used the words **"may be"** because this number is one of those many numbers which satisfy the conditions. We have used 1 as our final quotient and hence, got the number as 437. But for the other values, like 2, 3, ... the numbers will be different. And surprisingly, all of them give the same result. You may verify it for yourself!

Ex. 15: A watch ticks 90 times in 95 seconds and another watch tick 315 times in 323 seconds. If both the watches are started together, how many times will they tick together in the first hour?

Soln: The first watch ticks every $\frac{95}{90}$ seconds and the second watch ticks every $\frac{323}{315}$ seconds.

They will tick together after (LCM of $\frac{95}{90}$ and $\frac{323}{315}$) seconds.

Now, LCM of $\frac{95}{90}$ and $\frac{323}{315} = \frac{\text{LCM of } 95, 323}{\text{HCF of } 90, 315} = \frac{19 \times 5 \times 17}{45}$

The number of times they will tick in the first 3600 seconds

$= 3600 \div \frac{19 \times 5 \times 17}{45}$

$= \frac{3600 \times 45}{19 \times 5 \times 17} = 100\frac{100}{323}$

Once they have already ticked in the beginning; so in 1 hour they will tick 100 + 1 = 101 times

Ex. 16: By what number less than 1000 must 43259 be multiplied so that the last three figures to the right of the product may be 437?

Soln: It is clear that the required number is of three digits. Let that number be abc. Then we have,

```
  4 3 2 5 9
      a b c
-----------
  -------- 7
 --------
--------
-----------
      4 3 7
```

It is clear that c = 3; thus,

```
  4 3 2 5 9
      a b 3
-----------
  ----7 7 7
 --------
--------
-----------
      4 3 7
```


Now, we see that the second digit (from right) of product is 3. This is possible only when 7 is added to 6. And for this 'b' must be equal to 4. Thus,

```
  4 3 2 5 9
      a 4 3
-----------
      7 7 7
      3 6
    ---
-----------
      4 3 7
```

Now, for the third digit (4), we see that the third column has 1 (carried) + 7 + 3 = 11 and it needs 3 more. This is possible only when 'a' is equal to 7. Thus, we finally see

```
  4 3 2 5 9
      7 4 3
-----------
  ---- 7 7 7
  ---- 3 6
  ---- 3
-----------
      4 3 7
```

Hence, the required number is 743.

Note: The above question is given in detail so that you can grasp each step, but once you understand the method, you can solve it in a single step. The only thing which should be kept in mind is that when the last three digits of the product are given, you should not calculate beyond the third column for any row.

Ex.17: Find the least number by which 19404 must be multiplied or divided so as to make it a perfect square?

Soln: $19404 = 2\times2\times3\times3\times7\times7\times11 = 2^2 \times 3^2 \times 7^2 \times 11$

Thus, if the number is multiplied or divided by 11, the resultant number will be a perfect square. Therefore, the required number is 11.

Ex.18: Fill in the blank indicated by a star in the number 4 * 56 so as to make it divisible by 33.

Soln: The number should be divisible by 3 and 11. To make the number divisible by 3, as the digit-sum should be divisible by 3, we may put * = 0, 3, 6, or 9. We also know that a number is divisible by 11 if the sums of alternate digits differ by either 0 or a number divisible by 11. We have

$S_1 = 4 + 5 = 9$ (sum of digits at odd places)

Thus, S_2 should be 9 and hence * = 3.

Note: S_2 cannot be 9 + 11x (ie, 20, 31, 42,....) because in that case * becomes a double-digit number.

Theorem: *When two numbers, after being divided by a third number leave the same remainder, the difference of those two numbers must be perfectly divisible by the third number.*

Proof: Let two such numbers be A and B; and the divisor (the third number) and the remainder be x and y respectively. Then we have

$nx + y = A$ ------- (1)

$mx + y = B$ -------(2)

Subtracting (2) from (1), we get.

$x(n - m) = A - B$

Thus, A - B is perfectly divisible by x.

Ex. 19: 24345 and 33334 are divided by a certain number of three digits and the remainder is the same in both the cases. Find the divisor and the remainder.

Soln: By the above theorem, the difference of 24345 and 33334 must be perfectly divisible by the divisor. We have the difference

$= 33334 - 24345 = 8989 = 101 \times 89$

Thus, the three-digit number is 101.

The remainder can be obtained by dividing one of the numbers by 101. If we divide 24345 by 101, the remainder is 4.

Ex. 20: 451 and 607 are divided by a number and we get the same remainder in both the cases. Find all the possible divisors (other than 1).

Soln: By the above theorem;

607 - 451 = 156 is perfectly divisible by those numbers (divisors).

Now, $156 = 2 \times 2 \times 3 \times 13$

Thus, 1-digit numbers = 2, 3, 2×2, 2×3 = 2, 3, 4, 6

2-digit numbers = 12, 13, 26, 39, 52, 78

3-digit number = 156

Ex. 21: The sum of two numbers is 14 and their difference is 10. Find the product of the two numbers.

Soln: Let the two numbers be x and y, then, $x + y = 14$ & $x - y = 10$

Now, we have, $(x+y)^2 = (x-y)^2 + 4xy$

or, $(14)^2 = (10)^2 + 4xy$

$\therefore\ xy = \frac{(14)^2 - (10)^2}{4} = \frac{96}{4} = 24$

Direct Formula:

$$\text{Product} = \frac{(\text{Sum} + \text{Difference})(\text{Sum} - \text{Difference})}{4}$$

$$= \frac{(14+10)(14-10)}{4} = 24$$

Note: The numbers can be found by the direct formula

$$x = \frac{\text{Sum} + \text{Difference}}{2} = \frac{14+10}{2} = 12$$

$$y = \frac{\text{Sum} - \text{Difference}}{2} = \frac{14-10}{2} = 2$$

Ex. 22: The sum of two numbers is twice their difference. If one of the numbers is 10, find the other number.

Soln: Let the numbers be x and y.

From the question, we have

$x + y = 2(x - y)$

or, $x = 3y$ we are given, $y = 10$

$\therefore$ the other number, $x = 3 \times 10 = 30$

Ex. 23: Two numbers are said to be in the ratio 3 : 5. If 9 be subtracted from each, they are in the ratio of 12 : 23. Find the numbers.

Soln: Let the numbers be 3x and 5x. Then, by the question

$\frac{3x-9}{5x-9} = \frac{12}{23}$

or, $69x - 9 \times 23 = 60x - 12 \times 9$

or, $9x = 207 - 108 = 99$

$\therefore\ x = 11$

or, $3x = 33$ & $5x = 55$

Therefore, the numbers are 33 and 55.

Ex.24: A boy was asked to find $\frac{7}{9}$ of a fraction. He made a mistake of dividing the fraction by $\frac{7}{9}$ and so got an answer which exceeded the correct answer by $\frac{8}{21}$. Find the correct answer.

Soln: Let the fraction be x. Then,

$x \div \frac{7}{9} - x \text{ of } \frac{7}{9} = \frac{8}{21}$

or, $\frac{9x}{7} - \frac{7x}{9} = \frac{8}{21}$

or, $\frac{x(81-49)}{7 \times 9} = \frac{8}{21}$

$\therefore$ $x = \frac{8}{21}\left(\frac{7 \times 9}{32}\right) = \frac{3}{4}$

$\therefore$ The correct answer $= \frac{3}{4} \times \frac{7}{9} = \frac{7}{12}$.

Ex. 25: Four-fifths of a number is more than three-fourths of the number by 4. Find the number.

Soln: $\frac{4}{5} - \frac{3}{4} = 4$

or, $\frac{1}{20} = 4$

or, $1 = 20 \times 4 = 80$

Therefore, the required number is 80.

Ex. 26: If one-fifth of one-third of one-half of number is 15, find the number.

Soln: Let the number be x. Then we have,

$x\left(\frac{1}{5}\right)\left(\frac{1}{3}\right)\left(\frac{1}{2}\right) = 15$

$\therefore$ $x = 15 \times 5 \times 3 \times 2 = 450$

Direct Method:

(*) The required number $= 15\left(\frac{5}{1}\right)\left(\frac{3}{1}\right)\left(\frac{2}{1}\right) = 450$

Note: (*) The resultant should be multiplied by the reverse of each fraction.

Ex. 27: If the numerator of a fraction be increased by 12% and its denominator decreased by 2%, the value of the fraction becomes $\frac{6}{7}$. Find the original fraction.

Soln: Let the fraction be $\frac{x}{y}$. Then we have, $\frac{112\% \text{ of } x}{98\% \text{ or } y} = \frac{6}{7}$

$\therefore$ $\frac{x}{y} = \frac{98\% \text{ of } 6}{112\% \text{ of } 7} = \frac{98 \times 6}{112 \times 7} = \frac{3}{4}$

Ex. 28: The sum of the digits of a two-digit number is 8. If the digits are reversed, the number is decreased by 54. Find the number.

Soln: Let the two-digit number be $10x + y$.

Then, we have; $x + y = 8$ ----- (1) and

$10y + x = 10x + y - 54$

or, $x - y = \frac{54}{9} = 6$ ----- (2)

From equations (1) and (2)

$x = \frac{8+6}{2} = 7$ and $y = 1$

$\therefore$ the required number $= 7 \times 10 + 1 = 71$

Direct Formula:

The required number =

$5\left[\text{Sum of digits} + \frac{\text{Decrease}}{9}\right] + \frac{1}{2}\left[\text{Sum of digits} + \frac{\text{Decrease}}{9}\right]$

$= 5(8+6) + \frac{1}{2}(8-6) = 70 + 1 = 71$

Ex. 29: Three numbers are in the ratio of 3 : 4 : 5. The sum of the largest and the smallest equals the sum of the third and 52. Find the smallest number.

Soln: From the question, it is clear that the sum of the largest and the smallest is 52 more than the third. Thus we have,

$3 + 5 - 4 \rightarrow 52$

$4 \rightarrow 52$

$\therefore$ $1 \rightarrow 13$

Therefore, the smallest number is $3 \times 13 = 39$

Ex. 30: If 40% of a number is 360, what will be 15% of 15% of that number?

Soln: Let the number be x. Then we have

40% of x = 360

$\therefore$ $x = \frac{360 \times 100}{40} = 900$

Now, 15% of $x = \frac{15}{100} \times 900 = 135$

Again, 15% of $135 = \frac{15}{100} \times 135 = 20.25$

Direct Method:

40% = 360

$\therefore$ 15% of 15% $= \frac{360}{40\%} \times 15\%$ of 15%

$= \frac{360}{40} \times 15$ of 15% $= \frac{360}{40} \times \frac{15 \times 15}{100} = 20.25$

Ex. 31: The ratio of the sum and the difference of two numbers is 7 : 1. Find the ratio of those two numbers.

Soln: Let the two numbers be x and y. Then we have

$$\frac{x+y}{x-y} = \frac{7}{1}$$

$\Rightarrow x + y = 7x - 7y$

or, $6x = 8y \quad \therefore \frac{x}{y} = \frac{8}{6} = \frac{4}{3} = 4:3$

Quicker Method: (Using the rule of componendo-dividendo)

$$\frac{x+y}{x-y} = \frac{7}{1}$$

or, $\frac{(x+y)+(x-y)}{(x+y)-(x-y)} = \frac{7+1}{7-1}$ or, $\frac{2x}{2y} = \frac{8}{6} \quad \therefore \frac{x}{y} = \frac{4}{3} = 4:3$

Ex. 32: The difference between a two-digit number and the number obtained by interchanging the digits is 27. What are the sum and the difference of the two digits of the number?

Soln: Let the number by $10x + y$. Then we have

$$(10x + y) - (10y + x) = 27 \text{ or, } 9(x - y) = 27 \quad \therefore x - y = \frac{27}{9} = 3$$

Thus, the difference is 3, but we cannot get the sum of two digits.

Direct Formula:

$$\text{Difference of two digits} = \frac{\text{Diff. in original \& interchanged number}}{9}$$

$$= \frac{27}{9} = 3$$

Ex. 33: The digit at the unit's place of a 2-digit number is increased by 50%. And the digit at the ten's place of the same number is increased by 100%. Now, we find that the new number is 33 more than the original number. Find the original number.

Soln: Let the number be $10x + y$.

The new number is $10\left[x\left(\frac{200}{100}\right)\right] + y\left(\frac{150}{100}\right) = 20x + 1.5y$

Now, by the question, $(20x + 1.5y) - (10x + y) = 33$

or, $10x + 0.5y = 33 = 10 \times 3 + 3 \quad \therefore x = 3 \ \& \ y = 6$

Therefore, the number is 36.

Quicker Method: For the unit's place, we see that $50\% = 3$

$100\% = 6$

For the ten's place, we see that $100\% = 3 \quad \therefore$ the number is 36.

Ex. 34: It is given that $2^{32} + 1$ is exactly divisible by a certain number. Which one of the following is also divisible by the same number?

1) $2^{96} + 1$ 2) $2^{16} - 1$ 3) $2^{16} + 1$
4) 7×2^{33} 5) None of these

Soln: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$$2^{96} + 1 = \left(2^{32}\right)^3 + (1)^3 = (2^{32} + 1)\{(2^{32})^2 - 1 \times 2^{32} + 1\}$$

Thus $2^{96} + 1$ is divisible by $(2^{32} + 1)$ and hence also divisible by that number.

Ex. 35: When a certain number is divided by 223, the remainder is 79. When that number is divided by 179, the quotient is 315; then what will be the remainder?

Soln: When $179 \times 315 = 56385$ is divided by 223, we have the remainder 189. Therefore the required remainder will be $79 + (223 - 189) = 79 + 34 = 113$.

Ex. 36: The first 100 multiples of 10, i.e., 10, 20, 30, ... 1000 are multiplied together. How many zeroes will be there at the end of the product?

Soln : We are not able to find any quicker method for this question. We suggest that you understand the working of its detailed solution.

Total zeroes at the end are produced due to three factors:

1) Due to zeroes at the end of numbers.
2) Due to 5 at the ten-place of numbers followed by zero at the unit place.
3) Due to 5 at the hundred-place of numbers followed by zeroes at the end.

Due to factor (1) :

Number of numbers with single zero = 90
Number of numbers with double zeroes = 9
Number of numbers with triple zeroes = 1
Thus, the total no. of zeroes due to this factor $= 90 \times 1 + 9 \times 2 + 1 \times 3 = 111$

Due to factor (2):

When 5 is multiplied by any even number, a zero is produced. There are 10 numbers which have 5 at the ten-place followed by zero (like 50, 150, 250, ..., 950).

Thus, 10 zeroes are produced due to factor (2).

Due to factor (3) : Following the same reasoning, we may say that 500×200 will produce an extra zero.

Thus, there are total $111 + 10 + 1 = 122$ zeroes.

Ex. 37: The average of 7 consecutive integers is 7. Find the average of the squares of these integers.

Soln: Use the formula: [for odd number of consecutive integers]

Average of Squares

$$= \frac{1}{\text{No. of integers}} \times \left[\frac{n_1(n_1+1)(2n_1+1)}{6} - \frac{n_2(n_2+1)(2n_2+1)}{6}\right]$$

Where, $n_1 = \text{Average} + \frac{\text{No. of integers} - 1}{2}$

and $n_2 = \text{Average} - \frac{\text{No. of integers} + 1}{2}$

In the above case

$n_1 = 7 + \frac{7-1}{2} = 10$

$n_2 = 7 - \frac{7+1}{2} = 3$

$\therefore$ Average of squares $= \frac{1}{7}\left[\frac{10 \times 11 \times 21}{6} - \frac{3(4)(7)}{6}\right]$

$= \frac{1}{7}[385 - 14] = \frac{371}{7} = 53.$

Ex. 38: Find the least number which, when divided by 9, 11, and 13 leaves 1, 2 and 3 as the respective remainders.

Soln : In fact, there is no possible method for this question. The only thing you can do is to apply the hit-and-trial method by taking the choices one by one.

There may be such a number but it is impossible to find it. However, this is not so in all the cases. See the following example.

"Find the least number which when divided by 9, 11 and 13 leaves 1, 3, 5 as the respective remainders."

We see that $9 - 1 = 11 - 3 = 13 - 5 = 8$.

Now, we have an established method for this question.

LCM of 9, 11, and 13 = 1287

$\therefore$ the required least number $= 1287 - 8 = 1279$.

Note : 1. Find the least number which, when divided by 13, 15, and 19, leaves the remainders 2, 4 and 8 respectively. Can we find the specific solution?

Soln : YES. This question can be solved because

$13 - 2 = 15 - 4 = 19 - 8 = 11$

Now, LCM of 13, 15, 19 = 3705

$\therefore$ the required least number $= 3705 - 11 = 3694$.

2. Find the least number which, when divided by 13, 15 and 19, leaves the remainders 1, 2, 3 respectively. Can we find the solution?

Soln : No. But why? Because $13 - 1 \neq 15 - 2 \neq 19 - 3$.

Ex. 39: $4^{61} + 4^{62} + 4^{63} + 4^{64} + 4^{65}$ is divisible by

1) 3 2) 5 3) 11 4) 17 5) None of these

Soln : $4^{61}\left[1 + 4 + 4^2 + 4^3 + 4^4\right]$

$= 4^{61}[1 + 4 + 16 + 64 + 256] = 341 \times 4^{61}$

Since 341 is divisible by 11, the given expression is also divisible by 11.

Ex. 40: The ratio between a two-digit number and the sum of the digits of that number is 4 : 1. If the digit in the unit's place is 3 more than the digit in the ten's place, what is the number?

Soln: Suppose the two-digit number $= 10x + y$

Then we have $\frac{10x + y}{x + y} = \frac{4}{1}$

or, $10x + y = 4x + 4y$

or, $6x = 3y$ or, $2x = y$

or $x = y - x = 3$ (given) and $y = 6$

$\therefore$ the number is 36.

Ex. 41: Find the remainder when $7^{13} + 1$ is divided by 6.

Soln : See the following binomial expansion:

$(x+y)^n = x^n + {}^nc_1\, x^{n-1}y + {}^nc_2\, x^{n-2}y^2 + {}^nc_3 x^{n-3}y^3 + \ldots + {}^nc_{n-1}\, xy^{n-1} + y^n$

We find that each of the terms except the last term (y^n) contains x. It means each term except y^n is perfectly divisible by x. (Note: y^n may be perfectly divisible by x but we cannot say without knowing the values of x and y.)

Following the same logic:

$7^{13} = (6+1)^{13}$ has each term except 1^{13} exactly divisible by 6.
Thus, when 7^{13} is divided by 6 we have the remainder $1^{13} = 1$
and hence, when $7^{13} + 1$ is divided by 6 the remainder is $1 + 1 = 2$

Ex. 42: The product of two numbers is 7168 and their HCF is 16. Find the numbers.

Soln: The numbers must be multiples of their HCF.
So, let the numbers be 16a and 16b where a and b are two numbers prime to each other.
$\therefore$ $16a \times 16b = 7168$ $\therefore ab = 28$
Now, the pairs of numbers whose product is 28 are
(28, 1), (14, 2) and (7, 4).
(14, 2), which are not prime to each other, should be rejected. Hence, the required numbers are
$(28 \times 16, 1 \times 16)$ and $(7 \times 16, 4 \times 16)$
or, (448, 16) and (112, 64)

Note: (1) We see that there may be more than one pair of numbers.
(2) If you have understood the logic of working, you may simplify the task in the following way.

Step I: Find the value of $\dfrac{\text{Product}}{(\text{HCF})^2}$

Step II: Find the possible pairs of factors of value got in Step I

Step III: Multiply the HCF with the pair of prime factors obtained in Step II

For the above question:

Step I: $\dfrac{7168}{(16)^2} = 28$

Step II: (1, 28), (2, 14), (4, 7)

Step III: $(1 \times 16, 28 \times 16)$ and $(4 \times 16, 7 \times 16)$
or, (16, 448) and (64, 112)

Ex. 43: Find the greatest number that will divide 55, 127 and 175 so as to leave the same remainder in each case.

Soln: Let x be the remainder, then the numbers (55 - x), (127 - x) and (175 - x) must be exactly divisible by the required number.
Now, we know that if two numbers are divisible by a certain number, then their difference is also divisible by that number. Hence, the numbers
$(127 - x) - (55 - x)$, $(175 - x) - (127 - x)$ and $(175 - x) - (55 - x)$

or, 72, 48 and 120 are also divisible by the required number. HCF of 72, 48 and 120 is 24.
Therefore, the required number is 24.

Note: If you don't want to go into the details of the method, find the HCF of the positive differences of numbers. It will serve your purpose quickly.

Ex. 44: A number on being divided by 5 and 7 successively leaves the remainders 2 and 4 respectively. Find the remainder when the same number is divided by $5 \times 7 = 35$

Soln:

5	A	
7	B	2
	C	4

In the above arrangement, A is the number which, when divided by 5, gives B as a quotient and leaves 2 as a remainder. Again, when B is divided by 7, it gives C as a quotient and 4 as a remainder. For simplicity, we may take C = 1.
$\therefore B = 7 \times 1 + 4 = 11$
and $A = 5 \times 11 + 2 = 57$
Now, when 57 is divided by 35, we get 22 as the remainder.

Direct Formula: The required remainder $= d_1 \times r_2 + r_1$
Where, d_1 = the first divisor = 5
r_1 = the first remainder = 2
r_2 = the second remainder = 4
$\therefore$ the required remainder $= 5 \times 4 + 2 = 22$.

Ex. 45: In a long-division sum, the remainders from the first to the last are 221, 301, 334 and 280 respectively. Find the divisor and the quotient if the dividend is 987654.

Soln: Step I

$$\_\,\_\,\_)987654(\_$$
$$\underline{abc}$$
$$221$$

(a) Since we have four remainders, our multiplier (i.e., abc) should be a three-digit number. (Why?)
(b) Since we see that all the remainders are three-digit numbers, we may guess that our divisor is also a three-digit number. (It may not be true!)
(c) We can find abc = 987 - 221 = 766
Now, our divisor must be a factor of 766. Thus, it is either 766 or 383

Step II $\_\,\_\,\_)987654(\_\,\_$

$$\begin{array}{l} \underline{766} \\ 2216 \\ \underline{defg} \\ 301 \end{array}$$

$\therefore$ defg = 2216 - 301 = 1915

Now, we confirm our divisor by taking the HCF of 766 and 1915, which is 383. Once we get the divisor, we can complete the sum as:

$$\begin{array}{l} 383)987654(2578 \\ \underline{766} \\ 2216 \\ \underline{1915} \\ 3015 \\ \underline{2681} \\ 3344 \\ \underline{3064} \\ 280 \end{array}$$

Thus, our divisor is 383 and the quotient is 2578

Ex. 46: Find the number of zeroes at the end of the products:

(a) $12 \times 18 \times 15 \times 40 \times 25 \times 16 \times 55 \times 105$

(b) $5 \times 10 \times 15 \times 20 \times 25 \times 30 \times 35 \times 40 \times 40 \times 45$

Soln: We must know that zeroes are produced only due to the following reasons:

1) If there is any zero at the end of any multiplicand.

2) If 5 or multiple of 5 are multiplied by any even number.

To generalise the above two statements, we may say that:

$(5)^n (2)^m$ has n zeroes if n < m; or m zeroes if m < n.

Thus, write the product in the form $\{2^m \times 5^n \times .....\}$.

(a) $12 \times 18 \times 15 \times 40 \times 25 \times 16 \times 55 \times 105$

$= 12 \times 18 \times 16 \times 40 \times 15 \times 25 \times 55 \times 105$

$= (2^2 \times 3) \times (2 \times 9) \times (2)^4 \times (2^3 \times 5) \times (5 \times 3) \times (5)^2 \times (5 \times 11) \times (5 \times 21)$

$= (2)^{10} \times (5)^6 \times .....$

(Since numbers other than 2 and 5 are useless.)

Since 6 < 10, there are 6 zeroes at the end of the product.

(b) $5 \times (2 \times 5)(3 \times 5)(2^2 \times 5)(5)^2 (2 \times 3 \times 5)(5 \times 7)(2^3 \times 5)(2^3 \times 5)(5 \times 9)$

$= (2)^{10} \times (5)^{11} \times ....$

Since 10 < 11, there are 10 zeroes at the end of the product.

Note: This is the easiest way to count the number of zeroes in the chain of products. By this method, you can easily find that the product of $1 \times 2 \times 3 .... \times 100$ contains 24 zeroes. Try it.

To find the number of different divisors of a composite number

Rule: *Find the prime factors of the number and increase the index of each factor by 1. The continued product of increased indices will give the result including unity and the number itself.*

For example

Ex. 1: Find the number of different divisors of 50, besides unity and the number itself.

Soln: If you solve this problem without knowing the rule, you will take the numbers in succession and check the divisibility. In doing so, you may miss some numbers. It will also take more time.

Different divisors of 50 are: 1, 2, 5, 10, 25, 50

If we exclude 1 and 50, the number of divisors will be 4.

By rule: $50 = 2 \times 5 \times 5 = 2^1 \times 5^2$

$\therefore$ the number of total divisors = (1+1) × (2+1) = 2×3 = 6

or, the number of divisors excluding 1 and 50 = 6 - 2 = 4

Ex. 2: The number of divisors of 40, except unity, is ____

Soln: $40 = 2 \times 2 \times 2 \times 5 = 2^3 \times 5^1$

Total number of divisors = (3+1) (1+1) = 8

$\therefore$ number of divisors excluding unity = 8-1 = 7

Ex. 3: Find the different divisors of 37800, excluding unity.

Soln: $37800 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 \times 7$

$= 2^3 \times 3^3 \times 5^2 \times 7^1$

Total number of divisors = (3+1) (3+1) (2+1) (1+1) = 96

$\therefore$ the number of divisors excluding unity = 96 - 1 = 95

To find the number of numbers divisible by a certain integer

Ex. 1: How many numbers up to 100 are divisible by 6?

Soln : Divide 100 by 6. The quotient obtained is the required number of numbers.

$100 = \underline{16} \times 6 + 4$

Thus, there are 16 numbers.

Ex. 2 : How many numbers up to 200 are divisible by 4 and 3 together?

Soln : LCM of 4 and 3 = 12

Now, divide 200 by 12 and the quotient obtained is the required number of numbers.

$$200 = \underline{16} \times 12 + 8$$

Thus, there are 16 numbers.

Ex. 3 : How many numbers between 100 and 300 are divisible by 7?

Soln : Up to 100, there are 14 numbers which are divisible by 7 (since $100 = \underline{14} \times 7 + 2$). Up to 300, there are 42 numbers which are divisible by 7 (since $300 = \underline{42} \times 7 + 6$)

Hence, there are 42-14 =28 numbers.

Other method:

Step I: First find the range of the limit. In this case, range = 300 - 100 = 200

Step II: Divide the range by 7 and get the quotient as your required answer. Here, $200 \div 7 = 28$ as quotient.

$\therefore$ There are 28 numbers.

Note: The above method is not applicable in all cases. Sometimes it fails to give the correct answer, as in the following example.

Ex. 4: How many numbers between 100 and 300 are divisible by 13?

Soln: **Method I:** Up to 100, there are 7 numbers divisible by 13 because $100 = \underline{7} \times 13 + 9$

Upto 300, there are 23 numbers divisible by 13 because $300 = \underline{23} \times 13+1$

$\therefore$ there are 23 - 7 =16 numbers.

Method II: Range = 300 - 100 = 200

and $200 = \underline{15} \times 13 + 5$

$\therefore$ there are 15 numbers, which is not true.

Note: You can see how the two methods give different answers. We suggest that you solve these questions by Method I only.

Ex. 5: By what number less than 1000 must 43521 be multiplied so that the last three figures at the right end of the product may be 791?

Soln : The last digit of the product is 1, so the multiplier's last digit should be 1. For the second digit in the product follow the following: let the second digit in multiplier be x. Then by the rule of multiplication (cross-multiplication of last two digits of multiplicand and multiplies):

$$\begin{array}{r} 43521 \\ \underline{\times\ x1} \\ 791 \end{array}$$

$1 \times 2 + 1 \times x = 9 \quad \therefore x = 7$

Now, for the third digit in multiplier :

$$\begin{array}{r} 43521 \\ \underline{\times\ x71} \\ 791 \end{array}$$

$5 \times 1 + 2 \times 7 + 1 \times x = \_7$ or, $19 + x = \_7 \quad \therefore x = 8$

Thus, we see that the three-digit number is 871.

Ex. 6: What would be the maximum value of Q in the following equation?

$$5P9 + 3R7 + 2Q8 = 1114$$

1) 8 2) 7 3) 5 4) 4 5) None of the above

Soln: 5P9 + 3R7 + 2Q8 = 1114

For the maximum value of Q, the values of P and R should be the minimum, i.e. zero each

Now, 509 + 307 + 2Q8 = 1114

or, 816 + 2Q8 = 1114

or, 2Q8 = (1114 - 816 =) 298

So, the reqd value of Q is 9.

Ex. 7: If the places of last two digits of a three-digit number are interchanged, a new number greater than the original number by 54 is obtained. What is the difference between the last two digits of that number?

1) 9 2) 12 3) 6

4) Data inadequate 5) None of these

Soln: Let the three-digit number be 100x + 10y + z.

According to the question,

(100x + 10z + y) - (100x + 10y + z) = 54

or, 9z - 9y = 54 or, z - y = 6

Note: Remember that the difference between last two digits in such case is $\frac{\text{Difference in two values}}{9} = \frac{54}{9} = 6$

EXERCISES

1. How many numbers up to 120 are divisible by 8 ?
2. How many numbers between 200 and 500 are divisible by 13 ?
3. How many numbers between 100 and 300 are multiples of 13 ?

4. If we write the numbers from 1 to 201, what is the sum of all the odd numbers ?
5. If we write the numbers from 101 to 309, what is the sum of all the even numbers?
6. If we write the numbers from 50 to 151, what is the difference between the sum of all the odd and even numbers ?
7. How many different numbers of 7 digits are there ?
8. How many numbers up to 200 are divisible by 5 and 7 together ?
9. How many numbers between 200 and 400 are divisible by 3, 4 and 5 together ?
10. The sum of two numbers is 22 and their difference is 14. Find the product of the numbers.
11. The sum of two numbers is 30 and their difference is 6. Find the difference of their squares.
12. The product of two terms is 39 and their difference is 28. Find the difference of their reciprocals.
13. What is the number just greater than 9680 exactly divisible by 71 ?
14. What is the difference between the largest and the smallest numbers written with all the four digits 7, 3, 1 and 4 ?
15. What is the least number which is a perfect square and contains 3675 as its factor ?
16. What is the least number which must be subtracted from 9600 so that the remaining number becomes divisible by 78 ?
17. Find the least number which when added to 3000 becomes a multiple of 57.
18. In a division sum, the quotient is 105, the remainder is 195, and the divisor is the sum of the quotient and the remainder. What is the dividend?
19. Find the sum of all the numbers between 200 and 600 which are divisible by 16.
20. Is 1001 a prime number ?
21. Is 401 a prime number ?
22. What is the sum of all the prime numbers between 60 and 80 ?
23. When a certain number is multiplied by 13, the product consists entirely of sevens. Find the smallest such number.
24. Find the number which when multiplied by 16 is increased by 225.
25. How many times shall the keys of a typewriter have to be pressed in order to write first 200 counting numbers, i.e., to write 1,2,3,.... up to 200?

26. In a division sum, t e divisor is 4 times the quotient and 3 times the remainder. What is the dividend if the remainder is 4 ?
27. How many times must 79 be subtracted from 10,000 in order to leave remainder 6445 ?
28. Find the total number of prime numbers which are contained in $(30)^6$.
29. What is the number that added to itself 20 times, gives 861 as result?
30. If 97 be multiplied by a certain number, that number is increased by 7584. Find that number.
31. A certain number when successively divided by 3 and 5 leaves remainder 1 and 2. What is the remainder if the same number be divided by 15 ?
32. A certain number when successively divided by 7 and 9 leaves remainder 3 and 5 respectively. Find the smallest value of such a number.
33. A certain number when divided by 36 leaves a remainder 21. What is the remainder when the same number be divided by 12 ?
34. When a certain number is multiplied by 13, the product consists entirely of fives. What is the smallest such number ?
35. If a=16 and b=15, then what is the value of $\dfrac{a^2 + b^2 + ab}{a^3 - b^3}$?
36. What is the largest natural number by which the product of three consecutive even natural numbers is always divisible ?
37. If $\dfrac{x}{y} = \dfrac{3}{4}$, then the value of $\dfrac{6}{7} + \dfrac{y-x}{y+x}$ is ______.
38. If $\sqrt{\left(1 + \dfrac{27}{169}\right)} = \left(1 + \dfrac{x}{13}\right)$, then the value of x is ______.
39. What least value must be given to * so that the number 6135*2 is exactly divisible by 9 ?
40. What least value must be given to * so that the number 97215*6 is divisible by 11 ?
41. What least value must be given to * so that the number 91876*2 is divisible by 8 ?
42. What is the largest number of four digits which is exactly divisible by 88 ?
43. Write down the first prime number.
44. If the number $(10^n - 1)$ is divisible by 11 then n is
1) odd number 2) even number 3) any number 4) multiple of 11.

45. If $\frac{a}{b} = \frac{4}{3}$, then $\frac{3a+2b}{3a-2b} = ?$

46. $\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)\ldots\ldots\left(1-\frac{1}{n}\right) = ?$

47. $\left(2-\frac{1}{3}\right)\left(2-\frac{3}{5}\right)\left(2-\frac{5}{7}\right)\ldots\ldots\left(2-\frac{997}{999}\right) = ?$

48. $\frac{137\times137 + 137\times133 + 133\times133}{137\times137\times137 - 133\times133\times133} = ?$

49. $\frac{?}{54} = \frac{96}{?}$

50. What is the number of prime factors in the expression $(6)^{10} \times (7)^{17} \times (11)^{27}$?

SOLUTIONS (Hints)

1. $\frac{120}{8} = 15$

2. Number of numbers up to 200 which are divisible by 13

$= \frac{200}{13} = 15 + \frac{5}{13}$, i.e., 15

Number of numbers up to 500 which are divisible by 13

$= \frac{500}{13} = 38 + \frac{6}{13}$, i.e., 38

$\therefore$ the required numbers = 38 - 15 = 23

3. Number of numbers up to 100 which are multiples of 13

$= \frac{100}{13} = 7 + \frac{9}{13}$, i.e., 7

Number of numbers up to 300 which are multiples of 13

$= \frac{300}{13} = 23 + \frac{1}{13}$, i.e., 23 $\therefore$ the required numbers = 23 - 7 = 16

4. The number of numbers (201) is odd, hence there are $\frac{1}{2}(201+1) = 101$ odd numbers.

$\therefore$ the sum of the first 101 odd numbers = $(101)^2 = 10,201$

5. Number of even numbers up to 309 = $\frac{1}{2}(309 - 1) = 154$

$\therefore$ the sum of first 154 even numbers = 154 (154+1) = 23,870

Number of even numbers up to 101 = $\frac{1}{2}(101-1) = 50$

$\therefore$ the sum of first 50 even numbers = 50 (50+1) = 2,550

$\therefore$ the required sum = 23,870 - 2,550 = 21,320

6. If our series starts with an even number and ends with an odd number, then the required difference = $\frac{151-50+1}{2} = \frac{102}{2} = 51$

Note : 1) If our series starts with an odd number and ends with an odd number, then such difference = $\frac{\text{last number} + \text{first number}}{2}$

2) If our series starts with an even number and ends with an odd number, then such difference = $\frac{\text{last number} - \text{first number} + 1}{2}$

3) If our series starts with an even number and ends with an even number, then such difference = $\frac{\text{last number} + \text{first number}}{2}$

7. Least number of 7 digits = 10,00,000

Highest number of 7 digits = 99,99,999

Then the number of 7-digit numbers = 99,99,999 - 10,00,000 + 1

$= 9\times10^6$

8. Numbers which are divisible by 5 and 7 together are also divisible by their LCM.

LCM of 5 and 7 = 35

Therefore, the required number of numbers = $\frac{200}{35} = 5 + \frac{25}{35}$, i.e., 5

9. LCM of 3, 4 and 5 = 60

Number of numbers up to 200 which are divisible by 60

$= \frac{200}{60} = 3 + \frac{1}{3}$, i.e., 3

Number of numbers up to 400 which are divisible by 60

$= \frac{400}{60} = 6 + \frac{2}{3}$, i.e., 6 $\therefore$ the required number = 6 - 3 = 3

10. Let the numbers be x and y. Then $x + y = 22$ --------(1)

$x - y = 14$ -----------(2)

Squaring (1) and (2), we get

$x^2 + y^2 + 2xy = 484$ --------(3)

$x^2 + y^2 - 2xy = 196$ --------(4)

Subtracting (4) from (3), we get $4xy = 288 \quad \therefore xy = \frac{288}{4} = 72$

Direct formula : I: Product of two numbers

$$= \frac{(\text{Sum})^2 - (\text{Difference})^2}{4}$$

$$= \frac{(22)^2 - (14)^2}{4} = \frac{36 \times 8}{4} = 72$$

II: $x = \frac{22 + 14}{2} = 18$

$y = \frac{22 - 14}{2} = 4 \quad \therefore xy = 18 \times 4 = 72$

11. Let the two numbers be x and y.

Then $x+y=30$ and $x-y=6$

or, $(x+y)(x-y) = 180$ or, $x^2 - y^2 = 180$

12. Let the two terms be x and y. We are given $x-y = 28$ ----------(1)

and $xy = 39$ ----------(2)

Dividing (1) by (2), we get $\frac{x}{xy} - \frac{y}{xy} = \frac{28}{39}$

or, $\frac{1}{y} - \frac{1}{x} = \frac{28}{39}$ Ans.

13. On dividing 9680 by 71, we get a remainder of 24.

Now, 9680 needs (71 - 24 =) 47 more to be divisible by 71.

$\therefore$ the required number = 9680 + 47 = 9727

14. The largest number = 7431 and the smallest number = 1347

$\therefore$ the required difference = 7431 - 1347 = 6084

15. $3675 = 3 \times 5 \times 5 \times 7 \times 7 = 3 \times 5^2 \times 7^2$. See that all the factors except 3 are squares. So, if we multiply 3675 by 3, the obtained number will be a perfect square and also have 3675 as its factor. Thus, the required least number = $3675 \times 3 = 11025$

16. On dividing 9600 by 78, we get 6 as remainder. If we subtract 6 from 9600, the obtained number will have no remainder.

Thus, the required least number = 6

17. When we divide 3000 by 57, we get 36 as remainder. Then the required least number = 57 - 36 = 21, which when added to 3000, the obtained number becomes a multiple of 57, i.e., that number is perfectly divisible by 57.

18. Q = 105, R = 195, D = Q+R = 105+195 = 300

$\therefore$ Dividend = D × Q + R = 31695

19. The least such number = 16 × 13 = 208

The highest such number = 16 × 37 = 592

$\therefore$ the required sum = $16 \times 13 + 16 \times 14 + \ldots + 16 \times 37$

$= 16\,(13+14+\ldots+37)$

$= 16\,[(1 + 2 + \ldots + 37) - (1 + 2 + \ldots + 12)]$

$= 16\left[\frac{37 \times 38}{2} - \frac{12 \times 13}{2}\right] = 16\,[703 - 78] = 10{,}000$

20. The approximate square root of 1001 is 32. The prime numbers which are less than 32 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 and 31. We see that 1001 is divisible by 7; so it is not a prime number.

21. The approximate square root of 401 is 20. The prime numbers which are less than 20 are 2, 3, 5, 7, 11, 13, 17, 19. We see that 401 is not divisible by any of the above prime numbers. So it is a prime number.

22. Sum = 61+67+71+73+79 = 351

23. Write a number which consists of two sevens only (77). Divide the number by 13. If it is perfectly divisible, then the quotient obtained by division is the required number. If the number is not divisible, then add one more seven and check its divisibility by 13. If it is perfectly divisible, then the quotient is the required number. If it is not divisible, then add one more seven and .... Moving the same way, we see that

$777777 \div 13 = 59829$

24. Let that number be x. Then $16x - x = 225 \quad \therefore x = \frac{225}{15} = 15$

Note : Thus in this case: the required number = $\frac{\text{Increased value}}{\text{Multiplier} - 1}$

25. Up to 200, the number of one-, two- and three-digit numbers are 9, 90 and 101 respectively.

$\therefore$ the number of times the keys of the typewriter to be pressed

$= 9 \times 1 + 90 \times 2 + 101 \times 3 = 9+180+303 = 492$

26. $R = 4,\ D = 3 \times R = 12,\ Q = \frac{D}{4} = \frac{12}{4} = 3$

$\therefore$ Dividend = DQ + R = 12 × 3+4 = 40

27. The required number of times = $\frac{10{,}000 - 6{,}445}{79} = 45$

28. $(30)^6 = (2 \times 3 \times 5)^6 = 2^6 \times 3^6 \times 5^6$

2, 3 and 5 are repeated 6 times each, so there are 6+6+6 = 18 prime numbers.

29. The required number $= \frac{861}{20+1} = \frac{861}{21} = 41$

30. The required number $= \frac{7584}{97-1} = 79$

31. By Quicker Method : The required remainder $= d_1 \times r_2 + r_1$
$= 3 \times 2 + 1 = 7$

Note: It is a very important method. It should be remembered.

32. Start with the last quotient, i.e., 1

7	* * *	
9	* *	3
	1	5

$** = 9 \times 1 + 5 = 14$ $\quad *** = 7 \times 14 + 3 = 101$

33. The number $= 36x + 21 = 36x + 12 + 9 = (36x + 12) + 9$
As $(36x+12)$ is divisible by 12, the remainder will be 9

Note : When the first divisor is divisible by the second, the required remainder will be obtained by dividing the first remainder by second divisor.

34. Same as in Ex. 23.

35. $\frac{a^2+b^2+ab}{a^3-b^3} = \frac{1}{a-b} = \frac{1}{16-15} = 1$

36. The largest such number $= (2 \times 4 \times 6) = 48$

37. $\frac{x}{y} = \frac{3}{4}$

Using the rule of componendo-dividendo : $\frac{y-x}{y+x} = \frac{4-3}{4+3} = \frac{1}{7}$

Then $\frac{6}{7} + \frac{y-x}{y+x} = \frac{6}{7} + \frac{1}{7} = \frac{7}{7} = 1$

38. $\sqrt{1+\frac{27}{169}} = \sqrt{\frac{196}{169}} = \frac{14}{13} = 1 + \frac{1}{13} \quad \therefore x = 1$

39. A number is divisible by 9, when its digit-sum is divisible by 9. Digit-sum of the given number (excluding $*$) is 17. If we put $*=1$, the number will be perfectly divisible by 9.

40. 97215 $*$ 6

Digit-sum of odd positions $= 9+2+5+6 = 22$

Digit-sum of even positions (excluding $*$) $= 7+1 = 8$

The difference of the two should be either 0 or divisible by 11. So $*=3$

41. The last three digits should be divisible by 8. So $* = 3$

42. The largest number of 4 digits = 9999
On dividing 9999 by 88, we get a remainder of 55. Now, if this remainder is subtracted from 9999, the remaining number will be exactly divisible by 88.
$\therefore$ the required number $= 9999 - 55 = 9944$.

43. 2

44. n should be even number.

45. $\frac{a}{b} = \frac{4}{3}$, then by the rule of componendo - dividendo,

$\frac{3a+2b}{3a-2b} = \frac{3\times4+2\times3}{3\times4-2\times3} = \frac{18}{6} = 3$

Other way: $\frac{a}{b} = \frac{4}{3}, \frac{3a}{2b} = \frac{4}{3} \times \frac{3}{2} = \frac{2}{1} \quad \therefore \frac{3a+2b}{3a-2b} = \frac{2+1}{2-1} = 3$

46. $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \ldots\ldots \frac{n-1}{n} = \frac{2}{n}$

47. $\frac{5}{3} \times \frac{7}{5} \times \frac{9}{7} \times \ldots\ldots \times \frac{1001}{999} = \frac{1001}{3}$

48. Use the formula $a^3 - b^3 = (a-b)(a^2+ab+b^2)$

Then the given expression $= \frac{1}{137-133} = \frac{1}{4}$

49. $(?)^2 = 54 \times 96 \quad \therefore ? = \sqrt{9\times6\times6\times16} = 3\times6\times4 = 72$

50. $(6)^{10} \times (7)^{17} \times (11)^{27} = (2)^{10} \times (3)^{10} \times (7)^{17} \times (11)^{27}$
$\therefore$ total number of prime factors $= 10 + 10 + 17 + 27 = 64$

Binary System

Number System is a system which represents different numbers in different ways. There are many number systems. All the number systems have different bases. **Base** denotes the number of symbols in the number system. For example, in the decimal system base, the is 10 and it has 10 number-symbols (0, 1, 2, 3, ... 9). Some more examples of number systems are given below in tabular form:

Number System	**Base**	**Representation of Symbols**
Quinary No. System	5	0, 1, 2, 3 and 4
Octal No. System	8	0, 1, 2, 3, ... 7
Hexadecimal No. System	16	0, 1, 2, ...9, A(10), B(11), C(12),... F(15)
Binary No. System	2	0 and 1

Binary No. System was introduced by JV Newman in 1946.

Conversion of Decimal Number into its Binary equivalent:

To find the binary equivalent of a decimal number, we go on dividing the decimal number by the constant divisor 2 till the last quotient 1 is obtained. For example we convert 89 into its binary equivalent.

2	89	$2 \times q_1$ (i.e. 44) = 88; 89 - 88 = 1 (r_1) ↑
	44	$2 \times q_2$ (i.e. 22) = 44; 44 - 44 = 0 (r_2)
	22	$2 \times q_3$ (i.e. 11) = 22; 22 - 22 = 0 (r_3)
	11	$2 \times q_4$ (i.e. 5) = 10; 11 - 10 = 1 (r_4)
	5	$2 \times q_5$ (i.e. 2) = 4; 5 - 4 = 1 (r_5)
	2	$2 \times q_6$ (i.e. 1) = 2; 2 - 2 = 0 (r_6)
	1 (last quotient)	

q_1, q_2,..... are the first/ second/ .... quotients and r_1, r_2,.... are the first/ second/ .... remainders. For all the stages of division the common divisor 2 remains unchanged and the quotient obtained becomes the next dividend. The process continues till the last quotient (1) is obtained.

Here, after dividing the real dividend 89 by 2, the first quotient q_1 (=44) becomes our next dividend. Now after dividing 44 by 2, the second quotient q_2 (= 22) becomes our next dividend. And so on. Every time the remainder is noted down carefully. When the last quotient as 1 is obtained we finally note down the remainders (including the last quotient, 1) strictly in accordance with the arrow mark, i.e. we note down finally in the way:

First of all the last quotient, then the last remainder, then the secondlast remainder, ..., third remainder, then second remainder, and finally the first remainder. That is, the binary number equivalent to 89 is 1011001.

Or, by notation, $(89)_{10} = (1011001)_2$

The base 10 stands for decimal system and the base 2 stands for binary system.

Conversion of Binary Number to its Decimal equivalent:

In binary system the value of 1 doubles itself every time it shifts one place to the left, and wherever '0' occurs its value becomes zero.

Let us convert 1011001 into its decimal equivalent. To understand easily, we write each digit of 1011001 inside each box in the following way and the value of each box is written above it.

2^6	2^5	2^4	2^3	2^2	2^1	2^0
1	0	1	1	0	0	1

Now $(1011001)_2$

$= 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

$= 64 + 0 + 16 + 8 + 0 + 0 + 1 \; (\because 2^0 = 1)$

$= 64 + 16 + 8 + 1 = 89$

Some solved examples:

Ex. 1: Convert 90 into its Binary equivalent.

2	90	0	(r_1)
	45	1	(r_2)
	22	0	(r_3)
	11	1	(r_4)
	5	1	(r_5)
	2	0	(r_6)
	1		(Q)

Here we get $(90)_{10} = (1011010)_2$

Verification:

2^6	2^5	2^4	2^3	2^2	2^1	2^0
1	0	1	1	0	1	0

$(1011010)_2 = 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$

$= 64 + 0 + 16 + 8 + 0 + 2 + 0$

$= 64 + 16 + 8 + 2 = 90$

Ex. 2: Convert 88 into its Binary equivalent.

2	88	0
	44	0
	22	0
	11	1
	5	1
	2	0
	1	

Here we get $(88)_{10} = (1011000)_2$

Verification:

2^6	2^5	2^4	2^3	2^2	2^1	2^0
1	0	1	1	0	0	0

$(1011000)_2 = 2^6 + 2^4 + 2^3 = 64 + 16 + 8 = 88$

Ex. 3: Convert the following numbers into their binary equivalents:

(i) 2 **(ii)** 3 **(iii)** 4 **(iv)** 5 **(v)** 6 **(vi)** 7 **(vii)** 8

Also verify your answers.

Soln: (i) 2 | 2 | 0
1

i.e., $(2)_{10} = (10)_2$

(ii) 2 | 3 | 1
1

i.e., $(3)_{10} = (11)_2$

(iii) 2 | 4 | 0
2 | 0
1

i.e., $(4)_{10} = (100)_2$

(iv) 2 | 5 | 1
2 | 0
1

i.e., $(5)_{10} = (101)_2$

(v) 2 | 6 | 0
3 | 1
1

i.e., $(6)_{10} = (110)_2$

(vi)

2	7	1
	3	1
	1	

i.e., $(7)_{10} = (111)_2$

(vii)

2	8	0
	4	0
	2	0
	1	

i.e., $(8)_{10} = (1000)_2$

Verification:

(i) $(10)_2 = 1 \times 2^1 + 0 \times 2^0 = 2 + 0 = 2$

(ii) $(11)_2 = 1 \times 2^1 + 1 \times 2^0 = 2 + 1 = 3$

(iii) $(100)_2 = 1 \times 2^2 + 0 \times 2^1 + 0 \times 2^0 = 4 + 0 + 0 = 4$

(iv) $(101)_2 = 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 4 + 0 + 1 = 5$

(v) $(110)_2 = 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 4 + 2 + 0 = 6$

(vi) $(111)_2 = 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 4 + 2 + 1 = 7$

(vii) $(1000)_2 = 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 0 \times 2^0$
$= 8 + 0 + 0 + 0 = 8$

Ex. 4: Directions: In a certain code, symbol of 0 is ⊕ and 1 is □. There are no other symbols for other numbers greater than 1 and are written using these two symbols only. The value of the symbol □ doubles itself every time it shifts one place to the left.

0 is written as ⊕
1 is written as □
2 is written as □⊕
3 is written as □□
4 is written as □⊕⊕

Now answer the following questions.

(i) What is the value of $2^3 + 2^2 \times 1.5 \times \frac{45}{54} \times \frac{3}{5} - 1^3$?

a) ⊕□□⊕ b) □⊕⊕□ c) □⊕□⊕
d) □□⊕⊕ e) None of these

(ii) If □□□□ be multiplied by □□⊕□, what will be the result?

a) 165 b) 180 c) 175 d) 200 e) None of these

(iii) Which of the following will represent the value of 37.5% of 56?

a) □□⊕⊕□ b) □⊕□⊕□ c) □⊕⊕□□
d) □⊕□⊕⊕ e) None of these

(iv) Which of the following will represent 42?

a) □⊕⊕□□⊕ b) □⊕□□⊕⊕ c) □□⊕⊕□⊕
d) □⊕□⊕□⊕ e) None of these

(v) Which of the following pairs have the same numbers?

a) 3^3, □⊕⊕□□ b) 4^2, □⊕⊕□⊕
c) $2^3 \times 3^2$, □⊕⊕□⊕⊕⊕ d) $2^2 \times 3^2$, □⊕⊕□□⊕
e) None of these

(vi) If □⊕⊕□⊕⊕⊕⊕ be divided by □⊕⊕□⊕, the result is

a) □□⊕ b) □□⊕□ c) □⊕⊕□
d) □⊕⊕⊕ e) None of these

(vii) HCF of □□⊕⊕□ and □□□□ is

a) □□⊕ b) ⊕⊕□ c) □⊕□
d) ⊕□⊕ d) None of these

(viii) □⊕□ − □⊕⊕ × □□⊕ = ?

a) 20 b) 21 c) -19
d) 22 e) None of these

Soln: Obviously, these questions are based on Binary Number System.

(i) $2^3 + 2^2 \times 1.5 \times \frac{45}{54} \times \frac{3}{5} - 1^3 = 8 + 4 \times 1.5 \times \frac{9}{18} - 1 = 7 + 3 = 10$

Now we have to find the binary equivalent of 10.

2	10	0
	5	1
	2	0
	1	

i.e., $(10)_{10} = (1010)_2$

Now, using the given symbols for 0 and 1, we get

$(1010)_2$ = □⊕□⊕ ⇒ c) answer

(ii) Using the given symbols we get,

□□□□ $= (1111)_2 = 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$
$= 8 + 4 + 2 + 1 = 15$

and □□⊕□ $= (1101)_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$
$= 8 + 4 + 0 + 1 = 13$

Now, $15 \times 13 = 195 \Rightarrow$ e) answer

(iii) 37.5% of 56 $= \frac{3}{8} \times 56 = 21$

Now, the binary equivalent of 21 is

$$\begin{array}{r|r|r}2 & 21 & 1\\ \hline & 10 & 0\\ \hline & 5 & 1\\ \hline & 2 & 0\\ \hline & 1 & \end{array}$$

i.e. $(21)_{10} = (10101)_2 = \square \oplus \square \oplus \square \Rightarrow$ b) answer

(iv)

$$\begin{array}{r|r|r}2 & 42 & 0\\ \hline & 21 & 1\\ \hline & 10 & 0\\ \hline & 5 & 1\\ \hline & 2 & 0\\ \hline & 1 & \end{array}$$

i.e., $(42)_{10} = (101010)_2 = \square \oplus \square \oplus \square \oplus \Rightarrow$ d) answer

(v) (a) $\square \oplus \oplus \square\square = (10011)_2 = 2^4 + 0 + 0 + 2^1 + 2^0$

$= 16 + 2 + 1 = 19 \neq 27 (= 3^3)$

(b) $\square \oplus \oplus \square \oplus = (10010)_2 = 2^4 + 0 + 0 + 2^1 + 0$

$= 16 + 2 = 18 \neq 16 (= 4^2)$

(c) $\square \oplus \oplus \square \oplus \oplus \oplus$

$\Rightarrow (1001000)_2 = 2^6 + 0 + 0 + 2^3 + 0 + 0 + 0$

$= 64 + 8 = 72 = 8 \times 9 = 2^3 \times 3^2 \Rightarrow$ c) answer

(d) $\square \oplus \oplus \square\square \oplus = (100110)_2 = 2^5 + 0 + 0 + 2^2 + 2^1 + 0$

$= 32 + 4 + 2 = 38 \neq 36 (= 2^2 \times 3^2)$

(vi) $\square \oplus \oplus \square \oplus \oplus \oplus \oplus$

$\Rightarrow (10010000)_2 = 2^7 + 0 + 0 + 2^4 + 0 + 0 + 0 + 0$

$= 128 + 16 = 144$

$\square \oplus \oplus \square \oplus = (10010)_2 = 2^4 + 0 + 0 + 2^1 + 0 = 16 + 2 = 18$

Now $144 \div 18 = 8 = (1000)_2 = \square \oplus \oplus \oplus \Rightarrow$ d) answer

(vii) $\square\square \oplus \oplus \square = (11001)_2$

$= 2^4 + 2^3 + 0 + 0 + 2^0 = 16 + 8 + 1 = 25$

$\square\square\square\square = (1111)_2 = 2^3 + 2^2 + 2^1 + 2^0 = 8 + 4 + 2 + 1 = 15$

HCF of 25 and 15 = 5 = $(101)_2 = \square \oplus \square$ $\therefore$ Ans = (c)

(viii) $\square \oplus \square = 5; \square \oplus \oplus = 4; \square\square \oplus = 6$

$\therefore 5 - 4 \times 6 = -19$ $\therefore$ Ans = (c)

Permutation and Combination

To understand permutation and combination, let us take two examples:

Ex. 1: *How many triangles can be formed with four points (A, B, C & D) in a plane? It is given that no three points are collinear. From the three points A, B and C, have only one triangle with these points.*

It is irrespective of the fact where he starts. Although the arrangement of points may be in different orders like ABC, ACB, BAC, BCA, CAB and CBA, but in all these cases the triangles formed ΔABC, ΔACB, ΔBAC, ΔBCA, ΔCAB and ΔCBA are exactly the same triangle.

With the 4 points A, B, C and D we can form maximum 4 triangles namely ΔABC, ΔABD, ΔACD and ΔBCD.

Ex. 2: *How many number plates of 3 digits can be formed with four digits 1, 2, 3 and 4?*

Here, the order of arrangement of digits does matter.

For the digits 1, 2 and 3 the different arrangements are: 123, 132, 213, 231, 312 and 321.

Here the vehicles having the number plates 123, 132, 213, 231, 312 and 321 are 6 different vehicles but in Ex 1 the six triangles were the same.

The total no. of 3-digits number plates will be $= 4 \times 3 \times 2 = 24$.

(The 3-digit number plates will bear the numbers: 123, 132, 124, 142, 134, 143, 213, 231, 214, 241, 234, 243, 312, 321, 314, 341, 324, 342, 412, 421, 413, 431, 423 and 432).

Note: Ex.1 is the case of **combination** and Ex.2 is the case of **permutation.**

In Ex. 1, total number of triangles $= {}^4C_3 = \dfrac{4!}{3!\,(4-3)!} = 4$

Factorial notation: The product of n consecutive positive integers beginning with 1 is denoted by n! or $\underline{|n}$ and is read as factorial n.

(5! or $\underline{|5}$ is read as factorial five, 13! as factorial thirteen, etc.)

$\therefore$ $n! = 1 \times 2 \times \ldots\ldots \times (n-2) \times (n-1) \times n$

$= n \times (n-1) \times (n-2) \times \ldots . \times 2 \times 1.$

For example, $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

$4! = 4 \times 3 \times 2 \times 1 = 24$, $3! = 3 \times 2 \times 1 = 6$, $2! = 2 \times 1 = 2$

$$\frac{9!}{4!} = \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1}$$

$$= \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4!}{4!} = 9 \times 8 \times 7 \times 6 \times 5 = 15120$$

$$\frac{3!}{7!} = \frac{3!}{7 \times 6 \times 5 \times 4 \times 3!} = \frac{1}{7 \times 6 \times 5 \times 3} = \frac{1}{840}$$

If r and x be two positive integers such that $r < n$ then

$$\frac{n!}{r!} = \frac{n \times (n-1) \times (n-2) \times \dots \times 2 \times 1}{r \times (r-1) \times \dots \times 2 \times 1}$$

$$= n \times (n-1) \times \dots \times (r+1)$$

Similarly, $\frac{n!}{(n-r)!} = n \times (n-1) \times \dots \times (n-r+1)$

Arrangement: Suppose we have four different objects A, B, C and D. We have to form a group of two objects out of these four objects. In other words, we have to form a group of four different objects taken two at a time. Clearly, we will have six such groups: (i) A and B, (ii) A and C, (iii) A and D, (iv) B and C, (v) B and D, (vi) C and D. By the notational representation, the total no. of such groups $= {}^4C_2 = \frac{4!}{2!\,(4-2)!} = \frac{4!}{2! \times 2!} = 6$

Now, the two objects in each of these groups can be arranged in two different ways, namely

(i) A and B & B and A, (ii) A and C & C and A, and so on.

Thus, there are a total of twelve such arrangements.

Total no. of arrangements = total no. of groups × r!

Where r is the no. of objects in each group.

In the above example, total no. of arrangements

$= 6 \times 2! = 6 \times (2 \times 1) = 12.$

Definition of permutation: Each of the different arrangements which can be made by taking some or all of the given things or objects at a time is called a *permutation*. The symbol ${}^nP_r$ denotes the no. of permutations of n different things taken r at a time. The letter P stands for permutation.

Also, ${}^nP_r = \frac{n!}{(n-r)!}$

Thus, the symbol ${}^9P_4$ denotes the no. of permutations or arrangements of 9 different things taken 4 at a time and

$${}^9P_4 = \frac{9!}{(9-4)!} = \frac{9!}{5!}$$

$= 9 \times (9-1) \times \dots (5+1) = 9 \times 8 \times 7 \times 6 = 3024$

Definition of combination: Each of the different selections or groups which can be made by taking some or all of a no. of given things or objects at a time is called a *combination*. The symbol ${}^nC_r$ denotes the no. of combinations of n different things taken r at a time. The letter C stands for combination. Also, ${}^nC_r = \frac{n!}{r!\,(n-r)!}$

Thus, the symbol ${}^9C_4$ denotes the no. of selections, or groups of 9 different things taken 4 at a time and ${}^9C_4 = \frac{9!}{4!\,(9-4)!} = \frac{9!}{4!\,5!}$

$$= \frac{9 \times 8 \times 7 \times 6 \times 5!}{4 \times 3 \times 2 \times 1 \times 5!} = 126$$

Note: In the topic **Arrangement** we have,

Total no. of arrangements = total no. of groups or selection × r!

where r is the no. of objects in each group or selection.

So, ${}^nP_r = {}^nC_r \times r!$

For example, ${}^9P_4 = 4! \times {}^9C_4$

Some Fundamental Principles of Counting

1. Multiplication Rule:

Suppose one starts his journey from place X and has to reach place Z via a different place Y.

For Y, there are three means of transport - bus, train and aeroplane - from X. From Y, the aeroplane service is not available for Z. Only either by a bus or by a train can one reach Z from Y. Also, there is no direct bus or train service for Z from X. We want to know the maximum possible no. of ways by which one can reach Z from X.

For each means of transport from X to Y there are two means of transport for going from Y to Z. Thus, for going from X to Z via Y there will be 2 (firstly, by bus to Y and again by bus to Z; secondly, by bus to Y and thereafter by train to Z.)

+2 (firstly, by train to Y and thereafter by bus to Z; secondly, by train to Y and thereafter again by train to Z.)

+2 (firstly by aeroplane to Y and thereafter by bus to Z, secondly by aeroplane to Y and thereafter by train to Z.) $= 3 \times 2 = 6$ possible ways.

We conclude:

If a work A can be done in *m* ways and another work B can be done in *n* ways and C is the final work which is done only when

both A and B are done, then the no. of ways of doing the final work, $C = m \times n$.

In the above example, suppose the work to reach Y from X = the work A $\rightarrow$ in m i.e. 3 ways. The work to reach Z from Y = the work B $\rightarrow$ in n i.e. 2 ways. Then the final work to reach Z from X = the final work C $\rightarrow$ in $m \times n$, i.e. $3 \times 2 = 6$ ways.

II. Addition rule: *Suppose there are 42 men and 16 women in a party. Each man shakes his hand only with all the men and each woman shakes her hand only with all the women. We have to find the maximum no. of handshakes that taken place at the party.*

From each group of two persons we have one handshake.

Case 1: Total no. of handshakes among the group of 42 men

$$= {}^{42}C_2 = \frac{42!}{2!\,(42-2)!} = \frac{42!}{2!\,40!} = \frac{42 \times 41 \times 40!}{2 \times 1 \times 40!}$$

$= 21 \times 41 = 861$

Case 2: Total no. of handshakes among the group of 16 women

$$= {}^{16}C_2 = \frac{16!}{2!\,(16-2)!} = \frac{16 \times 15 \times 14!}{2 \times 1 \times 14!} = 8 \times 15 = 120$$

$\therefore$ maximum no. of handshakes = 861 + 120 = 981.

To find the no. of permutations or arrangements of n different things, all the n things taken at a time.

Suppose a student has 3 books (B_1, B_2 and B_3) and his book-rack has 3 shelves. He has to arrange the books in the shelves.

Case I: He puts one book in each shelf:

He can put anyone of the 3 books in the first shelf.

He is left with 2 books and he can put anyone of the remaining 2 books in 2 ways in the second shelf. Now, he is left with a single book which can be put in 1 way in the third shelf.

$\therefore$ Total no. of way in which he can put the books = $3 \times 2 \times 1 = (3! =) 6$

Case II: He puts 2 books together in one shelf and the remaining 1 book in another shelf.

He can put 2 books together out of the 3 different books in ${}^3P_2$ ways in one shelf. The remaining one books can be put in 1 way in anyone of the remaining two shelves.

$\therefore$ Total no. of ways of putting the books = ${}^3P_2 \times 1 = {}^3P_2 = \frac{3!}{(3-2)!} = \frac{3!}{1!} = 3! = 6$

Case III: He puts all the 3 books together in one shelf.

He can put all the 3 books in any one of the shelves in any one of the following sequences:

When the book B_1 is at the top, B_2 and B_3 can be arranged in two ways: B_2 in the middle and B_3 at the bottom, and B_3 in the middle and B_2 at the bottom. So, we see if the book B_1 is at the top in anyone of the shelves there are 2 ways of arrangement.

Similarly, when B_2 is at the top, thee are 2 ways of arrangement and when B_3 is at the top there are 2 ways of arrangement.

$\therefore$ Total no. of ways of putting the books = 2 + 2 + 2

$= 3 \times 2 \; (= 3!) = 6$

We see in all the above three cases, total no. of ways of putting all the 3 books = 3!

Thus, we conclude that total no. of arrangements of n different things, all (the n) things taken at a time = ${}^nP_n = n!$ ..... (i)

We have ${}^nP_r = \frac{n!}{(n-r)!}$

$\therefore {}^nP_n = \frac{n!}{(n-n)!} = \frac{n!}{0!}$ ..... (ii)

So, $n! = \frac{n!}{0!}$ [equating (i) and (ii)]

or, $0! = \frac{n!}{n!} = 1$

Thus, we get $0! = 1$

To find the no. of permutations or arrangements of n different things taken r at a time when each thing can be repeated any no. of times.

Note: ${}^nP_r$, the no. of permutations or arrangements of n different things taken r at a time = $\frac{n!}{(n-r)!}$, when repetition is not allowed.

Now, suppose a painter has to paint a 4-digit number on a number plate of vehicles using the digits 1, 2, ..., 9 and repetition of digits is allowed (i.e. he can paint the numbers 1111, 1112, 1211, 1121, 1221, 2121, etc.).

thousands place	hundreds place	tens place	units place
any one of the 9 digits 1, 2, 3, ...9 in 9 ways			

He can mark any one digit out of the 9 digits 1, 2, 3, ... 9 at thousands place on the number plate in 9 ways.

After marking at the thousands place he has again 1, 2, ..., 9 (total 9) digits (as repetition of digits is allowed). So, he can mark the hundreds place in 9 ways. Similarly, each of tens place and units place can be marked in 9 ways.

Thus, he can mark a total of $9 \times 9 \times 9 \times 9$ (= 9×9 .... 4 times = 9^4) = 6561, number plates

Now, we conclude the no. of permutations or arrangements of n different things taken r at a time, when repetition is allowed = $n \times n \times n$ ..... r times = n^r ways.

Now, suppose the painter has to paint 4-digit numbers on the number plates using all the ten digits (0, 1, 2, ... 9) and repetition of digits is allowed.

thousands place	hundreds place	tens place	units place
any one of the 9 digits 1, 2, ... 9 in 9 ways	any one of the 10 digits 0, 1, 2, ... 9 in 10 ways	Similarly in 10 ways	Similarly in 10 ways

Note: If he puts 0 at thousands place, the 4-digit no. will reduce to a 3-digit no. Thus he cannot do so.

$\therefore$ Reqd. total no. = $9 \times 10 \times 10 \times 10 = 9000$

From the examination point of view the following few results are useful. Without going into details you should simply remember the following results:

I. If ${}^nC_x = {}^nC_y$ then either $x = y$ or $x + y = n$

II. No. of permutations of n things out of which P are alike and are of one type, q are alike and are of the other type, and the remaining all are different $= \frac{n!}{p!\,q!}$

III. No. of selections of r things ($r \le n$) out of n identical things is 1.

IV. Total no. of selections of zero or more things from n identical things = n + 1.

V. Total no. of selections of zero or more things from n different things $= {}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC = 2n$

VI. No. of ways to distribute (or divide) n identical things among r persons where any person may get any no. of things $= {}^{n+r-1}C_{r-1}$.

Solved Examples

Ex. 1: If ${}^nP_3 = 210$, find n.

Soln: $\frac{n!}{(n-3)!} = 210$

or, $n \times (n-1) \times (n-2) = 7 \times 6 \times 5 \quad \therefore \; n = 7$

Ex. 2: If ${}^7P_r = 210$, find r.

Soln: $\frac{7!}{(7-r)!} = 210$

or, $7 \times 6 \times .... \times (7 - r + 1) = 7 \times 6 \times 5$

$\Rightarrow 7 - r + 1 = 5$ or, $8 - r = 5$

$\therefore \; r = 8 - 5 = 3$

Ex. 3: If ${}^{m+n}P_4 = 3024$ and ${}^{m-n}P_4 = 120$, find m and n.

Soln: ${}^{m+n}P_4 = \frac{(m+n)!}{(m+n-4)!}$

$= (m + n) \times (m + n - 1) \times (m + n - 2) \times (m + n - 3)$

$= 3024 = 9 \times 8 \times 7 \times 6$

$\therefore \; m + n = 9$ ..... (i)

Again, ${}^{m-n}P_4 = (m - n) \times (m - n - 1) \times (m - n - 2) \times (m - n - 3)$

$= 120 = 5 \times 4 \times 3 \times 2$

$\Rightarrow m - n = 5$ ...... (ii)

From the equations (i) and (ii), we get m = 7 and n = 2

Ex. 4: If ${}^nC_2 = {}^nC_5$, find n.

Soln: $\frac{n!}{2!\,(n-2)!} = \frac{n!}{5!\,(n-5)!}$

or, $5!\,(n-5)! = 2!\,(n-2)!$

or, $5 \times 4 \times 3 \times 2 \times (n-5)! = 2 \times (n-2) \times (n-3) \times (n-4) \times (n-5)!$

or, $5 \times 4 \times 3 = (n-2) \times (n-3) \times (n-4)$

$\therefore \; n - 2 = 5$ or, $n = 7$

Note: Whenever ${}^nC_x = {}^nC_y$ and $x \ne y$, then n must be equal to x + y.

Here $2 \ne 5 \quad \therefore \; n = 2 + 5 = 7$

Ex. 5: How many quadrilaterals can be formed by joining the vertices of an octagon?

Soln: A quadrilateral has 4 sides or 4 vertices whereas an octagon has 8 sides or 8 vertices.

$\therefore$ Reqd no. of quadrilaterals $= {}^8C_4 = \frac{8!}{4!\,(8-4)!}$

$= \frac{8 \times 7 \times 6 \times 5}{24} = 70$

Ex. 6: How many numbers of five digits can be formed with the digits 1, 3, 5, 7 and 9, no digit being repeated?

Soln: The given no. of digits = 5

$\therefore$ Reqd no. $= {}^5P_5 = 5! = 120$

Note: If repetition of digits be allowed, then reqd no. $= 5^5 = 3125$.

Ex. 7: How many numbers of five digits can be formed with the digits 0, 2, 4, 6 and 8?

Soln:

ten thousands place	thousands place	hundreds place	tens place	units place
in ${}^4P_1$ i.e. 4 ways (any one of 2/4/6/8)	After filling up ten thousands place we are left with 4 digits, including 0, and the number of blank places is 4. So, in ${}^4P_4 = 4! = 24$ ways			

$\therefore$ Required number $= 4 \times 24 = 96$

Note: If repetition of digits be allowed then reqd. no. $= 4 \times 5^4 = 2500$ (For ten thousands place, we can't consider 0.

Ex. 8: How many numbers of five digits can be formed with the digits 0, 1, 2, 3, 4, 6 and 8?

Soln: Here nothing has been said about the repetition of digits. So, it is understood that repetition of digits is not allowed.

ten thousands place	thousands place	hundreds place	tens place	units place
${}^6P_1 = 6$ ways (exclude 0)	After filling up ten thousands place we are left with 6 digits (including 0) and the blank places are 4. So, in ${}^6P_4 = 6 \times 5 \times 4 \times 3 = 360$ ways			

$\therefore$ Reqd no. $= 6 \times 360 = 2160$

Ex. 9: How many even numbers of three digits can be formed with the digits 0, 1, 2, 3, 4, 5 and 6?

Soln: Case (i): When 0 occurs at units place:

hundreds place	tens place	units place
${}^6P_2 = 6 \times 5 = 30$ ways		Only 0, i.e. in 1 way

Total of such numbers $= 30 \times 1 = 30$

Case (ii): When 0 does not occur at units place:

hundreds place	tens place	units place
After filling of units place we are left with 6 digits but 0 cannot occur at hundreds place. We are finally left with 5 digits, so in ${}^5P_1 = 5$ ways.	After filling up units place and hundreds place we are left with 5 digits (including 0) so in 5 ways.	any one of 2/4/6 in 3 ways

Total of such numbers $= 5 \times 5 \times 3 = 75$

$\therefore$ Reqd no. $= 30 + 75 = 105$

Ex. 10: How many nos. greater than 800 and less than 4000 can be made with the digits 0, 1, 2, 4, 5, 7, 8, 9, no number (digit) occurring more than once in the same number?

Soln: Case 1: 3-digit numbers:

hundreds place	tens place	units place
either 8 or 9, i.e. in 2 ways	${}^7P_2 = 7 \times 6 = 42$ ways	

$\therefore$ Total no. $= 2 \times 42 = 84$

Case 2: 4-digit numbers:

thousands place	hundreds place	tens place	units place
Either 1 or 2, i.e. in 2 ways	${}^7P_3 = 7 \times 6 \times 5 = 210$ ways		

$\therefore$ Total no. $= 2 \times 210 = 420$

$\therefore$ Reqd no. $= 84 + 420 = 504$

Ex. 11: Find the number of words formed with the letters of the word 'DELHI' which

(i) begins with D, (ii) ends with I,

(iii) has the letter L always in the middle, and

(iv) begins with D & ends with I

Soln: There are 5 letters in the word 'DELHI'.

Case (i):

D				
1 way	$^4P_4 = 24$ ways			

$\therefore$ Reqd no. $= 1 \times 24 = 24$

Case (ii):

				I
$^4P_4 = 24$ ways				1 way

$\therefore$ Reqd no. $= 1 \times 24 = 24$

Case (iii):

		L		
		1 way		

Remaining 4 places will be filled in $= {}^4P_4 = 24$ ways

$\therefore$ Reqd no. $= 24$

Case (iv):

D					I
1 way	$3P_3 = 6$ ways				1 way

$\therefore$ Reqd no. $= 6 \times 1 \times 1 = 6$

Ex. 12: How many words can be formed with the letters of the wor 'EQUATION'?

Soln: No. of permutations or arrangements of n different things, take all at a time, i.e. $^nP_n = n!$

Here, there are 8 letters in the word EQUATION.

$\therefore$ Reqd no. of words $= 8! = 40320$

Ex. 13: How many words beginning with vowels can be formed wit the letters of the word EQUATION?

Soln: There are 8 letters in the word EQUATION.

A/E/I/O/U							
in 5 ways	in $^7P_7 = 7! = 5040$						

$\therefore$ Reqd. no. $= 5 \times 5040 = 25200$

Ex. 14: How many words can be formed with the letters of the word INTERNATIONAL?

Soln: There are 13 letters in the word INTERNATIONAL, of which N occurs thrice, each of I, T and A occurs twice, and the rest are different.

$\therefore$ reqd. no. $= \dfrac{13!}{3!\ 2!\ 2!\ 2!}$

$= \dfrac{13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2}{6 \times 2 \times 2 \times 2}$

$= 13 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 3 \times 2 = 129729600$

Ex. 15: In how many ways can 4 boys and 5 girls be seated in a row so that they are alternate?

Soln:

G	B	G	B	G	B	G	B	G

The diagram shows the possible arrangement of sitting of boys and girls.

Now, 4 boys can be seated in 4 places in $^4P_4 = 4!$ and 5 girls in 5 places in 5! ways.

$\therefore$ Reqd no. of ways $= 4! \times 5! = 24 \times 120 = 2880$

Ex. 16: There are 4 boys and 4 girls. In how many ways can they be seated in a row so that all the girls do not sit together?

Soln: Total no. of persons $= 4 + 4 = 8$

When there is no restriction they can be seated in a row in 8! ways. But when all the 4 girls sit together, we can consider the group of 4 girls as one person. Therefore, we have only 4 (no. of boys) + 1 = 5 persons, who can be arranged in a row in 5! ways. But the 4 girls can be arranged among themselves in $^4P_4 = 4!$ ways.

$\therefore$ No. of ways when all the 4 girls are together $= 5! \times 4!$

$\therefore$ Reqd no. of ways in which all the 4 girls do not sit together

$= 8! - 5! \times 4!$

$= 8 \times 7 \times 6 \times 5! - 5! \times 24 = 5!(336 - 24) = 120 \times 312 = 37440$

Ex. 17: How many different words can be formed with the letters of the word EQUATION without changing the relative order of the vowels and consonants?

Soln: In the word EQUATION, the 5 vowels E, U, A, I and O occupy the 5 places 1, 3, 4, 6 and 7 respectively whereas the 3 consonants Q, T and N occupy the 3 places 2, 5, and 8 respectively. All the letters of the word are different i.e. there is no repetition of any letter.

The 5 vowels can be arranged in the 5 places in $^5P_5 = 5! = 120$ ways whereas the 3 consonants can be arranged in the 3 places in $^3P_3 = 3! = 6$ ways.

$\therefore$ Reqd no. $= 120 \times 6 = 720$

Ex. 18: How many words can be formed out of the letters of the word BANANA so that the consonants occupy the even places?

Soln:

1	2	3	4	5	6

The word BANANA contains 6 letters out of which A occurs thrice and N occurs twice.

The 3 consonants B and N (which occurs twice) can be arranged at the 3 even places 2, 4 and 6 in $\frac{3!}{2!} = 3$ ways

The remaining 3 odd places can be arranged with triple A in $\frac{3!}{3!}$ = 1 way

$\therefore$ Reqd no. of words = 3 × 1 = 3

Ex. 19: Find the no. of ways in which 4 identical balls can be distributed among 6 identical boxes, if not more than one ball goes into a box?

Soln: No. of identical balls = 4 and no. of identical boxes = 6

Now, distributing 4 identical balls among 6 identical boxes when not more than one ball goes into a box, implies to select 4 boxes from among the 6 boxes, which can be done in ${}^6C_4 = \frac{6!}{4!\ 2!} = 15$ ways.

Ex. 20: Find the no. of triangles formed by joining the vertices of a polygon of 12 sides.

Soln: A polygon of m sides will have m vertices. A triangle will be formed by joining any three vertices of the polygon.

$\therefore$ No. of triangles formed $= {}^mC_3 = \frac{m!}{3!\ (m-3)!}$

$= \frac{m \times (m-1) \times (m-2) \times (m-3)!}{6 \times (m-3)!} = \frac{m \times (m-1) \times (m-2)}{6}$

Putting m = 12, we get

Reqd. no. of triangles $= \frac{12 \times 11 \times 10}{6} = 220$

Ex. 21: Find the no. of diagonals of a polygon of 12 sides.

Soln: A polygon of m sides will have m vertices. A diagonal or a side of the polygon will be formed by joining any two vertices of the polygon.

No. of diagonals of the polygon + no. of sides of the polygon (= m) $= {}^mC_2$

$\therefore$ No. of diagonals of the polygon $= {}^mC_2 - m$

$= \frac{m!}{2!\ (m-2)!} - m = \frac{m \times (m-1)}{2} - m$

$= \frac{m(m-1) - 2m}{2} = \frac{m(m-3)}{2}$

Putting m = 12, we get the reqd. no. of diagonals $= \frac{12 \times 9}{2} = 54$

Ex. 22: In a party every person shakes hand with every other person. If there was a total of 210 handshakes in the party, find the no. of persons who were present in the party.

Soln: For each selection of two persons there will be one handshake. So, no. of handshakes in the party $= {}^nC_2$, where n = no. of persons.

Now, ${}^nC_2 = 210$ (given)

or, $\frac{n \times (n-1)}{2} = 210$

or, $n \times (n-1) = 2 \times (2 \times 3 \times 5 \times 7) = 21 \times 20$

$\therefore$ n = 21

Ex. 23: There are 5 members in a delegation which is to be sent abroad. The total no. of members is 10. In how many ways can the selection be made so that a particular member is always (i) included (ii) excluded?

Soln: (i) Selection of one particular member can be done in $= {}^1C_1 = 1$ way. After the selection of the particular member, we are left with 9 members and for the delegation, we need 4 members more. So selection can be done in ${}^9C_4$ ways.

$\therefore$ Reqd no. of ways of selection $= {}^1C_1 \times {}^9C_4$

$= \frac{1 \times 9 \times 8 \times 7 \times 6}{24} = 126$

(ii) When one particular person has to be always excluded from the 5-member delegation, we are left with 10 - 1 = 9 persons. So selection can be done in ${}^9C_5$ ways.

$\therefore$ Reqd no. $= {}^9C_5 = 126$

Ex. 24: Find the no. of triangles formed by the 11 points (out of which 5 are collinear) in a plane.

Soln: Let us suppose that the 11 points are such that no three of them are collinear. Now a triangle can be formed by joining any three of these 11 points. So, selection of any 3 out of the 11 points can be done in ${}^{11}C_3$ ways.

No. of triangles formed by 5 points when none of the 3 or more points are collinear = $^5C_3$

But from 3 or more than 3 collinear points no triangle can be formed.

$\therefore$ Reqd. no. of triangles = $^{11}C_3 - {^5C_3} = \frac{11 \times 10 \times 9}{6} - \frac{5 \times 4}{2}$

$= 165 - 10 = 155$

Ex. 25: A person has 12 friends out of which 7 are relatives. In how many ways can he invite 6 friends such that at least 4 of them are relatives?

Soln: No. of non-relative friends = 12 - 7 = 5

He may invite 6 friends in following ways:

I: 4 relatives + 2 non-relatives $\Rightarrow$ $^7C_4 \times {^5C_2}$

II: 5 relatives + 1 non-relative $\Rightarrow$ $^7C_5 \times {^5C_1}$

III: 6 relatives + 0 non-relative $\Rightarrow$ $^7C_6$

$\therefore$ Reqd. no. of ways = $^7C_4 \times {^5C_2} + {^7C_5} \times {^5C_1} + {^7C_6}$

$= 35 \times 10 + 21 \times 5 + 7 = 462$

Ex. 26: In an examination, a minimum of marks is to be scored in each 6 subjects to pass. In how many ways can a student fail?

Soln: The student will fail if he fails in one or more subjects out of 6 different subjects, i.e. $^6C_1 + {^6C_2} + {^6C_3} + {^6C_4} + {^6C_5} + {^6C_6} = ({^6C_0} + {^6C_1} + ... + {^6C_6}) - {^6C_0} = 2^6 - 1 = 64 - 1 = 63$ ways

Other approach: He fails if he fails in any of the 6 subjects. With each paper there two possibilities: either fail or pass. This way, for 6 subjects there are $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6 = 64$ possible cases. This also includes the case when he passes in all the 6 subjects. Thus, he can fail in 64 - 1 = 63 ways.

Note: There are 6 questions in a question paper. In how many ways can a student solve one or more questions? This is the same equation. So, answer is 63.

Ex. 27: In how many ways can 12 different books be divided equally among (a) 4 persons (b) 3 persons?

Soln: (a) Each person will get $12 \div 4 = 3$ books.

Now, first person can be given 3 books out of 12 different books in $^{12}C_3$ ways. Second person can be given 3 books out of the rest (12 - 3 =) 9 books in $^9C_3$ ways. Similarly, third person in $^6C_3$ and the fourth person in $^3C_3$ ways.

$\therefore$ Reqd. no. of ways = $^{12}C_3 \times {^9C_3} \times {^6C_3} \times {^3C_3}$

$= \frac{12!}{3!\,9!} \times \frac{9!}{3!\,6!} \times \frac{6!}{3!\,3!} \times \frac{3!}{3!\,0!}$

$= \frac{12!}{(3!)^4} = \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3!}{3! \times 6 \times 6 \times 6} = 369600$

(b) Now each person will get $12 \div 3 = 4$ books.

Similarly, required no. of ways = $^{12}C_4 \times {^8C_4} \times {^4C_4}$

$= \frac{12!}{4!\,8!} \times \frac{8!}{4!\,4!} \times \frac{4!}{4!\,0!} = \frac{12!}{(4!)^3}$

$= \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4!}{4! \times 24 \times 24} = 34650$

Ex. 28: In how many ways can 12 different books be divided equally among (a) 4 sets or groups; (b) 3 sets or groups?

Soln: (a) Reqd. no. of ways = $\frac{^{12}C_3 \times {^9C_3} \times {^6C_3} \times {^3C_3}}{4!}$

$= \frac{12!}{4!\,(3!)^4} = 15400$

(b) Required no. of ways = $\frac{^{12}C_4 \times {^8C_4} \times {^4C_4}}{3!} = \frac{12!}{3!\,(4!)^3} = 5775$

Ex. 29: How many different letter arrangements can be made from the letters of the word EXTRA in such a way that the vowels are always together?

Soln: Considering the two vowels E and A as one letter, the total no. of letters in the word 'EXTRA' is 4 which can be arranged in $^4P_4$, i.e. 4! ways and the two vowels can be arranged among themselves in 2! ways.

$\therefore$ reqd no. = $4! \times 2! = 4 \times 3 \times 2 \times 1 \times 2 \times 1 = 48$

Ex. 30: Letters of the word DIRECTOR are arranged in such a way that all the vowels come together. Find out the total number of ways for making such arrangement.

1) 4320 2) 2720 3) 2160
4) 1120 5) None of these

Soln: Taking all vowels (IEO) as a single letter (since they come together) there are six letters among which there are two R.

Hence no. of arrangements = $\frac{6!}{2!} \times 3! = 2160$

Three vowels can be arranged in 3! ways among themselves, hence multiplied with 3!. Hence, answer is (3).

Ex. 31: How many different letter arrangements can be made from the letters of the word RECOVER?

1) 1210 2) 5040 3) 1260
4) 1200 5) None of these

Soln: 3; Possible arrangements are : $\frac{7!}{2!\,2!} = 1260$

[division by 2 times 2! is because of the repetition of E and R]

Ex. 32: 4 boys and 2 girls are to be seated in a row in such a way that the two girls are always together. In how many different ways can they be seated?

1) 1200 2) 7200 3) 148 4) 240 5) None of these

Soln: 4; Assume the 2 given students to be together (i.e one). Now there are five students.

Possible ways of arranging them are = 5! = 120

Now they (two girls) can arrange themselves in 2! ways.

Hence total ways = 120 × 2 = 240

Ex. 33: On the occasion of a certain meeting each member gave shake-hand to the remaining members. If the total shakehands were 28, how many members were present for the meeting?

1) 14 2) 7 3) 9 4) 8 5) None of these

Soln: 4; A combination of 2 persons gives a result of one handshake. If we suppose that there are x persons then there are total ${}^xC_2$ handshakes, i.e. ${}^xC_2 = 28$

or, $\frac{x(x-1)}{2!} = 28$

or, $x(x-1) = 56 = 7 \times 8$

$\therefore x = 8$

Ex. 34: How many different numbers of six digits (without repetition of digits) can be formed from the digits 3, 1, 7, 0, 9, 5?

(i) How many of them will have 0 in the unit place?

(ii) How many of them are divisible by 5?

(iii) How many of them are not divisible by 5?

Soln: The total numbes of 6 digit numbers = 6 ! - 5! = 600.

[Note that 5 ! numbers are for those having 0 in first place which will be excluded.]

(i) 5 ! = 120

(ii) Numbers are divisible by 5 if

(a) they will have zero in the unit place and hence the remaining 5 can be arranged in 5 ! = 120 ways.

(b) they will have 5 in the last place and as above we will have 5 ! = 120 ways. These will also include numbers which will have zero in the first place (ie, number of 5 digits). Therefore the numbers having zero in 1st and 5 in unit place will be 4 !.

$\therefore$ Therefore 6 digit numbers having 5 in the end will be

5 ! - 4 ! = 120 - 24 = 96.

Therefore the total number of 6 digits numbers divisible by 5 is 120 + 96 = 216.

(iii) Not divisible by 5.

Total - (divisible by 5) = 600 - 216 = 384.

Ex. 35: Find the total number of 9 digits numbers which have all different digits.

Soln: Total No. of 9 digit numbers = ${}^{10}P_9 - {}^9P_8 = \frac{10!}{1!} - \frac{9!}{1!}$

$= 9\,!\,(10 - 1) = 9\,(9\,!)$

$= 9 \times (9 \times 8 \times 7 \times 6\,!)$

$= 81 \times 56 \times 720 = 3265920.$

Alternative. The number is to be of 9 digits. The first place (from left) can be filled in 9 ways only (as zero can not be in the first place). Having filled up the first place the remaining 8 places can be filled up by the remaining 9 digits in ${}^9P_8 = 9\,!$ ways. Hence the total is $9 \times 9\,!$.

Ex. 36: (a) How many different arrangements can be made by using all the letters in the word MATHEMATICS? How many of them begin with C? How many of them begin with T?

(b) How many words can be formed dy taking 4 letters at a time out of the letters of the word MATHEMATICS.

Soln: (a) There are 11 letters Two M, Two A, Two T, H, E, I, C, S.

(ii) Hence the number of words by taking all at a time

$= \frac{11\,!}{2\,!\,2\,!\,2\,!} = \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6\,!}{2 \times 2 \times 2}$

$= 990 \times 7 \times 720 = 990 \times 5040 = 4989600.$

To Begin with C.

Having fixed C at first place we have 10 letters in which 2 are M, 2 are A and 2 are T and the rest 4 are different.

Hence the number of words will be

$$\frac{10!}{2!\,2!\,2!} = \frac{10 \times 9 \times 8 \times 7 \times 6!}{2 \times 2 \times 2} = 90 \times 7 \times 720$$
$$= 630 \times 720 = 453600.$$

To Begin with T.

Having fixed T in first place we will have only 10 letters out of which 2 are M_s and 2 are A_s and rest six are H, E, I, C, S and T. Hence the number of words is

$\frac{10!}{2!\,.\,2!} = 907200.$ (i.e. double of part (ii))

(b) We can choose 4 letters from the 11 listed in part (a) as under.

All the four different

We have 8 different types of letters and out of these 4 can be chosen in $^8P_4 = \frac{8!}{4!} = 8 \times 7 \times 6 \times 5 = 1680$

Two different and two alike.

We have 3 pairs of like letters out of which one pair can be chosen in $^3C_1 = 3$ ways. Now we have to choose two out of the remaining 7 different types of letters which can be done in

$= {}^7C_2 = \frac{7!}{5!\,2!} = \frac{7 \times 6}{2} = 21$ ways

Hence the total number of groups, of 4 letters in which 2 are different and 2 are alike is $3 \times 21 = 63$

Let one such group be M, H, M, I.

Each such group has 4 letters out of which 2 are alike, can be arranged amongst themselves in $\frac{4!}{2!} = 12$ ways.

Hence the total number of words is $63 \times 12 = 756$.

Two alike of one kind and two alike of other kind.

Out of 3 pairs of like letters we can choose 2 pairs in $^3C_2$ ways = 3 ways.

One such group is MM AA.

These four letters out of which 2 are alike of one kind and 2 are alike of other kind, can be arranged in $\frac{4!}{2!\,2!} = 6$ ways.

Hence the total number of words of this type is $3 \times 6 = 18$.

Therefore total number of 4 letter words is $1680 + 756 + 18 = 2454$.

EXERCISES

1. If $^{15}C_{r-1} : {}^{15}C_r = 5 : 11$, find r.
2. If $^nC_{n-6} = 462$, find n.
3. How many numbers of five digits can be formed with the digits 0, 1, 2, 4, 6 and 8?
4. How many odd numbers of three digits can be formed with the digits 0, 1, 2, 3, 4, 5 and 6?
5. How many numbers of 4 digits, divisible by 5, can be formed with the digits 0, 2, 5, 6 and 9?
6. How many words of 4 letters beginning with either A or E can be formed with the letters of the word EQUATION?
7. In how many ways can be the letters of the word INTERMEDIATE be arranged?
8. How many words can be formed out of the letters of the word ARTICLE so that the vowels occupy the even places?
9. How many words can be formed with the letters used in EQUATION when any letter may be repeated any no. of times?
10. How many different words of 5 letters can be formed with the letters of the word EQUATION so that the vowels occupy odd places?
11. If 7 parallel lines are intersected by another 7 parallel lines, find the no. of parallelograms thus formed.
12. There are five students A, B, C, D and E.
    (i) In how many ways can they sit so that B and C do not sit together?
    (ii) In how many ways can a committee of 3 members be formed so that A is always included and E is always excluded?
13. There are 12 points in a plane out of which 5 are collinear. Find the no. of straight lines formed by joining them.
14. A candidate is required to answer 6 out of 10 questions which are divided into groups, each containing five questions. In how may ways can he answer the questions, if he is not allowed to attempt more than 4 questions from a group?
15. A committee of 8 students is to be formed out of 5 boys and 8 girls. In how many ways can it be done so that the no. of girls is not less than the no. of boys?
16. From 6 gentlemen and 4 ladies a committee of 5 is to be formed. In how many ways can this be done if the committee is to include at least one lady?
17. A candidate is required to answer 6 out of 10 questions which are divided into two groups each containing 5 questions and he is not permitted to

attempt more than 4 from each group. In how many ways can he make up his choice?

18. How many different groups can be selected for playing tennis out of 4 ladies and 3 gentlemen there being one lady and one gentleman on each side?

Solutions

1. Answer = 5
2. Answer = 11
3. Required no. of numbers $= 5 \times {}^5P_4 = 5 \times 5! = 5 \times 120 = 600$
4. Required no. of numbers $= 5 \times 5 \times 3 = 75$
5. For a digit to be divisible by 5, its unit digit must be either 0 or 5.
   When there is 0 at the unit place, the number of numbers $= {}^4P_3 \times 1 = 24$
   When there is 5 at the unit place, the number of numbers
   $= 3 \times {}^3P_2 \times 1 = 3 \times 6 \times 1 = 18$
   $\therefore$ total required numbers $= 24 + 18 = 42$
6. Required no. of such words $= {}^2P_1 \times {}^7P_3 = 2 \times (7 \times 6 \times 5) = 420$
7. There are 12 letters in the given word, out of which E occurs thrice, each of I and T occurs twice, and the rest occur only once.
   $\therefore$ total no. of such words $= \dfrac{12!}{3! \times 2! \times 2!} = 19958400$
8. There are three vowels and four consonants. So, the three vowels can be put in 3 even places in ${}^3P_3 = 6$ ways. And the four consonants can be arranged in 4 odd places in ${}^4P_4 = 24$ ways.
   $\therefore$ total no. of words $= 6 \times 24 = 144$.
9. Required no. of words $= 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8$
10. The three odd places can be occupied by 5 vowels in ${}^5P_3$
    $= 5 \times 4 \times 3 = 60$ ways.
    Whereas the two even places can be occupied by 3 consonants in ${}^3P_2$
    $= 3 \times 2 = 6$ ways.
    $\therefore$ Required no. of words $= 60 \times 6 = 360$.
11. No. of such parallelograms $= {}^7C_2 \times {}^7C_2 = 21 \times 21 = 441$
12. (i) No. of ways in which A and B sit together $= 2 \times 4! = 48$
    $\therefore$ No. of ways in which A and B do not sit together $= 5! - 48 = 120 - 48 = 72$

    (ii) After selection of A, we are left with 3 persons (excluding E) out of which 2 are to be selected.
    $\therefore$ total no. of required ways $= 1 \times {}^3C_2 = 1 \times 3 = 3$.
13. Required no. of straight lines $= {}^{12}C_2 - {}^5C_2 + 1 = 57$
14. Total no. of ways $= \left({}^5C_2 \times {}^5C_4\right) + \left({}^5C_3 \times {}^5C_3\right) + \left({}^5C_4 \times {}^5C_2\right) = 200$
15. Total no. of ways
    $= {}^8C_8 + \left({}^5C_1 \times {}^8C_7\right) + \left({}^5C_2 \times {}^8C_6\right)$
    $+ \left({}^5C_3 \times {}^8C_5\right) + \left({}^5C_4 \times {}^8C_4\right) = 1230$
16. 6 Gentlemen, 4 Ladies; Committee of 5.
    At least one lady to be included; the combinations are:
    (1L, 4G), or (2L, 3G), or (3L, 2G), or (4L, 1G)
    ${}^4C_1 \times {}^6C_4 + {}^4C_2 \times {}^6C_3 + {}^4C_3 \times {}^6C_2 + {}^4C_4 \times {}^6C_1$
    $= 60 + 120 + 60 + 6 = 246$.

Quicker Method:

[Total no. of committees (from 6 men & 4 ladies)] - [No. of committee without lady (only men)]

$= {}^{10}C_5 - {}^6C_5 = \dfrac{10 \times 9 \times 8 \times 7 \times 6}{1 \times 2 \times 3 \times 4 \times 5} - \dfrac{6}{1} = 252 - 6 = 246$

17. Group A and group B consists of 5 questions each out of which 6 are to be attempted but not more than 4 from any group
    (4A, 2B), (3A, 3B), (2A, 4B)
    ${}^5C_4 \times {}^5C_2 + {}^5C_3 \times {}^5C_3 + {}^5C_2 \times {}^5C_4 = 50 + 100 + 50 = 200$.
18. 4 Ladies 3 Gentlemen
    A B
    1L, 1G 1L, 1G
    Selection of side A $= {}^4C_1 \times {}^3C_1 = 4 \times 3 = 12$
    After selecting side A, we are left with 3L and 2G from which one each is to be chosen for side B.
    Selection of side B $= {}^3C_1 \times {}^2C_1 = 3 \times 2 = 6$
    Hence the number of ways of selection for the team $= 12 \times 6 = 72$.

Probability

Probability is a measurement of uncertainty. In this chapter chances of the happening of events are considered.

Terminology

Random Experiment. It is an experiment which if conducted repeatedly under homogeneous conditions does not give the same result. The result may be any one of the various possible 'outcomes'. Here the result is not unique (or the same every time). For example, if an unbiased dice is thrown it will not always fall with any particular number up. Any of the six numbers on the dice can come up.

Trial and Event. The performance of a random experiment is called a **trial** and the outcome an **event**. Thus throwing of a dice would be called a trial and the result (falling of any one of the six numbers 1, 2, 3, 4, 5, 6) an event.

Events could be either **simple or compound** (also called **composite**). An event is called **simple** if it corresponds to a single possible outcome. Thus in tossing a dice, the chance of getting 3 is a **simple event** (because 3 occurs in the dice only once). However, the chance of getting an odd number is a **compound event** (because odd numbers are more than one, i.e. 1, 3 and 5).

Exhaustive Cases. All possible outcomes of an event are known as *exhaustive cases*. In the throw of a single dice the exhaustive cases are 6 as the dice has only six faces each marked with a different number. However, if 2 dice are thrown the exhaustive cases would be 36 (6×6) as there are 36 ways in which two dice can fall. Similarly, the number of exhaustive cases in the throw of 2 coins would be four (2×2), i.e. HH, TT, HT and TH (where H stands for head and T for tail).

Favourable Cases. The number of outcomes which result in the happening of a desired event are called *favourable cases*. Thus in a single throw of a dice the number of favourable cases of getting an odd number is three, i.e. 1, 3 and 5. Similarly, in drawing a card from a pack, the cases favourable to getting a spade are 13 (as there are 13 spade cards in the pack).

Mutually Exclusive Events. Two or more events are said to be *mutually exclusive* if the happening of any one of them excludes the happening of all others in a single (i.e. same) experiment. Thus in the throw of a single dice the events 5 and 6 are mutually exclusive because if the event 5

happens no other event is possible in the same experiment. Here *one and only one* of the events can take place at a time, excluding all others.

Equally Likely Cases. Two or more events are said to be *equally likely* if the chances of their happening are equal, i.e. there is no preference of any one event to the other. Thus in a throw of an unbiased dice, the coming up of 1, 2, 3, 4, 5, or 6 is equally likely. In the throw of an unbiased coin, the coming up of head or tail is equally likely.

Independent and Dependent Events. An event is said to be *independent* if its happening is not affected by the happening of other events and if it does not affect the happening of other events. Thus in the throw of a dice repeatedly, coming up of 5 on the first throw is independent of coming up of 5 again in the second throw.

However if we are successively drawing cards from a pack (without replacement) the events would be dependent. The chance of getting a King on the first draw is 4/52 (as there are 4 Kings in a pack). If this card is not replaced before the second draw, the chance of getting a King again is 3/51 as there are now only 51 cards left and they contain only 3 Kings.

If, however, the card is replaced after the first draw, i.e. before the second draw, the events would remain independent. In each of the two successive draws the chance of getting a king would be 4/52.

While tossing a coin you are not at all sure that Head will come. Tail may also come. However, you are sure that whatever will come, will be any one of the two: either Head or Tail.

Let us see the following trials:

i) A coin is tossed. The outcomes (results) may be {H, T} where H is Head (of the coin) and T is Tail (of the coin).

ii) Two coins are tossed. The outcomes may be {(H, H), (H, T), (T, H), (T, T)}.

iii) A dice is thrown. The outcomes may be {1, 2, 3, 4, 5, 6}.

iv) A person is selected randomly and is asked the day of the week on which he was born. The outcomes may be {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}.

v) A candidate appears at a certain examination. The outcomes may be {Pass, Fail}.

vi) One person is to form triangles from the 4 non-collinear points A, B, C and D. The outcomes may be {ΔABC, ΔABD, ΔACD, ΔBCD}.

vii) One person is to form 3-digit numbers from the given 4 digits 1, 2, 3, 4. The outcomes may be {123, 132, 124, ...}.

A set containing all possible outcomes of a random experiment is known as **Sample Space**.

For the above-mentioned trial (i), number of **Sample Space** n(S) = 1 (for H) + 1 (for T) = 2

Similarly, for trial (ii), n(S) = 1 + 1 + 1 + 1 = 4; for (iii), n(s) = 6; for (iv), it is 7; for (v), it is 2; for (vi) it is $^4C_3 = 4$; for (vii), it is $^4P_3 = 4 \times 3 \times 2 = 24$

Each outcome of a Sample Space is called an **Event**. Thus, in the experiment (iv), {Sunday}, {Monday},..... {Saturday} are events.

We also see that total no. of events = n(S).

In all the above mentioned experiments it is reasonable to assume that each outcome is as likely to occur as any other outcome. While tossing a coin the chance of Head to come is the same as the chance of Tail.

Now, Probability of an event (E) is denoted by P(E) and is defined as $P(E) = \frac{n(E)}{n(S)} = \frac{\text{no. of desired events}}{\text{total no. of events (i.e. no. of Sample Space)}}$

When a coin is tossed, as for example, probability of Head coming, $P(H) = \frac{1}{2} = P(T)$, probability of Tail coming.

When two coins are tossed, probability for Heads coming on both the coins $= \frac{1}{4}$

Probability of at least one Tail coming $= \frac{1+1+1}{4} = \frac{3}{4}$

Solved Examples:

Ex. 1: A dice is thrown. What is the probability that the number shown on the dice is (i) an even no.; (ii) on odd no.; (iii) a no. divisible by 2; (iv) a no. divisible by 3; (v) a no. less than 4; (vi) a no. less than or equal to 4; (vii) a no. greater than 6; (viii) a no. less than or equal to 6.

Soln: In all the above cases, S = {1, 2, 3, 4, 5, 6}, n (S) = 6.

(i) E (an even no.) = {2, 4, 6}, n (E) = 3

$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

(ii) E (an odd no.) = {1, 3, 5}, n(E) = 3

$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

(iii) E (a no. divisible by 2) = {2, 4, 6}, n(E) = 3 $\therefore$ $P(E) = \frac{3}{6} = \frac{1}{2}$

(iv) E (a no. divisible by 3) = {3, 6}, n(E) = 2 $\therefore$ $P(E) = \frac{2}{6} = \frac{1}{3}$

(v) E (a no. less than 4) = {1, 2, 3}, n(E) = 3 $\therefore$ $P(E) = \frac{3}{6} = \frac{1}{2}$

(vi) E (a no. less than or equal to 4) = {1, 2, 3, 4} n(E) = 4

$\therefore P(E) = \frac{4}{6} = \frac{2}{3}$

(vii) E (a no. greater than 6) = {}, i.e. there is no number greater than 6 in the Sample Space, $\therefore P(E) = \frac{0}{6} = 0$

Probability of an impossible event = 0

(viii) E (a no. less than or equal to 6) = {1, 2, 3, 4, 5, 6}, n(E) = 6

$\therefore P(E) = \frac{6}{6} = 1$

Probability of a certain event = 1.

Note: $0 \le$ Probability of an event ≤ 1.

Ex. 2: Two coins are tossed. What is the probability of the appearing of (i) at most one head (ii) at most two heads?

Soln: n(S) = 4 = {(T, T), (H, T), (T, H), (H, H)}

For (i), E (of appearing at most one head) = {HT, TH, TT}, n(E) = 3

$\therefore P(E) = \frac{3}{4}$

For (ii), E (of appearing atmost two heads) = {HH, HT, TH, TT}, n (E) = 4

$\therefore P(E) = \frac{4}{4} = 1$

Ex. 3: A positive integer is selected at random and is divided by 7. What is the probability that the remainder is (i) 1; (ii) not 1?

Soln: When a positive integer is divided by 7, the remainder may be 0 or 1 or 2 or 3 or 4 or 5 or 6; n(S) = 7

For (i), E(1) = {1}, n(E) = 1.

$\therefore P(E) = \frac{1}{7}$

For (ii), E (not 1) = {0, 2, 3, 4, 5, 6}, n(E) = 6

$\therefore P(E) = \frac{6}{7}$

Note: We see that $E(1) + E(\text{not } 1) = \frac{1}{7} + \frac{6}{7} = 1$. If we represent an event by A then the event "not A" is represented by A′ or $\overline{A}$ or A^c and is known as *complement of an event* A. P(A) + P(A′) = 1, or, P(A′) = 1 - P(A)

Ex. 4: A dice is thrown. What is the probability that the number shown on the dice is not divisible by 3?

Soln: S = {1, 2, 3, 4, 5, 6}; n(S) = 6

E (not divisible by 3) = {1, 2, 4, 5}, n(E) = 4

$\therefore$ P (not divisible by 3) $= \frac{4}{6} = \frac{2}{3}$

Other Method: E (divisible by 3) = {3, 6}, n(E) = 2

$\therefore$ P (divisible by 3) $= \frac{2}{6} = \frac{1}{3}$

$\therefore$ P (not divisible by 3) = 1 - P (divisible by 3) $= 1 - \frac{1}{3} = \frac{2}{3}$

Ex. 5: (i): What is the chance that a leap year selected randomly will have 53 Sundays?

(ii) What is the chance, if the year selected in not a leap year?

Soln: (i): A leap year has 366 days so it has 52 complete weeks and 2 more days. The two days can be {Sunday and Monday, Monday and Tuesday, Tuesday and Wednesday, Wednesday and Thursday, Thursday and Friday, Friday and Saturday, Saturday and Sunday}, i.e. n(S) = 7.

Out of these 7 cases, cases favorable for more Sundays are {Sunday and Monday, Saturday and Sunday}, i.e., n(E) = 2

$\therefore P(E) = \frac{2}{7}$

(ii) When the year is not a leap year, it has 52 complete weeks and 1 more day that can be {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}, n(S) = 7

Out of these 7 cases, cases favorable for one more Sunday is {Sunday}, n(E) = 1 $\therefore P(E) = \frac{1}{7}$

Chart I: When two dices are thrown:

S = {(1, 1}, (1, 2), ..., (1, 6), (2, 1), (2, 2), ..., (2, 6), (3, 1), (3, 2), ..., (3, 6), (4, 1), ..., (4, 6), (5, 1), ..., (5, 6), (6, 1), ..., (6, 6)}

n(S) = 6 × 6 = 36

Sum of the numbers of the two dice		n(S)	Events	
(i)	(ii)		(i)	(ii)
2	12	1	{1,1}	{6,6}
3	11	2	{1,2}, {2,1}	{6,5}, {5,6}
4	10	3	{1,3}, {3,1}, {2,2}	{6,4}, {4,6}, {5,5}
5	9	4	{1,4}, {4,1}, {2,3}, {3,2}	{6,3}, {3,6}, {5,4}, {4,5}
6	8	5	{1,5}, {5,1}, {2,4}, {4,2}, {3,3}	{6,2}, {2,6}, {5,3}, {3,5}, {4,4
7		6	{1,6}, {6,1}, {2,5}, {5,2}, {4,3}, {3,4}	

Chart II: A pack of cards has a total of 52 cards:

Red suit (26)		Black suit (26)	
Diamond (13)	Heart (13)	Spade (13)	Club (13)

The numbers in the brackets show the respective no. of cards in th
category.

Each of Diamond, Heart, Spade and Club contains nine digit-cards
3, 4, 5, 6, 7, 8, 9 and 10 (a total of 9 × 4 = 36 digit-cards) along w
four Honour cards Ace, King, Queen and Jack (a total of 4 × 4 =
Honour cards).

Ex. 6: When two dice are thrown, what is the probability that

(i) sum of numbers appeared is 6 and 7?
(ii) sum of numbers appeared ≤ 8?
(iii) sum of numbers is an odd no?
(iv) sum of numbers is a multiple of 3?
(v) numbers shown are equal?
(vi) the difference of the numbers is 2?

Soln: Hint - use Chart I

(i) For 6, reqd probability $= \frac{n(E)}{n(S)} = \frac{5}{36}$

For 7, reqd probability $= \frac{6}{36} = \frac{1}{6}$

(ii) Desired sums of the numbers are 2, 3, 4, 5, 6, 7 and 8;
$n(S) = 1 + 2 + 3 + 4 + 5 + 6 + 5 = 26$

$\therefore$ reqd probability $= \frac{26}{36} = \frac{13}{18}$

(iii) Desired sums of the numbers are 3, 5, 7, 9 and 11;
$n(S) = 2 + 4 + 6 + 4 + 2 = 18 \quad \therefore$ reqd probability $= \frac{18}{36} = \frac{1}{2}$

(iv) Desired sums of the numbers are 3, 6, 9 and 12;
$n(S) = 2 + 5 + 4 + 1 = 12 \quad \therefore$ reqd probability $= \frac{12}{36} = \frac{1}{3}$

(v) Events = {1, 1}, {2, 2}, {3, 3}, {4, 4}, {5, 5}, {6, 6}; n(S) = 6

$\therefore P(E) = \frac{6}{36} = \frac{1}{6}$

(vi) Events = {3, 1}, {4, 2}, {5, 3}, {6, 4}, {4, 6}, {3, 5}, {2, 4}, {1, 3} or, n(S) = 8 $\quad \therefore P(E) = \frac{8}{36} = \frac{2}{9}$

Ex. 7: A card is drawn from a pack of cards. What is the probability that it is

(i) a card of black suit?
(ii) a spade card?
(iii) an honours card of red suit?
(iv) an honours card of club?
(v) a card having the number less than 7?
(vi) a card having the number a multiple of 3?
(vii) a king or a queen?
(viii) a digit-card of heart?
(ix) a jack of black suit?

Soln: (Hint - Use Chart II)

For all the above cases $n(S) = {}^{52}C_1 = 52$

(i) $\frac{26}{52} = \frac{1}{2}\left[\text{or, } \frac{{}^{26}C_1}{{}^{52}C_1} = \frac{26}{52} \; (\because {}^{n}C_1 = n)\right]$ (ii) $\frac{13}{52} = \frac{1}{4}$

(iii) $\frac{4 \times 2}{52} = \frac{2}{13}$ (iv) $\frac{4}{52} = \frac{1}{13}$

(v) $\frac{5 \times 4}{52} = \frac{5}{13}$ (vi) $\frac{3 \times 4}{52} = \frac{4}{13}$

(vii) P (a king) $= \frac{4}{52} = \frac{1}{13}$; P(a queen) $= \frac{4}{52} = \frac{1}{13}$

$\therefore$ P(a king or a queen) $= \frac{1}{13} + \frac{1}{13} = \frac{2}{13}$

(viii) $\frac{9}{52}$ (ix) $\frac{2}{52} = \frac{1}{26}$

Ex. 8: From a pack of 52 cards, 2 cards are drawn. What is the probability that it has

(i) both the Aces?
(ii) exactly one queen?

(iii) no honours card?
(iv) no digit-card?
(v) One King and one Queen?

Soln: For all the above cases, $n(S) = {}^{52}C_2 = \frac{52 \times 51}{2} = 26 \times 51$

(i) Total no. of Aces = 4

$\therefore n(E) = {}^4C_2 = \frac{4 \times 3}{2} = 6 \qquad \therefore P(E) = \frac{6}{26 \times 51} = \frac{1}{221}$

(ii) Total no. of Queens = 4

Selection of 1 Queen card out of 4 can be done in ${}^4C_1 = 4$ ways. He can select the remaining 1 card from the remaining (52 - 4 =) 48 cards. Now, cards in ${}^{48}C_1 = 48$ ways.

$\therefore n(E) = 4 \times 48 \qquad \therefore P(E) = \frac{4 \times 48}{26 \times 51} = \frac{32}{221}$

(iii) Total no. of honours card = 16

To have no honours card, he has to select two cards out of the remaining 52 - 16 = 36 cards which he can do in ${}^{36}C_2$ = $\frac{36 \times 35}{2} = 18 \times 35$ ways $\qquad \therefore P(E) = \frac{18 \times 35}{26 \times 51} = \frac{105}{221}$

(iv) $P(E) = \frac{{}^{16}C_2}{26 \times 51} = \frac{8 \times 15}{26 \times 51} = \frac{20}{221}$

(v) $n(E) = {}^4C_1 \times {}^4C_1 = 4 \times 4 = 16 \qquad \therefore P(E) = \frac{16}{26 \times 51} = \frac{8}{663}$

Ex. 9: From a pack of 52 cards, 3 cards are drawn. What is the probability that it has
(i) all three aces?
(ii) no queen?
(iii) one ace, one king and one queen?
(iv) one ace and two jacks?
(v) two digit-cards and one honours card of black suit?

Soln: For all the above cases, n(S)

$$= {}^{52}C_3 = \frac{52 \times 51 \times 50}{3 \times 2} = 26 \times 17 \times 50$$

(i) $n(E) = {}^4C_3 = 4 \qquad \therefore P(E) = \frac{4}{26 \times 17 \times 50} = \frac{1}{5525}$

(ii) $n(E) = {}^{48}C_3 = 8 \times 47 \times 46 \qquad \therefore P(E) = \frac{8 \times 47 \times 46}{26 \times 17 \times 50} = \frac{4324}{5525}$

(iii) $n(E) = {}^4C_1 \times {}^4C_1 \times {}^4C_1 = 4 \times 4 \times 4$

$\therefore P(E) = \frac{4 \times 4 \times 4}{26 \times 17 \times 50} = \frac{16}{5525}$

(iv) $n(E) = {}^4C_1 \times {}^4C_2 = 4 \times 6 \qquad \therefore P(E) = \frac{4 \times 6}{26 \times 17 \times 50} = \frac{6}{5525}$

(v) $n(E) = {}^{36}C_2 \times {}^8C_1 = 18 \times 35 \times 8$

$\therefore P(E) = \frac{18 \times 35 \times 8}{26 \times 17 \times 50} = \frac{252}{1105}$

Ex. 10: A bag contains 3 red, 5 yellow and 4 green balls. 3 balls are drawn randomly. What is the probability that the balls drawn contain
(i) balls of different colours?
(ii) exactly two green balls?
(iii) no yellow ball?

Soln: Total no. of balls = 3 + 5 + 4 = 12;

$n(S) = {}^{12}C_3 = \frac{12 \times 11 \times 10}{3 \times 2} = 220$

(i) In order to have 3 different-coloured balls, the selection of one ball of each colour is to be made.

$n(E) = {}^3C_1 \times {}^5C_1 \times {}^4C_1 = 3 \times 5 \times 4$

$\therefore P(E) = \frac{3 \times 5 \times 4}{220} = \frac{3}{11}$

(ii) 2 green balls can be selected from 4 green balls in ${}^4C_2$ ways and the rest one ball can be selected from the remaining (12 - 4 =) 8 balls in ${}^8C_1$ ways.

$n(E) = {}^4C_2 \times {}^8C_1 = 6 \times 8 = 48 \qquad \therefore P(E) = \frac{48}{220} = \frac{12}{55}$

(iii) 3 balls can be selected from 3 (red) + 4 (green) = 7 balls in ${}^7C_3$ ways.

$n(E) = {}^7C_3 = \frac{7 \times 6 \times 5}{3 \times 2} = 35 \qquad \therefore P(E) = \frac{35}{220} = \frac{7}{44}$

Ex. 11: If the letters of the word EQUATION be arranged at random, what is the probability that
(i) there are exactly six letters between N and E?
(ii) all vowels are together?
(iii) all vowels are not together?

Soln: There are eight different letters in the given word

$\therefore$ Total no. of arrangements, $n(S) = {}^8P_8 = 8!$

(i) If N occupies first place, E must occupy last place and vice versa so th there are exactly six letters in between the letters N and E. N and E c be arranged in ${}^2P_2 = 2$ ways and the rest six places can be filled by th remaining six letters (Q, U, A, T, I and O) in ${}^6P_6 = 6!$ ways.

$\therefore n(E) = 2 \times 6!$

$\therefore p(E) = \frac{2 \times 6!}{8!} = \frac{2}{8 \times 7} = \frac{1}{28}.$

(ii) Considering all the five vowels as one letter we have a total o (consonants) + 1 = 4 letters which can be arranged in ${}^4P_4 = 4!$ wa But the five vowels can also be arranged in 5! ways among themselv So, in $4! \times 5!$ ways can the letters be arranged so that vowels are togeth

i.e., $n(E) = 4! \times 5!$ $\quad \therefore P(E) = \frac{4! \times 5!}{8!} = \frac{4 \times 3 \times 2}{8 \times 7 \times 6} = \frac{1}{14}$

(iii) P (All vowels are not together) = 1 - P(All vowels are together)

$= 1 - \frac{1}{14} = \frac{13}{14}$

Ex. 12: A three-digit number is formed with the digits 1, 3, 6, 4 and 5 a random. What is the chance that the number formed is
(i) divisible by 2?
(ii) not divisible by 2?
(iii) divisible by 5?

Soln: A three-digit number can be formed with the given five digits in ${}^5P_3$ ways, i.e. $n(S) = {}^5P_3 = 5 \times 4 \times 3$

(i) Any one of the two digits 4 and 6 should come at units place, which can be done in 2 ways. After filling up the units place, the remaining two places can be filled up with the remaining four digits in ${}^4P_2$ ways;

$n(E) = 2 \times {}^4P_2 = 2 \times 4 \times 3 \quad \therefore P(E) = \frac{2 \times 4 \times 3}{5 \times 4 \times 3} = \frac{2}{5}$

(ii) P (not divisible by 2) = 1 - P (divisible by 2) $= 1 - \frac{2}{5} = \frac{3}{5}$

(iii) A number is divisible by 5, when its units digit is either 0 or 5. We have not been provided the digit 0. So, the units place can be filled up with only 5, i.e. in 1 way. The rest two places can be filled up with

the remaining 4 digit in ${}^4P_2$ ways;

$n(E) = 1 \times {}^4P_2 = 4 \times 3 \quad \therefore P(E) = \frac{4 \times 3}{5 \times 4 \times 3} = \frac{1}{5}$

Ex. 13: There are 4 boys and 4 girls. They sit in a row randomly. What is the chance that all the girls do not sit together?

Soln: Try yourself. Answer $= \frac{13}{14}$.

Ex. 14: The letters of the word 'ARTICLE' is arranged in different ways randomly. What is the chance that the vowels occupy the even places?

Soln: The 7 different letters of the word ARTICLE can be arranged in 7! ways, i.e., $n(S) = 7!$

$n(E) = {}^3P_3 \times {}^4P_4 = 3! \times 4! = 6 \times 24 \quad \therefore P(E) = \frac{6 \times 24}{7!} = \frac{1}{35}.$

Ex. 15: A committee of 4 is to be formed from among 4 girls and 5 boys. What is the probability that the committee will have number of boys less than number of girls?

Soln: Selection of 1 boy and 3 girls in ${}^5C_1 \times {}^4C_3 = 5 \times 4 = 20$ ways

Selection of 4 girls and no boy in ${}^5C_0 \times {}^4C_4 = 1 \times 1 = 1$ way

$\therefore$ n (E) = total no. of ways = 21

Without any restriction, a committee of 4 can be formed from among 4 girls and 5 boys in ${}^9C_4 = \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2} = 9 \times 7 \times 2$ ways

$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{21}{9 \times 7 \times 2} = \frac{1}{6}.$

Ex. 16: A box contains 4 black balls, 3 red balls and 5 green balls. 2 balls are drawn from the box at random. What is the probability that both the balls are of the same colour?

1) $\frac{47}{68}$ 2) $\frac{1}{6}$ 3) $\frac{19}{66}$ 4) $\frac{2}{11}$ 5) None of these

Soln: Total no. of balls = 4 + 3 + 5 = 12

$n(S) = {}^{12}C_2 = \frac{12 \times 11}{2} = 66$

$n(E) = {}^4C_2 + {}^3C_2 + {}^5C_2 = \frac{4 \times 3}{2} + \frac{3 \times 2}{2} + \frac{5 \times 4}{2}$

$= 6 + 3 + 10 = 19$

$\therefore$ Required probability, $P(E) = \frac{n(E)}{n(S)} = \frac{19}{66}$

Ex. 17: In a box carrying one dozen of oranges, one-third have become bad. If 3 oranges are taken out from the box at random, what is the probability that at least one orange out of the three oranges picked up is good?

1) $\frac{1}{55}$ 2) $\frac{54}{55}$ 3) $\frac{45}{55}$ 4) $\frac{3}{55}$ 5) None of these

Soln: 2; P(At least one good) = 1 - P(All bad) ..... (*)

$= 1 - \frac{^4C_3}{^{12}C_3} = 1 - \frac{4}{220} = 1 - \frac{1}{55} = \frac{54}{55}$

Note: (*) See the following combinations of selection of 3 oranges out of 8 good and 4 bad oranges.

(i) All 3 are bad and 0 good.
(ii) 1 bad 2 good
(iii) 2 bad 1 good
(iv) 0 bad 3 good

Combination of (ii), (iii) and (iv) can be said to be "At least one good".

We have, P(i) + {P(ii) + P(iii) + P(iv)} = 1

or, P(i) + P{At least one good} = 1

$\therefore$ P{At least one good} = 1 - P(All 3 bad)

Ex. 18: A box contains 5 green, 4 yellow and 3 white marbles. 3 marbles are drawn at random. What is the probability that they are not of the same colour?

1) $\frac{13}{44}$ 2) $\frac{41}{44}$ 3) $\frac{13}{55}$ 4) $\frac{152}{55}$ 5) None of these

Soln: 2; Total no. of balls = 5 + 4 + 3 = 12

$n(S) = {}^{12}C_3 = \frac{12 \times 11 \times 10}{1 \times 2 \times 3} = 220$

i.e., 3 marbles out of 12 marbles can be drawn in 220 ways.

If all the three marbles are of the same colour, it can be done in

${}^5C_3 + {}^4C_3 + {}^3C_3 = 10 + 4 + 1 = 15$ ways

Now, P(all the 3 marbles of the same colour) +P(all the 3 marbles are not of the same colour) = 1

$\therefore$ P(all the 3 marbles are not of the same colour)

$= 1 - \frac{15}{220} = \frac{205}{220} = \frac{41}{44}$

Ex. 19: Out of 15 students studying in a class, 7 are from Maharashtra, 5 from Karnataka and 3 from Goa. Four students are to be selected at random. What are the chances that at least one is from Karnataka?

1) $\frac{12}{13}$ 2) $\frac{11}{13}$ 3) $\frac{100}{15}$ 4) $\frac{51}{15}$ 5) None of these

Soln: 2; P(At least one from Karnataka) = 1 - P(No one from Karnataka)

$= 1 - \frac{^{10}C_4}{^{15}C_4} = 1 - \frac{10 \times 9 \times 8 \times 7}{15 \times 14 \times 13 \times 12} = 1 - \frac{2}{13} = \frac{11}{13}$

Ex. 20: A bag contains 5 red and 8 black balls. Two draws of three balls each are made, the ball being replaced after the first draw. What is the chance that the balls were red in the first draw and black in the second?

Soln: Required probability $= \frac{^5C_3}{^{13}C_3} \times \frac{^8C_3}{^{13}C_3} = \frac{140}{20449}$

Ex. 21: A bag contains 5 black and 7 white balls. A ball is drawn out of it and replaced in the bag. Then a ball is drawn again. What is the probability that (i) both the balls drawn were black; (ii) both were white; (iii) the first ball was white and the second black; (iv) the first ball was black and the second white?

Soln: The events are independent and capable of simultaneous occurrence. The rule of multiplication would be applied.

The probability that

(i) **both** the balls were black $= \frac{5}{12} \times \frac{5}{12} = \frac{25}{144}$

(ii) **both** the balls were white $= \frac{7}{12} \times \frac{7}{12} = \frac{49}{144}$

(iii) the first was white **and** second black $= \frac{7}{12} \times \frac{5}{12} = \frac{35}{144}$

(iv) the first was black **and** second white $= \frac{5}{12} \times \frac{7}{12} = \frac{35}{144}$

Ex. 22: A bag contains 6 red and 3 white balls. Four balls are drawn out one by one and not replaced. What is the probability that they are alternately of different colours?

Soln: Balls can be drawn alternately in the following order:

Red, White, Red, White **OR** White, Red, White, Red

If Red ball is drawn first, the probability of drawing the balls alternately $= \frac{6}{9} \times \frac{3}{8} \times \frac{5}{7} \times \frac{2}{6}$ ... (I)

If White ball is drawn first the probability of drawing the balls alternately $= \frac{3}{9} \times \frac{6}{8} \times \frac{2}{7} \times \frac{5}{6}$ ... (II)

Required probability = (I) + (II) .... (*)

$$= \frac{6}{9} \times \frac{3}{8} \times \frac{5}{7} \times \frac{2}{6} + \frac{3}{9} \times \frac{6}{8} \times \frac{2}{7} \times \frac{5}{6} = \frac{5}{84} + \frac{5}{84} = \frac{5}{42}.$$

Important Note: In Ex. 20 & 21, the two events are independent and can occur simultaneously. So, we used "multiplication". In other words, since the word AND was used between two events, we used multiplication. Mark that both also means first **and** second. In Ex. 22, in (*) we used addition because the two events are joined with **OR**.

Ex. 23: A bag contains 4 white and 6 red balls. Two draws of one ball each are made without replacement. What is the probability that one is red and other white?

Soln: Such problems can be very easily solved with the help of the rules of permutation and combination.

Two balls can be drawn out of 10 balls in ${}^{10}C_2$ or $\frac{10!}{2!\,8!}$ or $\frac{10 \times 9}{2}$ or 45 ways.

One white ball can be drawn out of 4 white balls in ${}^{4}C_1$ or $\frac{4!}{1!\,3!}$ or 4 ways.

One red ball can be drawn out of 6 red balls in ${}^{6}C_1$ or 6 ways.

The total number of ways of drawing a white **and** a red ball are ${}^{4}C_1 \times {}^{6}C_1$ or $4 \times 6 = 24$. (See Important Note given above)

The required probability would be

$$= \frac{\text{No. of cases favourable to the event}}{\text{Total No. of ways in which the event can happen}}$$

$$= \frac{24}{45} = \frac{8}{15}.$$

Ex. 24: A bag contains 7 white and 9 black balls. Two balls are drawn in succession at random. What is the probability that one of them is white and the other black?

Soln: Total number of ways of drawing 2 balls from (7 + 9) or 16 balls is ${}^{16}C_2 = \frac{16!}{2!\,14!} = \frac{16 \times 15}{2} = 120$

Number of ways of drawing a white ball out of ${}^{7}C_1 = \frac{7!}{1!\,6!} = 7$

Number of ways of drawing a black ball out of 9 is

$${}^{9}C_1 = \frac{9!}{1!\,8!} = 9$$

Number of ways of drawing a white **and** a black ball would be

$${}^{7}C_1 \times {}^{9}C_1 = 7 \times 9 = 63$$

The required probability $= \frac{{}^{7}C_1 \times {}^{9}C_1}{{}^{16}C_2} = \frac{7 \times 9}{120} = \frac{21}{40}.$

Ratio and Proportion

The number of times one quantity contains another quantity of the same kind is called the **ratio** *of the two quantities.*

Clearly, the ratio of two quantities is equivalent to the fraction that one quantity is of the other.

Observe carefully that the two quantities must be of the same kind. There can be a ratio between Rs 20 and Rs 30, but there can be no ratio between Rs 20 and 30 mangoes.

The ratio 2 to 3 is written as 2 : 3 or $\frac{2}{3}$. 2 and 3 are called the **terms of the ratio**. 2 is the first term and 3 is the second term.

The first term of a ratio is called the **antecedent** and the second the **consequent.**

In the ratio 2 : 3, 2 is the antecedent and 3 is the consequent.

Note: The word 'antecedent' literally means 'that which goes before'. The word 'consequent' literally means 'that which goes after'.

2. Since the quotient obtained on dividing one concrete quantity by another of the same kind is an abstract number, the ratio between two concrete quantities of the same kind is an **abstract number.** Thus, the ratio between Rs 5 and Rs 7 is 5 : 7.

Since a fraction is not altered by multiplying or dividing both its numerator and denominator by the same number, a ratio which is also a fraction is not altered by multiplying or dividing both its terms by the same number.

Thus 3 : 5 is the same as 6 : 10, and 15 : 20 is the same as 3 : 4.

Compound Ratio

Ratios are **compounded** *by multiplying together the antecedents for a new antecedent, and the consequents for a new consequent.*

Ex.: Find the ratio compounded of the four ratios :

4 : 3, 9 : 13, 26 : 5 and 2 : 15

Soln: The required ratio $= \frac{4 \times 9 \times 26 \times 2}{3 \times 13 \times 5 \times 15} = \frac{16}{25}$

Note: When the ratio 4 : 3 is compounded with itself the resulting ratio is $4^2 : 3^2$. It is called the **duplicate ratio** of 4 : 3.

Similarly, $4^3 : 3^3$ is the **triplicate ratio** of 4 : 3.

$\sqrt{4} : \sqrt{3}$ is called the **subduplicate ratio** of 4 : 3.

$a^{1/3} : b^{1/3}$ is **subtriplicate ratio** of a and b.

Inverse Ratio

If 2:3 be the given ratio, then $\frac{1}{2}:\frac{1}{3}$ or 3:2 is called its **inverse or reciprocal ratio.**

If the antecedent = the consequent, the ratio is called the **ratio of equality,** such as 3:3.

If the antecedent > the consequent, the ratio is called the **ratio of greater inequality,** as 4:3.

If the antecedent < the consequent, the ratio is called the **ratio of less inequality,** as 3:4.

Ex.1: Divide 1458 into two parts such that one may be to the other as 2 : 7.

Soln: 1st part $= 2 \times \frac{1458}{2+7} = 2 \times \frac{1458}{9} = 324$

2nd part $= 7 \times \frac{1458}{9} = 1134$

Ex.2: Find three numbers in the ratio of 1 : 2 : 3, so that the sum of their squares is equal to 504.

Soln: Let the numbers be x, 2x, 3x. Then we have,

$x^2 + (2x)^2 + (3x)^2 = 504$ or, $14x^2 = 504$ $\therefore x = 6$

Hence, the required numbers are 6, 12 and 18.

Ex.3: A, B, C and D are four quantities of the same kind such that A : B = 3 : 4, B : C = 8 : 9, C : D = 15 : 16.

(i) Find the ratio A : D; (ii) Find A : B : C; and

(iii) Find A : B : C : D.

Soln: (i) $\frac{A}{B} = \frac{3}{4}, \frac{B}{C} = \frac{8}{9}, \frac{C}{D} = \frac{15}{16}$

$\therefore \frac{A}{D} = \frac{A}{B} \times \frac{B}{C} \times \frac{C}{D} = \frac{3}{4} \times \frac{8}{9} \times \frac{15}{16} = \frac{5}{8}$

$\therefore$ A : D = 5 : 8

(ii) A : $\underline{B}$ = 3 : 4 = 6 : $\underline{8}$

$\underline{B}$: C = $\underline{8}$: 9

$\therefore$ A : B : C = 6 : 8 : 9

(iii) We put down the first ratio in its original form and change the terms of the other ratios so as to make each antecedent equal to the preceding consequent.

$$\begin{aligned}
A : B &= 3 : 4 \text{ ---------------(1)}\\
B : C &= 8 : 9 \text{ ---------------(2)}\\
&= 1 : \tfrac{9}{8} \text{ (divided by 8)}\\
&= 4 : \tfrac{9}{8} \times 4 \text{ (multiplied by 4)}\\
&= 4 : \tfrac{9}{2} \text{ ---------------2(i)}\\
C : D &= 15 : 16 \text{ ---------------(3)}\\
&= 1 : \tfrac{16}{15} \text{ (divided by 15)}\\
&= \tfrac{9}{2} : \tfrac{16}{15} \times \tfrac{9}{2} \left(\text{multiplied by } \tfrac{9}{2}\right)\\
&= \tfrac{9}{2} : \tfrac{24}{5} \text{ ---------------3(i)}
\end{aligned}$$

$\therefore A : B : C : D = 3 : 4 : \frac{9}{2} : \frac{24}{5} = 30 : 40 : 45 : 48$

Note: 1) In the Equation (2), B = 8. To make the ratios equivalent, the '8' in (2) should be reduced to '4' (equivalent to B in (1)).

2) In the Equation (3), C = 15. To make the ratios equivalent, the '15' in (3) should be reduced to $\frac{9}{2}$ (equivalent to C in 2 (i)).

Ex.4: A : B = 1 : 2 , B : C = 3 : 4 , C : D = 6 : 9 and D : E = 12 : 16. Find A : B : C : D : E

Soln: A : B = 1 : 2 = 3 : 6

B : C = 3 : 4 = 6 : 8

C : D = 6 : 9 = 8 : 12

D : E = 12 : 16

$\therefore$ A : B : C : D : E = 3 : 6 : 8 : 12 : 16

Note: In the above example, we moved from below, because it made the calculations easier.

Theorem : If the ratio between the first and the second quantities is a : b and the ratio between the second and the third quantities is c : d, then the ratio among first, second and third quantities is given by

ac : bc : bd

The above ratio can be represented diagramatically as

a : b

c : d

ac : bc : bd

Proof : We have First : Second = a : b; Second : Third = c : d

To equate the two ratios, we need to equate the consequent (b) of the first ratio and antecedent (c) of the second ratio. So, we multiply the first ratio by c and the second ratio by b. Therefore,

First : Second = ac : bc
Second : Third = bc : bd
Then, First : Second : Third = ac : bc : bd

Ex. 5: The sum of three numbers is 98. If the ratio between the first and second be 2 : 3 and that between the second and third be 5 : 8, then find the second number.

Soln : The theorem does not give the direct value of the second number, but we can find the combined ratio of all the three numbers by using the above theorem.

The ratio among the three numbers is

2 : 3
5 : 8
10 : 15 : 24

$\therefore$ The second number $= \frac{98}{10+15+24} \times 15 = 30$

Ex. 6: The ratio of the money with Rita and Sita is 7 : 15 and that with Sita and Kavita is 7 : 16. If Rita has Rs 490, how much money does Kavita have ?

Soln : Rita : Sita : Kavita
7 : 15
7 : 16
49 : 105 : 240

The ratio of money with Rita, Sita and Kavita is 49 : 105 : 240
We see that 49 ≡ Rs 490 $\therefore$ 240 ≡ Rs 2400

Theorem: If the ratio between the first and the second quantities is a : b; the ratio between the second and the third quantities is c : d and the ratio between the third and the fourth quantities is e : f then the ratio among the first, second, third and fourth quantities is given by

1st : 2nd = a : b
2nd : 3rd = c : d
3rd : 4th = e : f

1st : 2nd : 3rd : 4th = ace : bce : bde : bdf

Proof: It is easy to prove this by the same method used in the previous theorem.

Ex. 7: If A : B = 3 : 4, B : C = 8 : 10 and C : D = 15 : 17
Then find A : B : C : D.

Soln: A : B = 3 : 4
B : C = 8 : 10
C : D = 15 : 17

A : B : C : D = 3 × 8 × 15 : 4 × 8 × 15 : 4 × 10 × 15 : 4 × 10 × 17
= 9 : 12 : 15 : 17

Note: 1. Ex. 3 can be solved with the help of above theorems. Try it.
2. If A : B = 1 : 2, B : C = 3 : 4, C : D = 2 : 3 and D : E = 3 : 4
Then find A : B : C : D : E.

Soln: A : B = 1 : 2
B : C = 3 : 4
C : D = 2 : 3
D : E = 3 : 4

A : B : C : D : E = 1 × 3 × 2 × 3 : 2 × 3 × 2 × 3 : 2 × 4 × 2 × 3
: 2 × 4 × 3 × 3 : 2 × 4 × 3 × 4
= 3 : 6 : 8 : 12 : 16

Ex. 8: A hound pursues a hare and takes 5 leaps for every 6 leaps of the hare, but 4 leaps of the hound are equal to 5 leaps of the hare. Compare the rates of the hound and the hare.

Soln: 4 leaps of hound = 5 leaps of hare

$\therefore$ 5 leaps of hound $= \frac{25}{4}$ leaps of hare

$\therefore$ the rate of hound : rate of hare $= \frac{25}{4} : 6 = 25 : 24$

or, Ratio of	Hound	Hare
leap frequency	5	6
leap length	4	5

Then the required ratio of speed is the ratio of the cross-product.
That is, speed of hound : speed of hare = 5 × 5 : 6 × 4 = 25 : 24

Ex 9: A can do a piece of work in 12 days. B is 60% more efficient than A. Find the number of days it takes B to do the same piece of work.

Soln :

	A : B
Efficiency	100 : 160
Days	160 : 100
or,	8 : 5

$\therefore$ the number of days taken by B $= \frac{12}{8} \times 5 = \frac{15}{2} = 7\frac{1}{2}$ days.

Or,

By the rule of fraction : As B is more efficient, it is clear that 'B' will complete the work in less days. So, the number of days (12) should be multiplied by a less-than-one fraction and that fraction is $\frac{100}{100+60}$, i.e., $\frac{100}{160}$. Therefore, our required answer is

$12 \times \frac{100}{160} = \frac{12 \times 5}{8} = \frac{15}{2} = 7\frac{1}{2}$ days.

Ex 10: One man adds 3 litres of water to 12 litres of milk and another 4 litres of water to 10 litres of milk. What is the ratio of the strengths of milk in the two mixtures?

Soln : Strength of milk in the first mixture $= \frac{12}{12+3} = \frac{12}{15}$

Strength of milk in the second mixture $= \frac{10}{10+4} = \frac{10}{14}$

$\therefore$ the ratio of their strengths $= \frac{12}{15} : \frac{10}{14}$

$= 12 \times 14 : 15 \times 10 = 28 : 25$

Ex 11: Rs 425 is divided among 4 men, 5 women and 6 boys such that the share of a man, a woman and a boy may be in the ratio of 9 : 8 : 4. What is the share of a woman?

Soln : The ratio of shares of group of men, women and boys $= 9 \times 4 : 8 \times 5 : 4 \times 6 = 36 : 40 : 24$

Share of 5 women $= \frac{425}{36+40+24} \times 40 =$ Rs 170

$\therefore$ the share of 1 woman $= \frac{170}{5} =$ Rs 34

Ex 12: If a carton containing a dozen mirrors is dropped, which of the following cannot be the ratio of broken mirrors to unbroken mirrors?

(1) 2 : 1 (2) 3 : 1 (3) 3 : 2
(4) 1 : 1 (5) 7 : 5

Soln : There are 12 mirrors in the carton. So, the sum of terms in the ratio must divide 12 exactly. We see that 2 + 1 = 3 divides 12 exactly. 3 + 1 = 4 also divides exactly. 3 + 2 = 5 doesn't divide 12 exactly. Thus, our answer is (3).

PROPORTION

Consider the two ratios:

1st ratio	2nd ratio
6 : 18	8 : 24

Since 6 is one-third of 18, and 8 is one-third of 24, the two ratios are equal. The equality of ratios is called **proportion.**

The numbers 6, 18, 8 and 24 are said to be in **proportion**.

The proportion may be written as

6 : 18 : : 8 : 24 (6 is to 18 as 8 is to 24)

or, $6 : 18 = 8 : 24$ or, $\frac{6}{18} = \frac{8}{24}$.

The numbers 6, 18, 8 and 24 are called the **terms. 6 is the first term**, 18 the **second**, 8 the **third**, and 24 the **fourth**. The first and fourth terms, i.e., 6 and 24 are called the **extremes** (end terms), and the second and the third terms, i.e., 18 and 8 are called the **means** (middle terms). 24 is called the **fourth** proportional.

1. *If four quantities be in* **proportion,** *the product of the extremes is equal to the product of the means.*

Let the four quantities 3, 4, 9 and 12 be in proportion.

We have $\frac{3}{4} = \frac{9}{12}$

Multiply each ratio by 4×12

$\therefore \frac{3}{4} \times 4 \times 12 = \frac{9}{12} \times 4 \times 12$

$\therefore 3 \times 12 = 4 \times 9$

2. *Three quantities of the same kind are said to be in* **continued proportion** *when the ratio of the first to the second is equal to the ratio of the second to the third.*

The second quantity is called the **mean proportional** *between the first and the third ; and the third quantity is called the* **third proportional** *to the first and second.*

Thus, 9, 6 and 4 are in continued proportion for 9 : 6 : :6 : 4.

Hence, 6 is the mean proportional between 9 and 4, and 4 is the third proportional to 9 and 6.

Ex.1: Find the fourth proportional to the numbers 6, 8 and 15.

Soln: If x be the fourth proportional, then $6 : 8 = 15 : x$

$\therefore x = \frac{8 \times 15}{6} = 20$

Ex.2: Find the third proportional to 15 and 20.

Here, we have to find a fourth proportional to 15, 20 and 20.

If x be the fourth proportional, we have $15 : 20 = 20 : x$

$\therefore x = \frac{20 \times 20}{15} = \frac{80}{3} = 26\frac{2}{3}$

Ex.3: Find the mean proportional between 3 and 75.

Soln: If x be the required mean proportional, we have

$3 : x :: x : 75$

$\therefore x = \sqrt{3 \times 75} = 15$

Note: It is evident that the mean proportional between two numbers is equal to the square root of their product. **(Remember)**

Consider the proportion 5 : 15 : : 8 : x. Here, the 1st, 2nd and 3rd terms are given, and the 4th term is unknown. The unknown term is denoted by x. We want to find x.

Now, the product of the means is equal to the product of the extremes.

$\therefore \quad 5 \times x = 15 \times 8$ or, $x = \frac{15 \times 8}{5} = 24$

Hence, **the 4th term** can be found by the following rule

Rule : Multiply the 2nd and 3rd terms together, and divide the product by the 1st term.

We shall now take examples concerning **concrete quantities.**

Direct Proportion: Consider the following example.

Ex.: If 5 balls cost Rs 8, what do 15 balls cost ?

Soln: It will be seen at once that if the number of balls be *increased* 2, 3, 4 ..., times, the price will also be *increased* 2, 3, 4.... times. Therefore, 5 balls is the same fraction of 15 balls that the cost of 5 balls is of the cost of 15 balls.

$\therefore$ 5 balls : 15 balls : : Rs 8 : required cost

$\therefore$ the required cost = Rs $\frac{15 \times 8}{5}$ = Rs 24

This example is an illustration of what is called **direct proportion.** In this case, the two given quantities are so related to each other

that if one of them is multiplied (or divided) by any number, the other is also multiplied (or divided) by the same number.

Inverse Proportion: Consider the following example.

Ex.: If 15 men can reap a field in 28 days, in how many days will 10 men reap it?

Soln: Here it will be seen that if the number of men be increased 2, 3, 4,... times, the number of days will be decreased 2, 3, 4..., times. Therefore, the inverse ratio of the number of men is equal to the ratio of the corresponding number of days.

$\therefore \quad \frac{1}{15} : \frac{1}{10} :: 28$: the required number of days

or, 10 : 15 : : 28 : the required number of days

$\therefore$ the required number of days = $\frac{15 \times 28}{10} = 42$

The above example is an illustration of what is called **inverse proportion.** In this case, the two quantities are so related that if one of them is multiplied by any number, the other is divided by the same number, and *vice versa.*

Note: The arrangement of figures may create a problem. To overcome this, we give you a general rule known as the RULE OF THREE.

The Rule of Three : *The method of finding the 4th term of a proportion when the other three are given is called* **Simple Proportion** *or the* **Rule of Three.**

In every question of simple proportion, two of the given terms are of the same kind, and the third term is of the same kind as the required fourth term.

Now, we give the rule of arranging the terms in a question of simple proportion.

Rule: I: Denote the quantity to be found by the letter 'x', and set it down as the 4th term.

II: Of the three given quantities, set down that for the third term which is of the same kind as the quantity to be found.

III: Now, consider carefully whether the quantity to be found will be greater or less than the third term; if greater, make the greater of the two remaining quantities the 2nd term, and the other 1st term, but if less, make the less quantity the second term, and the greater the 1st term.

IV: Now, the required value = $\frac{\text{Multiplication of means}}{\text{1st term}}$

After having the detailed knowledge about proportions and the Rule of Three, we now solve some of the examples which are usually solved by the unitary method.

Ex. 1 : If 15 books cost Rs 35 , what do 21 books cost ?

Soln : This is an example of direct proportion. Because if the number of books is increased, their cost also increases.

By the Rule of Three :

Step I : ... : ... = ... : Required cost

Step II : ... : ... = Rs 35 : Required cost

Step III : The required cost will be greater than the given cost; so the greater quantity will come as the 2nd term. Therefore,

15 books : 21 books = Rs 35 : Required cost

Step IV : $\therefore$ the required cost $= \frac{21 \times 35}{15} =$ Rs 49.

Ex 2: In a given time, 12 persons make 111 toys. In the same time, 148 toys are to be made. How many persons should be employed?

Soln:

Step I : ... : ... = ... : Number of persons

Step II : ... : ... = 12 : x

Step III : The required number of persons is more.

Hence, 111 : 148 = 12 : x

Step IV : $x = \frac{148 \times 12}{111} = 16$

Ex. 3: If 192 mangoes can be bought for Rs 15, how many can be bought for Rs 5?

Soln :

Step I : ... : ... = ... : the required number of mangoes

Step II : ... = ... = 192 : x

Step III : As the required quantity would be less,

15 : 5 = 192 : x

Step IV : $x = \frac{5 \times 192}{15} = 64$

Ex. 4: If 15 men can reap a field in 28 days, in how many days will 5 men reap it ?

Soln :

Step I : ... : ... = ... : Required number of days

Step II : ...: ... = 28 : x

Step III : The required number of days will be more, since 5 men will take more time than 15 men. Therefore, 5 : 15 = 28 : x

Step IV : $x = \frac{15 \times 28}{5} = 84$ days

Ex. 5: A fort had provisions for 150 men for 45 days. After 10 days, 25 men left the fort. How long will the food last at the same rate for the remaining men?

Soln : The remaining food would last for 150 men for (45-10 =) 35 days. But as 25 men have gone out, the remaining food would last for a longer period. Hence, by the **Rule of Three**, we have the following relationship.

125 men : 150 men = 35 days : the required no. of days.

$\therefore$ the required no. of days $= \frac{150 \times 35}{125} = 42$ days

Compound Proportion or Double Rule of Three

Ex. 6: If 8 men can reap 80 hectares in 24 days, how many hectares can 36 men reap in 30 days ?

Soln : We can resolve this problem into two questions.

1st : If 8 men can reap 80 hectares, how many hectares can 36 men reap?

8 men : 36 men = 80 hectares : the required no. of hectares

$\therefore$ the required no. of hectares $= \frac{36 \times 30}{8} = 360$ hectares

2nd : If 360 hectares can be reaped in 24 days, how many hectares can be reaped in 30 days ?

By the Rule of Three

24 days : 30 days = 360 hectares : the reqd. no. of hectares.

$\therefore$ the reqd. no. of hectares $= \frac{30 \times 360}{24} = 450$

We observe that the original number of hectares, namely 80, has been changed in the ratio formed by compounding the ratio $\frac{36}{8}$ and $\frac{30}{24}$

The above question can be solved in a single step. We arrange the figures in the following form :

$$\left.\begin{array}{l} 8 \text{ men} : 36 \text{ men} \\ 24 \text{ days} : 30 \text{ days} \end{array}\right] :: 80 \text{ hect} : \text{the reqd. no. of hectares}$$

The reqd. no. of hectares $= \frac{\text{Multiplication of means}}{\text{Multiplication of 1st terms}}$

$= \frac{80 \times 36 \times 30}{8 \times 24} = 450$

Ex 7: If 30 men working 7 hrs a day can do a piece of work in 18 days,

in how many days will 21 men working 8 hrs a day do the same piece of work ?

Soln : $\left.\begin{array}{l}21 \text{ men} : 30 \text{ men}\\ 8 \text{ hrs} : 7 \text{ hrs}\end{array}\right]$ :: 18 days : the reqd. no. of days

$\therefore$ the reqd. no. of days $= \dfrac{18 \times 30 \times 7}{21 \times 8} = 22\dfrac{1}{2}$ days

Note : Two lines of reasoning are used in the above case:
(1) Less men : more days.
(2) More working hrs : less days.

Ex. 8: If 15 men or 24 women or 36 boys do a piece of work in 12 days, working 8 hrs a day, how many men must be associated with 12 women and 6 boys to do another piece of work $2\frac{1}{4}$ times as great in 30 days working 6 hrs a day ?

Soln : Useful reasoning
(1) More days : less men.
(2) Less working hrs : more men.
(3) More work : more men.
Therefore, by the Rule of Three,

$\left.\begin{array}{l}30 \text{ days} : 12 \text{ days}\\ 6 \text{ hrs} : 8 \text{ hrs}\\ 1 \text{ work} : 2\frac{1}{4} \text{ works}\end{array}\right]$ :: 15 men : the reqd. no. of men

$\therefore$ the reqd. no. of men $= \dfrac{15 \times 12 \times 8 \times 2.25}{30 \times 6 \times 1} = 18$

Now, we have, 24 women = 15 men
$\therefore$ 12 women = 7.5 men
And also, 36 boys = 15 men
$\therefore$ 6 boys $= \dfrac{15}{6} = \dfrac{5}{2} = 2.5$ men
$\therefore$ 12 women + 6 boys = 7.5 + 2.5 = 10 men
So, 18 - 10 = 8 men must be associated.

Ex. 9 : A garrison of 2200 men is provisioned for 16 weeks at the rate of 45 dag per day per man. How many men must leave the garrison so that the same provisions may last 24 weeks at 33 dag per day per man ?

Soln : We use the following steps in reasoning :
(1) For more weeks, less men are needed.

(2) For less dag, more men are needed.
So, by the Rule of Three

$\left.\begin{array}{l}24 \text{ weeks} : 16 \text{ weeks}\\ 33 \text{ dag} : 45 \text{ dag}\end{array}\right]$ :: 2200 men : the reqd. no. of days

$\therefore x = \dfrac{2200 \times 16 \times 45}{24 \times 33} = 2000$

Hence, 2200 - 2000 = 200 men must leave the garrison.

Ex. 10 : Two cogged wheels, of which one has 16 cogs and the other has 27, work into each other. If the latter turns 80 times in three quarters of a minute, how often does the other turn in 8 seconds?

Soln : Reasoning to be used :
(1) Less cogs, more turns.
(2) Less time, less turns.

$\left.\begin{array}{l}16 \text{ cogs} : 27 \text{ cogs}\\ 45 \text{ sec} : 8 \text{ sec}\end{array}\right]$ :: 80 turns : x turns

By the Rule of Three

$x = \dfrac{80 \times 27 \times 8}{16 \times 45} = 24$

Ex. 11: If 30 men do a piece of work in 27 days, in what time can 18 men do another piece of work 3 times as great ?

Soln : $\left.\begin{array}{l}\text{Men } 18 : 30\\ \text{Work } 1 : 3\end{array}\right]$ :: 27 : the reqd. no. of days $\left[\begin{array}{l}\text{Less men, more days}\\ \text{More work, more days}\end{array}\right.$

$\therefore$ the reqd. no. of days $= \dfrac{27 \times 30 \times 3}{18 \times 1} = 135$ days

Ex. 12: If a family of 7 persons can live on Rs 840 for 36 days, how long can a family of 9 persons live on Rs 810 ?

Soln : $\left.\begin{array}{l}\text{persons } 9 : 7\\ \text{Rs } 840 : 810\end{array}\right]$ :: 36 : the reqd. no. of days $\left[\begin{array}{l}\text{more persons, less days}\\ \text{less money, less days}\end{array}\right.$

$\therefore$ the reqd. no. of days $= \dfrac{36 \times 7 \times 810}{9 \times 840} = 27$ days

Ex. 13 : If 1000 copies of a book of 13 sheets require 26 reams of paper, how much paper is required for 5000 copies of a book of 17 sheets?

Soln: Books 1000 : 5000] :: 26 : x [More books, more paper
Sheets 13 : 17] More sheets, more paper

$\therefore$ the quantity of paper $= \dfrac{26 \times 5000 \times 17}{1000 \times 13} = 170$ reams

Ex. 14 : If 6 men can do a piece of work in 30 days of 9 hours each, how many men will it take to do 10 times the amount of work if they work for 25 days of 8 hours?

Soln : We need three lines of reasoning in this question:

(1) Less days, more men (i.e., if a work is to be finished in less days, there should be more men at the work).

(2) Less working hours, more men (i.e., if the working hour is less, the number of persons at work should be more to complete the work in a stipulated time).

(3) More work, more men (i.e., if the work is more, the number of persons should be more so that all the work can be finished within the given time).

Following the Rule of Three :

Step I : Days: ... : ...
Hrs : ... : ...] : : 6 : the reqd. no. of men
Work: ... : ...

Step II: The **Rule of Three** states that :

(1) To do the work in less days (25 days) we need **more men** (Reasoning : Less days, more men), hence greater value will go at the second place and smaller value will go at the first place.

Like this :

Days : 25:30

(2) The working hours (8 hrs) is less now, so we need **more men.** Thus, the greater value will go at the 2nd place and the smaller value will go at the 1st place. Like:

Hrs : 8 : 9

(3) If there is more work (10 times) we need **more men.** Thus, greater value will go at the 2nd place and the smaller value will go at the 1st place. Like:

Work : 1:10

Thus, we reach the stage where all the blanks in Step I can be filled up.

Days 25 : 30
Hrs 8 : 9] :: 6:x
Work 1 : 10

Now, by the Rule of Three, we have

$x = \dfrac{\text{Third term} \times \text{Multiplication of means}}{\text{Multiplication of first terms}}$

or, $x = \dfrac{6 \times 30 \times 9 \times 10}{25 \times 8 \times 1} = 81$ men

Now, we go for the **Rule of Fractions,** which is very much similar to the Rule of Three in theory.

Some Basics of Fractions

(1) When a fraction has its numerator greater than the denominator, its value is greater than one. Let us call it **greater fraction.** Whenever a number (say x) is multiplied by a **greater fraction,** it gives a value greater than itself.

For example :

When 15 is multiplied by $\frac{4}{3}$ (greater fraction), we get 20, which is greater than 15.

(2) When a fraction has its numerator less than the denominator, its value is less than one. Let us call it **less fraction**. Whenever a number (say x) is multiplied by a **less fraction,** it gives a value less than itself.

For example :

$15 \times \frac{3}{5} = 9$, which is less than 15.

Note : We will use the above two basics as well as the reasoning used in **Rule of Three** while solving Ex. 14 by the **Rule of Fractions.**

Soln: Step I : We look for our required unit. It is the number of men. So, we write down the number of men given in the question. It is 6.

Step II : The number of days gets reduced from 30 to 25, so it will need **more men** (Reasoning : Less days, more men). It simply means that 6 should be multiplied by a **greater fraction** because we need a value greater than 6. So, we have : $6 \times \frac{30}{25}$

Step III : Following in the same way, we see that the above figure should be multiplied by a 'greater fraction', i.e., by $\frac{9}{8}$. So, we have:

$6 \times \frac{30}{25} \times \frac{9}{8}$

Step IV : Following in the same way, we see that the above figure should be multiplied by a 'greater fraction' i.e. by $\frac{10}{1}$. So, we have:

$6 \times \frac{30}{25} \times \frac{9}{8} \times \frac{10}{1} = 81$ men

Ex. 15 : In a given period, 9 persons can make 108 toys. How many persons are needed to make 48 toys in the same period ?

Soln : We see that we require **less number of toys**. So, **less number of persons** is needed. It means the given number of persons should be multiplied by a **less fraction**. Thus, our answer should be :

$$9 \times \frac{48}{108} = 4 \text{ persons.}$$

Ex. 16 : If 8 men can reap 80 hectares in 24 days, how many hectares can 36 men reap in 30 days ?

Soln : Step I : The required unit is hectare, so we write down the given number of hectares, i.e., 80.

Step II : The number of men increases, so now they will reap **more** hectares. So, 80 should be multiplied by a greater fraction, i.e., by $\frac{36}{8}$. Thus, we have $80 \times \frac{36}{8}$.

Step III : The number of days also increases and hence they will reap **more** hectares. Thus, we have :

$$80 \times \frac{36}{8} \times \frac{30}{24} = 450 \text{ hectares.}$$

PROPORTIONAL DIVISION

Proportion may be applied to divide a given quantity into parts which are proportional to the given numbers.

Ex.1: Divide Rs 1350 into three shares proportional to the numbers 2, 3 and 4.

Soln. 1st share = Rs $1350 \times \frac{2}{2+3+4} = 1350 \times \frac{2}{9} =$ Rs 300

2nd share = Rs $1350 \times \frac{3}{9} =$ Rs 450

3rd share = Rs $1350 \times \frac{4}{9} =$ Rs 600

Ex.2: Divide Rs 391 into three parts proportional to the fractions $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$

Soln: Multiplying these ratios by the LCM of the denominators 2, 3, 4 namely12, we get $\frac{1}{2} : \frac{2}{3} : \frac{3}{4} = 6 : 8 : 9$

Now $6 + 8 + 9 = 23$

$\therefore$ 1st part $= \frac{6}{23} \times 391 =$ Rs 102

2nd part $= \frac{8}{23} \times 391 =$ Rs 136

3rd part $= \frac{9}{23} \times 391 =$ Rs 153

Note : The third part may also be found by subtracting the sum of Rs 102 and Rs 136 from Rs 391.

Ex.3 : A certain sum of money is divided among A, B and C such that for each rupee A has, B has 65 paise and C has 40 paise. If C's share is Rs 8, find the sum of money.

Soln: Here A : B : C = 100 : 65 : 40 = 20 : 13 : 8

Now, $20 + 13 + 8 = 41$

As $\frac{8}{41}$ of the whole sum = Rs 8

$\therefore$ the whole sum = Rs $\frac{8 \times 41}{8}$ = Rs 41

Ex.4: Divide Rs 1540 among A, B, C so that A shall receive $\frac{2}{9}$ as much as B and C together, and B $\frac{3}{11}$ of what A and C together do.

Soln : A's share : (B + C)'s share = 2 : 9 ---------(1)

B's share : (A + C)'s share = 3 : 11 ---------(2)

Now, dividing Rs 1540 in the ratio of 2 : 9 and 3 : 11 ,

A's share $= \frac{2}{11}$ of Rs 1540 = Rs 280

B's share $= \frac{3}{14}$ of Rs 1540 = Rs 330

$\therefore$ C's share = Rs 1540 - (Rs 280 + Rs 330) = Rs 930

Ex.5: Divide 581 into three parts such that 4 times the first may be equal

to 5 times the second and 7 times the third.

Soln: 4 times the 1st part = 5 times the 2nd = 7 times the 3rd = 1 (say)

$\therefore$ 1st part $= \frac{1}{4}$, 2nd part $= \frac{1}{5}$, 3rd part $= \frac{1}{7}$

$\therefore$ 1st part : 2nd part : 3rd part $= \frac{1}{4} : \frac{1}{5} : \frac{1}{7} = 35 : 28 : 20$

Now, divide 581 in the proportion of these numbers.

Note: Remember that for such questions, the three parts are in the proportion of the reciprocals of the numbers 4, 5 and 7.

Ex.6: Divide Rs 2430 among three persons A, B and C such that if their shares be diminished by Rs 5, Rs 10, Rs 15 respectively, the remainders shall be in the ratio 3 : 4 : 5.

Soln: Rs 2430 - (Rs 5 + Rs 10 + Rs 15) = Rs 2400

Dividing Rs 2400 in the ratio 3 : 4 : 5, we get

A's share $= \frac{3}{3+4+5}$ of Rs 2400 + Rs 5 = Rs 605

B's share $= \frac{4}{3+4+5}$ of Rs 2400 + Rs 10 = Rs 810

C's share $= \frac{5}{3+4+5}$ of Rs 2400 + Rs 15 = Rs 1015

Ex.7: Divide Rs 1320 among 7 men, 11 women and 5 boys such that each woman may have 3 times as much as a boy, and a man as much as a woman and a boy together. Find how much each person receives.

Soln: 1 man = 1 woman + 1 boy

1 woman = 3 boys

$\therefore$ 1 man = 4 boys

$\therefore$ 7 man : 11 women : 5 boys = 28 boys : 33 boys : 5 boys = 28 : 33 : 5

Dividing Rs 1320 in the ratio of 28, 33 and 5, we have

7 men's share $= \frac{28}{66} \times 1320 =$ Rs 560

$\therefore$ 1 man's share $= \frac{560}{7} =$ Rs 80

4 boys' share = Rs 80 (As 1 man = 4 boys)

$\therefore$ 1 boy's share = Rs 20

and 1 woman's share = $3 \times$ Rs 20 = Rs 60.

Ex.8: How many one-rupee coins, fifty-paise coins and twenty-five-paise coins of which the numbers are proportional to $2\frac{1}{2}$, 3 and 4, are together worth Rs 210?

Soln: Here $2\frac{1}{2} : 3 : 4 = 5 : 6 : 8$

Their proportional value $= 5 \times 1 : 6 \times \frac{1}{2} : 8 \times \frac{1}{4} = 5 : 3 : 2$

Now, 5 + 3 + 2 = 10

$\therefore$ the value of rupees $= \frac{5}{10}$ of Rs 210 = Rs 105

The value of fifty-paise coins $= \frac{3}{10}$ of Rs 210 = Rs 63

The value of 25-paise coins $= \frac{2}{10}$ of Rs 210 = Rs 42

Therefore, there are 105 rupees, 126 fifty-paise coins and 168 twenty-five paise coins.

Miscellaneous Examples

Theorem : If in x litres mixture of milk and water, the ratio of milk and water is a : b, the quantity of water to be added in order to make this ratio c : d is $\frac{x(ad - bc)}{c(a + b)}$

Proof : Quantity of milk in the mixture $= \frac{x}{a+b} \cdot a$

Quantity of water in the mixture $= \frac{x}{a+b} \cdot b$

Suppose we added y litres of water to get our required ratio (c : d). Then we have

$$\left(\frac{ax}{a+b}\right) : \left(\frac{bx}{a+b} + y\right) = c : d$$

$$\text{or, } \left(\frac{ax}{a+b}\right) : \left(\frac{bx + y(a+b)}{a+b}\right) = c : d$$

$$\text{or, } \frac{ax}{bx + y(a+b)} = \frac{c}{d} \quad \text{or, } y = \frac{x(ad - bc)}{(a+b)c}$$

Note : The above result is very systematic. They should be remembered. It saves a lot of time.

Ex. 1: In 40 litres mixture of milk and water the ratio of milk and water

is 3 : 1. How much water should be added in the mixture so that the ratio of milk to water becomes 2 : 1 ?

Soln : Solving the above question by the direct formula given in the above theorem :

The quantity of water to be added to get the required ratio:

$$= \frac{40(3 \times 1 - 1 \times 2)}{(3+1)2} = \frac{40}{8} = 5 \text{ litres.}$$

Note : The above solution can be verified as follows :

In 40 litres of mixture, milk $= \frac{40}{3+1} \times 3 = 30$ litres

and water $= 40 - 30 = 10$ litres.

5 litres water is added; so in the new mixture, milk is 30 litres and water is $10 + 5 = 15$ litres.

Thus, the new ratio is 30 : 15 = 2 : 1. This ratio is the same as given in the question.

Ex. 2: In 30 litres mixture of milk and water, the ratio of milk and water is 7 : 3. Find the quantity of water to be added in the mixture in order to make this ratio 3 : 7.

Soln : Following the same theorem, we have,

The reqd. answer $= \frac{30(7 \times 7 - 3 \times 3)}{3(7+3)} = 40$ litres

Note : The above question is the special case of the above mentioned theorem. Here, we see that the first ratio is reversed in the second case. That is, a : b becomes b : a in the new mixture. Moreover, the total quality of initial mixture equals the denominator $[c(a+b)]$. In this case, the water to be added $= a^2 - b^2$

Theorem : A mixture contains milk and water in the ratio a : b. If x litres of water is added to the mixture, milk and water become in the ratio a : c. Then the quantity of milk in the mixture is given by $\frac{ax}{c-b}$ and that of water is given by $\frac{bx}{c-b}$

Proof.: Let the quantity of mixture be M litres.

Then the quantity of milk $= \frac{aM}{a+b}$ litres.

and the quantity of water $= \frac{bM}{a+b}$ litres.

When x litres of water are added to the mixture, we have

$$\frac{aM}{a+b} : \frac{bM}{a+b} + x = a : c$$

or, $\frac{aM}{a+b} : \frac{bM + x(a+b)}{a+b} = a : c$ or, $\frac{aM}{bM + x(a+b)} = \frac{a}{c}$

or, $cM = bM + x(a+b)$ $\therefore M = \frac{x(a+b)}{(c-b)}$

Thus, the quantity of milk in the mixture

$$= \frac{aM}{a+b} = \frac{ax(a+b)}{(a+b)(c-b)} = \frac{ax}{c-b}$$

Similarly, the quantity of water in the mixture

$$= \frac{bM}{a+b} = \frac{bx(a+b)}{(a+b)(c-b)} = \frac{bx}{c-b}$$

Ex. 3 : A mixture contains milk and water in the ratio of 3 : 2. If 4 litres of water is added to the mixture, milk and water in the mixture become equal. Find the quantities of milk and water in the mixture.

Soln : If we want to solve the above question by the theorem stated above, we will have to change the form of ratios to a : b and a : c. In the above question, the initial ratio is 3 : 2. Thus, to equate the antecedents of the ratio, we write the second ratio as 3 : 3. Then by the above direct formula :

The quantity of milk $= \frac{3 \times 4}{3-2} = 12$ litres.

and the quantity of water $= \frac{2 \times 4}{3-2} = 8$ litres.

Ex. 4 : A mixture contains milk and water in the ratio of 8 : 3. On adding 3 litres of water, the ratio of milk to water becomes 2 : 1. Find the quantity of milk and water in the mixture.

Soln : To follow the above theorem, we change the ratios in the form a : b and a : c. Then the ratios can be written as 8 : 3 and 8 : 4.

Thus, the quantity of milk in the mixture $= \frac{8 \times 3}{4-3} = 24$ litres

and the quantity of water in the mixture $= \frac{3 \times 3}{4-3} = 9$ litres.

Theorem : If two quantities X and Y are in the ratio x : y. Then

X + Y : X - Y :: x + y : x - y

Proof : The above theorem can be proved by the rule of componendo-dividendo.

We are given that $\frac{X}{Y} = \frac{x}{y}$

Then, by the rule of componendo-dividendo : $\frac{X+Y}{X-Y} = \frac{x+y}{x-y}$

or, $X + Y : X - Y :: x + y : x - y$

Ex. 5: A sum of money is divided between two persons in the ratio of 3 : 5. If the share of one person is Rs 20 less than that of the other, find the sum.

Soln : By the above theorem : $\frac{\text{Sum}}{20} = \frac{3+5}{5-3}$ $\therefore$ Sum $= \frac{8}{2} \times 20 =$ Rs 80

Note : The above question can also be solved as follows (this method is similar to the above theorem):

$5 - 3 \equiv$ Rs 20 $\quad \therefore 5 + 3 \equiv \frac{20}{5-3} \times (5+3) =$ Rs 80

Ex. 6: The prices of a scooter and a moped are in the ratio of 9 : 5. If a scooter costs Rs 4200 more than a moped, find the price of the moped.

Soln : Following the method mentioned in the above note, we have,

$9 - 5 \equiv$ Rs 4200 $\quad \therefore 5 \equiv \frac{4200}{9-5} \times 5 =$ Rs 5250

Theorem : In any two two-dimensional figure, if the corresponding sides are in the ratio a : b, then their areas are in the ratio $a^2 : b^2$.

Proof : To prove the above theorem, we take the case of a rectangle. This will be true for every other two-dimensional figure also.

Suppose we have a rectangle whose sides are x and y.

We are given that the sides of another rectangle are in the ratio a : b, therefore, the sides of second rectangle are $\frac{b}{a}x$ and $\frac{b}{a}y$.

Therefore, the ratio of the two areas $= xy : \frac{b^2}{a^2}xy = a^2 : b^2$

Ex. 7: The sides of a hexagon are enlarged by three times. Find the ratio of the areas of the new and old hexagons.

Soln: Following the above theorem, we see that the ratio of the corresponding sides of the two hexagons is a : b = 1 : 3.

Therefore, the ratio of their areas is given by

$a^2 : b^2 = 1^2 : 3^2 = 1 : 9.$

Ex. 8: The ratio of the diagonals of two squares is 2 : 1. Find the ratio of their areas.

Soln: We should follow the same rule when the ratio of diagonals is given instead of the ratio of sides. Thus, the ratio of their areas = $2^2 : 1^2 = 4 : 1$.

Ex. 9: The ratio of the radius (or diameter or circumference) of two circles is 3 : 4. Find the ratio of their areas.

Soln : Following the rule, we have,

ratio of their areas $= 3^2 : 4^2 = 9 : 16.$

Note : The above mentioned theorem is true for any two-dimensional figure and for any measuring length related to that figure (see Examples 8 and 9).

Theorem : In any two 3-dimensional figure, if the corresponding sides or other measuring lengths are in the ratio a : b, then their volumes are in the ratio $a^3 : b^3$.

Ex. 10: (a) The sides of two cubes are in the ratio 2 : 1. Find the ratio of their volumes.

(b) Each side of a parallelopipe is doubled find the ratio of volume of old to new parallelopipe.

Soln : (a) The required ratio $= (2)^3 : (1)^3 = 8 : 1$

(b) The required ratio $= (1)^3 : (2)^3 = 1 : 8$

Theorem : The ratio between two numbers is a : b. If each number be increased by x, the ratio becomes c : d. Then,

Sum of the two numbers $= \frac{x(a+b)(c-d)}{ad-bc}$

Difference of the two numbers $= \frac{x(a-b)(c-d)}{ad-bc}$

And the two numbers are given as $\frac{xa(c-d)}{ad-bc}$ and $\frac{xb(c-d)}{ad-bc}$

Proof : Let the sum of the two numbers be X.

Then the numbers are $\frac{aX}{a+b}$ and $\frac{bX}{a+b}$.

Now, when each number is increased by x then

$\frac{aX}{a+b} + x : \frac{bX}{a+b} + x = c : d.$

or, $\frac{aX + x(a+b)}{a+b} : \frac{bX + x(a+b)}{a+b} = c : d$

or, $\frac{aX + x(a+b)}{bX + x(a+b)} = \frac{c}{d}$ $\quad \therefore X = \frac{x(a+b)(c-d)}{ad-bc}$

And the numbers are $\frac{aX}{a+b} = \frac{xa(c-d)}{ad-bc}$ and $\frac{bX}{a+b} = \frac{xb(c-d)}{ad-bc}$

$\therefore$ the difference of the two numbers $= \frac{x(a-b)(c-d)}{ad-bc}$

Ex. 11: The ratio between two numbers is 3 : 4. If each number be increased by 2, the ratio becomes 7 : 9. Find the numbers.

Soln : Following the above theorem, the numbers are

$\frac{2 \times 3(7-9)}{3 \times 9 - 4 \times 7}$ and $\frac{2 \times 4(7-9)}{3 \times 9 - 4 \times 7}$

or, 12 and 16.

Ex. 12: The ratio between two numbers is 3 : 4. If each number be increased by 6, the ratio becomes 4 : 5. Find the two numbers.

Soln : The above question may be considered as a special case of the above theorem where $\quad c - a = d - b$

It is easy to distinguish this type of question. In such a question, there should be a uniform increase in ratio, i.e., the antecedent and consequent is increased by the same value.

In the above question, we see that both the antecedent and the consequent are increased by 1 each, and the numbers are increased by 6. Therefore, we may say that

$1 \equiv 6$

or, $3 \equiv 3 \times 6 = 18$ and $4 \equiv 4 \times 6 = 24$

Thus the numbers are 18 and 24.

Note : (1) The above general formula also works for the above example (Ex. 12). It is suggested that you apply that method because that method is universal and you should be familiar with it.

(2) The above question may be rewritten as :

The ratio of two numbers is 4 : 5. If each of them is decreased by 6, the ratio becomes 3 : 4. Find the two numbers.

Apply the same rule in this case also.

Ex. 13: The students in three classes are in the ratio 2 : 3 : 5. If 20 students are increased in each class, the ratio changes to 4 : 5 : 7. What was the total number of students in the three classes before the increase ?

Soln : In the above question also, we see that each term increases by the same value. That is,

4 - 2 = 5 - 3 = 7 - 5 = 2. Thus, we have

$2 \equiv 20$

$\therefore (2+3+5) \equiv \frac{20}{2} \times 10 = 100$ students.

Theorem : The incomes of two persons are in the ratio a : b and their expenditures are in the ratio c : d. If each of them saves Rs X, then their incomes are given by $\frac{Xa(d-c)}{ad-bc}$ and $\frac{Xb(d-c)}{ad-bc}$

Proof : Try this yourself, because the proof for this theorem is similar to the above theorems.

Ex. 14: The incomes of A and B are in the ratio 3 : 2 and their expenditures are in the ratio 5 : 3. If each saves Rs 2000, what is their income ?

Soln : According to the above theorem,

a : b = 3 : 2 (Income)

c : d = 5 : 3 (Expenditure)

X = 2000 (Savings)

Therefore, A's income $= \frac{Xa(d-c)}{ad-bc}$

$= \frac{2000 \times 3 \times (3-5)}{3 \times 3 - 2 \times 5} = $ Rs 12,000

and B's income $= \frac{Xb(d-c)}{ad-bc} = \frac{2000 \times 2 \times (3-5)}{3 \times 3 - 2 \times 5} = $ Rs 8,000

Note : I: If we are asked to find the expenditure, we have two options:

(1) Expenditure = Income - Saving

Thus, A's expenditure = Rs (12000 - 2000) = Rs 10,000

and B's expenditure = Rs (8000 - 2000) = Rs 6,000.

(2) The direct formula is given by :

A's expenditure $= \frac{Xc(b-a)}{ad-bc}$

B's expenditure $= \frac{Xd(b-a)}{ad-bc}$

II: If you note carefully, you will see the similarity between the direct formula for income and expenditure.

Ex. 15: The incomes of Ram and Shyam are in the ratio 8 : 11 and their expenditures are in the ratio 7 : 10. If each of them saves Rs 500, what are their incomes and expenditures ? *(Only by the direct formula)*

Soln : Using the theorem :

a : b = 8 : 11 (Income)

$c : d = 7 : 10$ (Expenditure)
$X = Rs\ 500$ (Savings)

Ram's income $= \frac{Xa(d-c)}{ad-bc} = \frac{500 \times 8(10-7)}{80-77} = Rs\ 4000$

Shyam's income $= \frac{Xb(d-c)}{ad-bc} = \frac{500 \times 11(10-7)}{80-77} = Rs\ 5500$

Ram's expenditure $= \frac{Xc(b-a)}{ad-bc} = \frac{500 \times 7(11-8)}{80-77} = Rs\ 3500$

Shyam's expenditure $= \frac{Xd(b-a)}{ad-bc} = \frac{500 \times 10(11-8)}{80-77} = Rs\ 5000$

Note : It is very easy to remember the above formulae. To be more familiar with these, you need only good practice. Use them whenever you find such a question. You need not write the formula each time, but do only the digital values for calculations. You will get the answer within seconds.

Theorem : If the ratio of any quantities be $a : b : c : d$, then the ratio of other quantities which are inversely proportional to that is given by $\frac{1}{a} : \frac{1}{b} : \frac{1}{c} : \frac{1}{d}$

Ex. 16: The speed of three cars are in the ratio 2 : 3 : 4. What is the ratio among the times taken by these cars to travel the same distance ?

Soln : We know that speed and time taken are inversely proportional to each other. That is, if speed is more the time taken is less and *vice versa*. So, we can apply the above theorem in this case.

Hence, ratio of time taken by the three cars $= \frac{1}{2} : \frac{1}{3} : \frac{1}{4}$

Now, multiply each fraction by the LCM of denominators i.e., the LCM of 2, 3, 4, i.e., 12. So, the required ratio is given by

$$\frac{12}{2} : \frac{12}{3} : \frac{12}{4} = 6 : 4 : 3$$

Ex. 17: The same type of work is assigned to three groups of men. The ratio of persons in the groups is 3 : 4 : 5. Find the ratio of days in which they will complete the works.

Soln : We see that in this case also, man and days are inversely proportional to each other. So, the above rule can be applied in this case also. Therefore, the required ratio is $\frac{1}{3} : \frac{1}{4} : \frac{1}{5}$

Multiplying the above fractions by the LCM of 3, 4 and 5, i.e., 60, we have, $\frac{60}{3} : \frac{60}{4} : \frac{60}{5} = 20 : 15 : 12$

Theorem : If the sum of two numbers is A and their difference is a, then the ratio of numbers is given by $A + a : A - a$

Proof : Let the two numbers be x and y.
Then we have,

$x + y = A$ --------(1)
and $x - y = a$ --------(2)
(1) + (2) gives $2x = A - a$

$\therefore x = \frac{A+a}{2}$

(1) - (2) gives $2y = A - a$

$\therefore y = \frac{A-a}{2}$

Now, the ratio of two numbers

$= x : y = \frac{A+a}{2} : \frac{A-a}{2} = A + a : A - a$

Ex. 18: The sum of two numbers is 40 and their difference is 4. What is the ratio of the two numbers ?

Soln : Following the above theorem, the required ratio of numbers

$= 40 + 4 : 40 - 4 = 44 : 36 = 11 : 9$

Theorem : A number which, when added to the terms of the ratio a: b makes it equal to the ratio c : d is $\frac{ad-bc}{c-d}$

Proof : Let the required number be x, then $\frac{a+x}{b+x} = \frac{c}{d}$

or, $ad + dx = bc + cx$
or, $x(c - d) = ad - bc$

$\therefore x = \frac{ad-bc}{c-d}$

Ex. 19: Find the number which, when added to the terms of the ratio 11 : 23 makes it equal to the ratio 4 : 7.

Soln : Following the above rule :

$a : b = 11 : 23$
$c : d = 4 : 7$

$\therefore$ the required number $= \frac{ad-bc}{c-d} = \frac{11 \times 7 - 23 \times 4}{4-7} = \frac{(-)15}{(-)3} = 5$

Theorem : A number which, when subtracted from the terms of the ratio a: b makes it equal to the ratio c : d is $\frac{bc-ad}{c-d}$

Proof : Try it yourself.

Ex. 20: Find the number which, when subtracted from the terms of the ratio 11 : 23 makes it equal to the ratio 3 : 7.

Soln : Here, a : b = 11 : 23
c : d = 3 : 7

$\therefore$ the required number $=\frac{bc-ad}{c-d}=\frac{23\times 3-11\times 7}{3-7}=\frac{8}{4}=2$

Ex. 21: The contents of two vessels containing water and milk are in the ratio 1 : 2 and 2 : 5 are mixed in the ratio 1 : 4. The resulting mixture will have water and milk in the ratio ______.

Soln: Change the ratios into fractions.

	Water	:	Milk
Vessel I	$\frac{1}{3}$		$\frac{2}{3}$
Vessel II	$\frac{2}{7}$		$\frac{5}{7}$

From Vessel I, $\frac{1}{5}$ is taken and from Vessel II, $\frac{4}{5}$ is taken.

Therefore, the ratio of water to milk in the new vessel

$=\left(\frac{1}{3}\times\frac{1}{5}+\frac{2}{7}\times\frac{4}{5}\right):\left(\frac{2}{3}\times\frac{1}{5}+\frac{5}{7}\times\frac{4}{5}\right)$

$=\left(\frac{1}{15}+\frac{8}{35}\right):\left(\frac{2}{15}+\frac{20}{35}\right)=\frac{31}{105}:\frac{74}{105}=31:74$

Ex. 22: The ratio of A's and B's income last year was 3 : 4. The ratio of their own incomes of last year and this year is 4 : 5 and 2 : 3 respectively. If the total sum of their present incomes is Rs 4160, then find the present income of A.

Soln : The ratio of present incomes $=3\times\frac{5}{4}:4\times\frac{3}{2}$

$=\frac{15}{4}:\frac{12}{2}=30:48=5:8$

$\therefore$ A's present income $=\frac{4160}{5+8}\times 5=$ Rs 1600

Ex. 23: Three glasses A, B and C with their capacities in the ratio 2 : 3 : 4 are filled with a mixture of spirit and water. The ratio of

spirit to water in A, B and C is 1 : 5, 3 : 5 and 5 : 7 respectively. If the contents of these glasses are mixed together, find the ratio of spirit to water in the mixture.

Soln :
A : B : C
2 : 3 : 4
Sp : W = 1 : 5 3 : 5 5 : 7

When they are mixed, the ratio of spirit to water

$=\left(2\times\frac{1}{1+5}+3\times\frac{3}{3+5}+4\times\frac{5}{5+7}\right)$

$:\left(2\times\frac{5}{1+5}+3\times\frac{5}{3+5}+4\times\frac{7}{5+7}\right)$

$=\left(\frac{1}{3}+\frac{9}{8}+\frac{5}{3}\right):\left(\frac{5}{3}+\frac{15}{8}+\frac{7}{3}\right)=\frac{25}{8}:\frac{47}{8}=25:47$

Ex. 24: 465 coins consist of rupee, 50 paise and 25 paise coins. Their values are in the ratio 5 : 3 : 1. Find the number of each coin.

Soln : The ratio of number of coins

$=5\times\frac{100}{100}:3\times\frac{100}{50}:1\times\frac{100}{25}=5:6:4$

$\therefore$ the number of one-rupee coins $=\frac{465}{5+6+4}\times 5=155$

The number of 50P coins $=\frac{465}{5+6+4}\times 6=186$

The number of 25P coins $=\frac{465}{5+6+4}\times 4=124$

Ex. 25: A sum of Rs 11.70 consists of rupee, 50 paise and 5 paise coins in the ratio 3 : 5 : 7. Find the number of each kind of coins.

Soln : This question is different from Ex. 24.

In Ex. 24, the ratio of values were given but in this case, the ratio of numbers is given. Now, the given ratio of numbers is to be changed in the ratio of values. (Whereas in the Ex. 24, the ratio of values was changed into ratio of numbers.)

$\therefore$ the ratio of values $=3\times\frac{100}{100}:5\times\frac{50}{100}:7\times\frac{5}{100}$

$=300:250:35=60:50:7$

$\therefore$ the value of 1-rupee coins $=\frac{11.70}{60+50+7}\times 60=6$ or 6 coins

The value of 50P coins $= \frac{11.70}{60+50+7} \times 50 = 5$ or 10 coins

The value of 5P coins $= \frac{11.70}{60+50+7} \times 7 = 0.7$

or $0.7 \times 20 = 14$ coins

Ex. 26. One year ago, the ratio between Laxman's and Gopal's salaries was 3 : 5. The ratio of their individual salaries of last year and present year are 2 : 3 and 4 : 5 respectively. If their total salaries for the present year are Rs 4300, find the present salary of Laxman.

Soln : The ratio of Laxman's salary for the two years = 2 : 3

The ratio of Gopal's salary for the two years = 4 : 5

We are also given that the ratio of their salary during the last year = 3 : 5

Now, we change the antecedents (2 and 4) of the first two ratios so that the antecedent in the first becomes 3 (antecedent of the third ratio) and the antecedent in the second becomes 5 (consequent of the third ratio).

Thus, $2 : 3 = \; = 3 : \frac{9}{2}$

and $4 : 5 = 4\left(\frac{5}{4}\right) : 5\left(\frac{5}{4}\right) = 5 : \frac{25}{4}$

Now, it is clear that the ratio of their salaries for the present year is $\frac{9}{2} : \frac{25}{4} = 18 : 25$

$\therefore$ the present salary of Laxman $= \frac{4300}{18+25} \times 18 =$ Rs 1800

Note : You should understand the above method clearly. Once you do that, you need very few calculations to reach the answer.

What happens when you are given:

"The present ratio between their salaries is 18 : 25, and the ratios of their individual salaries for the two years is 2 : 3 and 4 : 5. Their total salary for last year was Rs 3200. And you are asked to find the salary of Laxman for last year."

Now, we do the same for the consequents:

$2 : 3 = 2 \times 6 : 3 \times 6 = 12 : 18$

$4 : 5 = 4 \times 5 : 5 \times 5 = 20 : 25$

Thus, the ratio of their salaries last year was $12 : 20 = 3 : 5$

$\therefore$ Laxman's salary last year $= \frac{3200}{3+5} \times 3 =$ Rs 1200.

Ex. 27: A bucket contains a mixture of two liquids A and B in the proportion 7 : 5. If 9 litres of the mixture is replaced by 9 litres of liquid B, then the ratio of the two liquids becomes 7 : 9. How much of the liquid A was there in the bucket?

Soln: Detailed Method:

Suppose the two liquids A and B are 7x litres and 5x litres respectively.

Now, when 9 litres of mixture are taken out,

A remains $7x - 9\left(\frac{7}{7+5}\right) = 7x - \frac{9 \times 7}{12} = \left(7x - \frac{21}{4}\right)$ litres

and B remains $5x - 9\left(\frac{5}{7+5}\right) = 5x - \frac{9 \times 5}{12} = \left(5x - \frac{15}{4}\right)$ litres. Now, when 9 litres of liquid B are added,

$$\left(7x - \frac{21}{4}\right) : \left(5x - \frac{15}{4} + 9\right) = 7 : 9$$

or, $\frac{7x - \frac{21}{4}}{5x - \frac{15}{4} + 9} = \frac{7}{9}$ or, $63x - \frac{189}{4} = 35x - \frac{105}{4} + 63$

or, $28x = \frac{189}{4} - \frac{105}{4} + 63 = 21 + 63 = 84$

or, $x = \frac{84}{28} = 3$ $\quad\therefore\; 7x = 7 \times 3 = 21$ litres

Quicker Method: If we ignore the intermediate steps, we find a formula which is fast-working as well as easier to remember.

1st ratio = 7 : 5, 2nd ratio = 7 : 9

D = Difference of cross-products of ratios $= 7 \times 9 - 7 \times 5 = 63 - 35 = 28$

Now, the formula is:

Common factor of first ratio

$$= \left[\frac{\text{Quantity Replaced}}{\text{Sum of terms in 1st ratio}}\right] + \left[\frac{\text{Quantity replaced} \times \text{term A in 2nd ratio}}{D}\right]$$

$$=\left[\frac{9}{7+5}\right]+\left[\frac{9\times 7}{28}\right]=\frac{9}{12}+\frac{9}{4}=\frac{36}{12}=3$$

$\therefore$ Quantity of A = $7\times 3 = 21$ litres.

Similarly, quantity of B = $5\times 3 = 15$ litres.

Ex. 28: The employer decreases the number of his employees in the ratio 10 : 9 and increases their wages in the ratio 11 : 12. What is the ratio of his two expenditures ?

Soln: The required ratio = $10\times 11 : 9\times 12 = 55 : 54$

Ex. 29. A vessel contains liquids A and B in ratio 5 : 3. If 16 litres of the mixture are removed and the same quantity of liquid B is added, the ratio becomes 3 : 5. What quantity does the vessel hold?

Soln: Detailed Method:

Suppose the vessel contains 5x litres and 3x litres of liquids A and B respectively.

The removed quantity contains $\frac{16}{5+3}\times 5 = 10$ litres of A and $16 - 10 = 6$ litres of B. Now,

$(5x - 10) : (3x - 6 + 16) = 3 : 5$

or, $\frac{5x-10}{3x+10}=\frac{3}{5}$ or, $25x - 50 = 9x + 30$ or, $16x = 80$ $\therefore$ $x = 5$

$\therefore$ The vessel contains $8x = 8\times 5 = 40$ litres.

Quicker Method: When the ratio is reversed (i.e., 5 : 3 becomes 3 : 5), we can use the formula:

$$\text{Total quantity} = \frac{(5+3)^2}{5^2-3^2}\times \text{Quantity of A in the removed mixture}$$

$$=\frac{64}{16}\times 10 = 40 \text{ litres.}$$

Note: When the liquid B is used as a filler, the quantity of A is used in the formula.

Ex. 30: If $(a+b) : (b+c) : (c+a) = 6 : 7 : 8$ and $a + b + c = 14$, then find $a : b : c$ and the value of a, b and c.

Soln: We should know that

$$a+b=\frac{6}{6+7+8}[(a+b)+(b+c)+(a+c)]$$

$$=\frac{6}{21}[2(a+b+c)]=\frac{6}{21}\times 28 = 8$$

Similarly, $b+c=\frac{7}{6+7+8}[2(a+b+c)]=\frac{7}{21}\times 28=\frac{28}{3}$

and $a+c=\frac{8}{21}\times 28=\frac{32}{3}$

Now, $a = [(a+b+c) - (b+c)] = 14 - \frac{28}{3}=\frac{14}{3}$

Similarly, $b = 14 - \frac{32}{3}=\frac{10}{3}$ and $c = 14 - 8 = 6$

Thus, $a=\frac{14}{3}$, $b=\frac{10}{3}$ and $c = 6$

$\therefore$ $a : b : c = \frac{14}{3} : \frac{10}{3} : 6 = 14 : 10 : 18 = 7 : 5 : 9$

Quicker Method: $(a+b) : (b+c) : (c+a) = 6 : 7 : 8$

Now, $[(a+b)+(b+c)+(c+a)] : (a+b) : (b+c) : (c+a)$
$= (6+7+8) : 6 : 7 : 8$

or, $2(a+b+c) : (a+b) : (b+c) : (c+a) = 21 : 6 : 7 : 8$

or $(a+b+c) : (a+b) : (b+c) : (c+a) = 10.5 : 6 : 7 : 8$

Now, $a : b : c = (10.5 - 7) : (10.5 - 8) : (10.5 - 6)$
$= 3.5 : 2.5 : 4.5 = 7 : 5 : 9$

$\therefore$ $a=\frac{14}{7+5+9}\times 7=\frac{14}{3}$

$b=\frac{14}{7+5+9}\times 5=\frac{10}{3}$

$c=\frac{14}{7+5+9}\times 9 = 6$

EXERCISE

1. Form the compound ratio of the ratios 45 : 75, 3 : 4, 51 : 68 and 256 : 81.
2. If A : B = 6 : 7 and B : C = 8 : 9, find A : B : C.
3. The sum of two numbers is 20, and their difference is $2\frac{1}{2}$. Find the ratio of the numbers.
4. If 0.7 of one number be equal to 0.075 of another, what is the ratio of the two numbers ?
5. Find a fraction which shall bear the same ratio to $\frac{1}{27}$ that $\frac{3}{11}$ does to $\frac{5}{9}$.
6. Two sums of money are proportional to 8 : 9. If the first is Rs 20, what is the other ?

7. Find two numbers in the ratio of $5\frac{5}{7}$ to 5 such that when each is diminished by $12\frac{1}{2}$, they shall become in the ratio of $3\frac{2}{3}$ to 3.
8. Divide 37 into two parts such that 5 times one part and 11 times the other are together 227.
9. Find a ratio equal to $\frac{4}{5}$ whose antecedent is 9.
10. Find the value of x in the folowing proportions :
(i) $5 : 15 = 2 : x$. (ii) $75 : 3 = x : 9$.
11. Calculate a fourth proportional to the numbers :
(i) 1, 2, 3. (ii) 490, 70, 69. (iii) 2.5, 1.5, 1.5.
12. If 30 men do a piece of work in 27 days, in what time can 18 men do another piece of work 3 times as great?
13. When wheat is Rs 1.30 per kg, 60 men can be fed for 15 days at a certain cost. How many men can be fed for 45 days at the same cost, when wheat is Re 1 per kg?
14. If a family of 7 persons can live on Rs 840 for 36 days, how long can a family of 9 persons live on Rs 810?
15. If 5 horses eat 18 quintals of oats in 9 days, how long at the same rate will 66 quintals last for 15 horses?
16. If 1000 copies of a book of 13 sheets require 26 reams of paper, how much paper is required for 5000 copies of a book of 17 sheets?
17. If the carriage of 810 kg for 70 km costs Rs 45, what will be the cost of the carriage of 840 kg for a distance of 63 km at half the former rate?
18. If 300 men can do a piece of work in 16 days, how many men would do $\frac{1}{4}$ of the same work in 15 days?
19. Divide Rs 324.36 into three parts in the proportion of 5 : 6 : 7.
20. Divide Rs 53.95 between A, B and C such that A gets thrice as much as B, and C one-third as much as B.
21. Divide Rs 91.30 between A, B and C such that A gets $1\frac{1}{2}$ times as much as C and B $2\frac{1}{2}$ times as much as C.
22. Divide Rs 625 among A, B and C such that A gets $\frac{2}{9}$ of B's share and C gets $\frac{3}{4}$ of A's share.

23. Divide Rs 99 among A, B, C such that A may get 5 times as much as B, and C gets $\frac{1}{2}$ of what A and B together get.
24. Divide Rs 355 into three parts such that three times the first part may be equal to five times the second and seven times the third.
25. A body of 7300 troops is formed of 4 battalions, so that $\frac{1}{2}$ of the first, $\frac{2}{3}$ of the second, $\frac{3}{4}$ of the third and $\frac{4}{5}$ of the fourth are all composed of the same number of men. How many men are there in each battalion?
26. The estate of a bankrupt person worth of Rs 21000 is to be divided among four creditors whose debts are — A's to B's, as 2 : 3, B's to C's as 4 : 5, C's to D's as 6 : 7. What amount must each receive ?
27. How many one-rupee coins, 50 P coins and 25 P coins, of which the numbers are proportional to 4, 5 and 6 are together worth Rs 32 ?
28. A sum of Rs 3115 is divided among A, B and C such that if Rs 25, Rs 28 and Rs 52 be diminished from their shares respectively, the remainders shall be in the ratio of 8 : 15 : 20. Find the share of each.
29. What must be added to two numberes that are in the ratio of 3 : 4, so that they become in the ratio 4 : 5?
30. Find the number which, when subtracted from the terms of the ratio 19 : 23 makes it equal to the ratio of 3 : 4.
31. An employer reduces the number of his employees in the ratio 9 : 8 and increases their wages in the ratio 14 : 15. State whether his bill of total wages increases or decreases, and in what ratio.
32. Rs 50 is divided among 6 men, 12 women and 17 boys so that 2 men get as much as 5 boys and 2 women as much as 3 boys. Find the share of a boy.

ANSWERS

1. $\frac{45}{75} \times \frac{3}{4} \times \frac{51}{68} \times \frac{256}{81} = \frac{16}{15} = 16 : 15$

2. A : B $= 6 : 7$
B : C $= 8 : 9$
A : B : C $= 6 \times 8 : 7 \times 8 : 7 \times 9 = 48 : 56 : 63$

3. Ratio $= \dfrac{20+\frac{5}{2}}{20-\frac{5}{2}} = \dfrac{22.5}{17.5} = \dfrac{225}{175} = \dfrac{9}{7} = 9:7$

4. We have, $0.7x = 0.075y$

$\therefore \dfrac{x}{y} = \dfrac{0.075}{0.7} = \dfrac{75}{700} = \dfrac{3}{28} = 3:28$

5. $x : \dfrac{1}{27} = \dfrac{3}{11} : \dfrac{5}{9}$ or, $27x = \dfrac{3 \times 9}{11 \times 5}$ $\therefore x = \dfrac{1}{55}$

6. $\dfrac{20 \times 9}{8} = \dfrac{45}{2} =$ Rs 22.5

7. $\dfrac{40}{7}x - \dfrac{25}{2} : 5x - \dfrac{25}{2} = \dfrac{11}{3} : 3$

or, $\dfrac{\frac{40}{7}x - \frac{25}{2}}{5x - \frac{25}{2}} = \dfrac{11}{9}$

or, $\dfrac{40 \times 9x}{7} - \dfrac{225}{2} = 55x - \dfrac{275}{2}$ or, $\dfrac{385-360}{7}x = \dfrac{50}{2}$ or, $x = 7$

$\therefore$ Therefore, the numbers are $\dfrac{40}{7} \times 7$ and 5×7 or 40 and 35.

Quicker Method :

This question can be simplified if we change the form of ratio as follow:

$5\frac{5}{7} : 5 = \frac{40}{7} : 5 = 40 : 35 = 8 : 7$ and $3\frac{2}{3} : 3 = \frac{11}{3} : 3 = 11 : 9$

Common factor $= \dfrac{12.5(11-9)}{7 \times 11 - 8 \times 9} = 5$

Thus, the numbers are 8×5 and 7×5 or 40 and 35.

8. $x + y = 37$

$5x + 11y = 227$

Solving these two equations, $x = 30$ and $y = 7$

9. $\dfrac{4}{5} = \dfrac{9}{x}$ or, $x = \dfrac{45}{4}$ Therefore, ratio $= 9 : \dfrac{45}{4} = 4 : 5$

10. (i) $x = \dfrac{15 \times 2}{5} = 6$ (ii) $x = \dfrac{75 \times 9}{3} = 225$

11. (i) $1 : 2 :: 3 : x$ $\therefore x = \dfrac{2 \times 3}{1} = 6$

(ii) $490 : 70 :: 69 : x$ $\therefore x = \dfrac{70 \times 69}{490} = \dfrac{69}{7}$

(iii) $x = \dfrac{1.5 \times 1.5}{2.5} = 0.9$

12. By the Rule of Proportion :

18 men : 30 men

:: 27 : the reqd. no. of days

1 work : 3 work

Answer $= \dfrac{30 \times 3 \times 27}{18} = 135$ days.

By rule of Fraction : $27\left(\dfrac{30}{18}\right)\left(\dfrac{3}{1}\right) = 135$ days.

13. $60\left(\dfrac{15}{45}\right)\left(\dfrac{1.3}{1}\right) = 26$ men.

Note : I: Put the number of men (given) as you are asked to find the number of men.

II: When the number of days increases, less persons could be fed, so multiply by a less-than-one fraction, i.e., $\dfrac{15}{45}$

III: Since price decreases, more persons could be fed. Hence, multiply by a more-than-one fraction i.e., $\left(\dfrac{1.3}{1}\right)$

14. $36\left(\dfrac{7}{9}\right)\left(\dfrac{810}{840}\right) = 27$ days.

Follow the same reasoning as in Q. 13.

15. $9\left(\dfrac{66}{18}\right)\left(\dfrac{5}{15}\right) = 11$ days

16. $26\left(\dfrac{5000}{1000}\right)\left(\dfrac{17}{13}\right) = 170$ reams

17. $45\left(\dfrac{840}{810}\right)\left(\dfrac{63}{70}\right)\left(\dfrac{1}{2}\right) =$ Rs 21

18. $300\left(\dfrac{16}{15}\right)\left(\dfrac{1}{4}\right) = 80$ men.

19. Three parts are $\frac{324.36}{18} \times 5, \frac{324.36}{18} \times 6, \frac{324.36}{18} \times 7.$

$= 18.02 \times 5, 18.02 \times 6, 18.02 \times 7$ = Rs 90.10, Rs 108.12, Rs 126.14

20. A : B = 3 : 1

B : C = 3 : 1

$\therefore$ A : B : C = 3 × 3 : 1 × 3 : 1 × 1 = 9 : 3 : 1

Now, the process is the same as in Q. 19.

21. A : C = 3 : 2

C : B = 2 : 5

$\therefore$ A : C : B = 3 × 2 : 2 × 2 : 2 × 5 = 6 : 4 : 10 = 3 : 2 : 5

$\therefore$ A : B : C = 3 : 5 : 2

22. A : B = 2 : 9 ⇒ B : A = 9 : 2

A : C = 4 : 3

$\therefore$ B : A : C = 9 × 4 : 2 × 4 : 2 × 3 = 36 : 8 : 6 = 18 : 4 : 3

or, A : B : C = 4 : 18 : 3

Note : We have written the ratios in such a way that the **consequent** of the first ratio and the **antecedent** of the second ratio are the same.

Like : A : $\underline{B}$ & $\underline{B}$: C -----(1)

or, B : $\underline{A}$ & $\underline{A}$: C --------(2)

or, B : $\underline{C}$ & $\underline{C}$: A -------(3)

Then we apply the rule:

For (1)

A : B : C = A × B : B × B : B × C

For (2)

B : A : C = B × A : A × A : A × C

23. A = 5B ⇒ A : B = 5 : 1

$C = \frac{1}{2}(A + B) = \frac{1}{2}(6B) = 3B$

⇒ B : C = 1 : 3 $\therefore$ A : B : C = 5 × 1 : 1 × 1 : 1 × 3 = 5 : 1 : 3

24. Try it and yourself (follow the method used in Q. 23).

25. $\frac{x}{2} = \frac{2y}{3} = \frac{3z}{4} = \frac{4w}{5} = k$ (say)

$\therefore x = 2k; y = \frac{3k}{2}; z = \frac{4k}{3}; w = \frac{5k}{4}$

$\therefore x : y : z : w = 2 : \frac{3}{2} : \frac{4}{3} : \frac{5}{4} = 24 : 18 : 16 : 15$

Now, 24 + 18 + 16 + 15 = 73

$\therefore$ the four battlions have 2400, 1800, 1600 and 1500.

26. A : B = 2 : 3

B : C = 4 : 5

C : D = 6 : 7

A : B : C = 2 × 4 : 3 × 4 : 3 × 5 = 8 : 12 : 15

C : D = 6 : 7 = 15 : 17.5

$\therefore$ A : B : C : D = 8 : 12 : 15 : 17.5 = 16 : 24 : 30 : 35

Since 16 + 24 + 30 + 35 = 105

A's share = $\frac{21000}{105} \times 16$ = Rs 3200

B's share = 200 × 24 = Rs 4800

C's share = 200 × 30 = Rs 6000

D's share = 200 × 35 = RS 7000.

27. The ratio of values of a rupee, 50P and 25 P coins

= 4 × 100 : 5 × 50 : 6 × 25 = 8 : 5 : 3

Since 8 + 5 + 3 = 16

The value of Re coins = $\frac{32}{16} \times 8 = 16$

The value of 50 P coins = $\frac{32}{16} \times 5 = 10$

The value of 25 P coins = $\frac{32}{16} \times 3 = 6$

Therefore, the number of Re coins = 16 × 1 = 16

The number of 50 P coins = 10 × 2 = 20

The number of 25 P coins = 6 × 4 = 24

28. The total sum after deduction = 3115 - (25 + 28 + 52) = Rs 3010

Their diminished share is in the ratio 8 : 15 : 20

$\therefore$ A's diminished share $\frac{3010}{43} \times 8$ = Rs 560

B's diminished share = 70 × 15 = Rs 1050

C's diminished share = 70 × 20 = Rs 1400

$\therefore$ A's share = 560 + 25 = Rs 585

B's share = 1050 + 28 = Rs 1078

C's share = 1400 + 52 = Rs 1452

29. 3 : 4

4 : 5

$\therefore$ The number = $\frac{4 \times 4 - 3 \times 5}{5 - 4} = \frac{1}{1} = 1$

30. 19 : 23
3 : 4

$\therefore$ The number $= \frac{19 \times 4 - 23 \times 3}{4 - 3} = \frac{7}{1} = 7$

31. 9 : 8
14 : 15

We know that the total bill = wage per person × no. of total employees
Therefore, the ratio of change in bill

$= 9 \times 14 : 8 \times 15 = 126 : 120 = 21 : 20$

The ratio shows that there is a decrease in the bill.

Note : For a detailed method let the no. of employees in two cases = 9x & 8x. Wages in two cases be 14y & 15y

Initial wage $= 9x \times 14y = 126xy$

Changed wage $= 8x \times 15y = 120\,xy$

This shows the decrease in bill and ratio is 126xy : 120xy = 21 : 20.

32. $2m = 5b$
$2w = 3b$

Combining the two relations: (Follow the rule)

$2m = 5b$
$3b = 2w$

$2 \times 3m = 5 \times 3b = 5 \times 2w \Rightarrow 6m = 15b = 10w$

Now, to find the ratio of wages of a man, a woman and a boy, let

$6m = 15b = 10\,w = k$ (say)

$\therefore m = \frac{k}{6}; b = \frac{k}{15}; w = \frac{k}{10}$

$\therefore m : w : b = \frac{1}{6} : \frac{1}{10} : \frac{1}{15} = 5 : 3 : 2$

The ratio of wages of 6 men, 12 women and 17 boys

$= 6 \times 5 : 12 \times 3 : 17 \times 2 = 30 : 36 : 34$

$\therefore$ 17 boys get $\frac{50}{30 + 36 + 34} \times 34 =$ Rs 17

$\therefore$ 1 boys gets Re 1.

Note : For a quicker approach, if $6m = 10w = 15b$

$\therefore m : w : b = 10 \times 15 : 6 \times 15 : 6 \times 10 = 5 : 3 : 2$

$\therefore 6m : 12w : 17b = 6 \times 5 : 12 \times 3 : 17 \times 2 = 30 : 36 : 34$

$\therefore 17b = \frac{50}{30 + 36 + 34} \times 34 =$ Rs 17 $\quad \therefore b =$ Re 1.

Partnership

A **partnership** is an association of two or more persons who put their money together in order to carry on a certain business. It is of two kinds:

(i) Simple (ii) Compound

Simple partnership : If the capital of the partners are invested for the same period, the partnership is called **simple.**

Compound Partnership : If the capitals of the partners are invested for different lengths of time, the partnership is called **compound.**

In a group of n persons invested different amount for different period then their profit ratio is:

$$At_1 : Bt_2 : Ct_3 : Dt_4 \,\ldots\ldots\ldots : Xt_n$$

[Here first person invested amount A for t_1 period, second persons invested amount B for t_2 period, and so on.]

Ex. 1 : Three partners A, B and C invest Rs 1600, Rs 1800 and Rs 2300 respectively in business. How should they divide a profit of Rs 1938 ?

Soln : The profit should be divided in the ratios of the capitals, i.e. in the ratio 16:18:23.

Now, 16+18+23 = 57

A's share $= \frac{16}{57}$ of Rs 1938 = Rs 544

B's share $= \frac{18}{57}$ of Rs 1938 = Rs 612

C's share $= \frac{23}{57}$ of Rs 1938 = Rs 782

Ex. 2 : A, B and C enter into partnership. A advances Rs 1200 for 4 months, B Rs 1400 for 8 months, and C Rs 1000 for 10 months. They gain Rs 585 altogether. Find the share of each.

Soln : Rs 1200 in 4 months earns as much profit as Rs 1200 × 4 or Rs 4800 in 1 month.

Rs 1400 in 8 months earns as much profit as Rs 1400 × 8 or Rs 11200 in 1 month.

Rs 1000 in 10 months earns as much profit as Rs 1000 × 10 or Rs 10,000 in 1 month.

Therefore, the profit should be divided in the ratios of 4800, 11,200 and 10,000 i.e. in the ratios of 12, 28 and 25.

Now, $12 + 28 + 25 = 65$

A's share $= \frac{12}{65} \times 585 =$ Rs 108

B's share $= \frac{28}{65} \times 585 =$ Rs 252

C's share $= \frac{25}{65} \times 585 =$ Rs 225

Note : In compound partnership, the ratio of profits is directly proportional to both money and time, so they are multiplied together to get the corresponding shares in the ratio of profits.

Ex. 3 : A starts a business with Rs 2,000. B joins him after 3 months with Rs 4,000. C puts a sum of Rs 10,000 in the business for 2 months only. At the end of the year the business gave a profit of Rs 5600. How should the profit be divided among them ?

Soln : Ratio of their profits (A's:B's:C's)

$= 2 \times 12 : 4 \times 9 : 10 \times 2 = 6 : 9 : 5$

Now, $6 + 9 + 5 = 20$

Then A's share $= \frac{5600}{20} \times 6 =$ Rs 1680

B's share $= \frac{5600}{20} \times 9 =$ Rs 2520

C's share $= \frac{5600}{20} \times 5 =$ Rs 1400

Ex. 4 : A and B enter into a partnership for a year. A contributes Rs 1500 and B Rs 2000. After 4 months they admit C, who contributes Rs 2250. If B withdraws his contribution after 9 months, how would they share a profit of Rs 900 at the end of the year ?

Soln : A's share : B's share : C's share

$= 1500 \times 12 : 2000 \times 9 : 2250 \times 8$

$= 15 \times 12 : 20 \times 9 : 22.5 \times 8 = 180 : 180 : 180 = 1 : 1 : 1$

Therefore, each of them gets Rs $\frac{900}{3} =$ Rs 300.

Ex. 5 : A, B and C enter into partnership. A advances one-fourth of the capital for one-fourth of the time. B contributes one-fifth of the capital for half of the time. C contributes the remaining capital for the whole time. How should they divide a profit of Rs 1140 ?

Soln : A's share : B's share : C's share

$= \frac{1}{4} \times \frac{1}{4} : \frac{1}{5} \times \frac{1}{2} : \left\{1 - \left(\frac{1}{4} + \frac{1}{5}\right)\right\} \times 1 = \frac{1}{16} : \frac{1}{10} : \frac{11}{20}$

Multiplying each fraction by LCM of 16, 10 and 20, i.e., 80.

We have 5 : 8 : 44

$\therefore$ A's share $= \frac{1140}{57} \times 5 =$ Rs 100

B's share $= \frac{1140}{57} \times 8 =$ Rs 160

C's share $= \frac{1140}{57} \times 44 =$ Rs 880

Ex. 6 : A and B enter into a speculation. A puts in Rs 50 and B puts in Rs 45. At the end of 4 months A withdraws half his capital and at the end of 6 months B withdraws half of his capital. C then enters with a capital of Rs 70. At the end of 12 months, in what ratio will the profit be divided ?

Soln : A's share : B's share : C's share

$= 50 \times 4 + \frac{50}{2} \times 8 : 45 \times 6 + \frac{45}{2} \times 6 : 70 \times 6$

$= 400 : 405 : 420 = 80 : 81 : 84$

Therefore, the profit will be divided in the ratio of 80 : 81 : 84.

Now, you must have understood both simple partnership and compound partnership. The formula for compound partnership can also be written as

$$\frac{\text{A's Capital} \times \text{A's Time in partnership}}{\text{B's Capital} \times \text{B's Time in partnership}} = \frac{\text{A's Profit}}{\text{B's Profit}}$$

The above relationship should be remembered because it is used very often in some types of question.

Ex. 7 : A began a business with Rs 450 and was joined afterwards by B with Rs 300. When did B join if the profits at the end of the year were divided in the ratio 2 : 1?

Soln : Suppose B joined the business for x months.

Then using the above formula, we have $\frac{450 \times 12}{300 \times x} = \frac{1}{2}$

or, $300 \times 2x = 450 \times 12$ $\quad \therefore x = \frac{450 \times 12}{2 \times 300} = 9$ months

Therefore, B joined after (12-9 =) 3 months.

Ex. 8 : A and B rent a pasture for 10 months. A puts in 100 cows for 8 months. How many can B put in for the remaining 2 months, if he pays half as much again as A ?

Soln : Suppose B puts in x cows. The ratio of A's and B's rents

$= 1 : 1 + \frac{1}{2} = 1 : \frac{3}{2} = 2 : 3$

Then, $\frac{100 \times 8}{x \times 2} = \frac{2}{3}$ or, $x = \frac{100 \times 8 \times 3}{2 \times 2} = 600$ cows.

Ex 9 : A and B enter into a partnership with their capitals in the ratio 7 : 9. At the end of 8 months, A withdraws his capital. If they receive the profits in the ratio 8 : 9, find how long B's capital was used

Soln : Suppose B's capital was used for x months. Following the same rule, we have, $\frac{7 \times 8}{9 \times x} = \frac{8}{9}$ or, $x = \frac{7 \times 8 \times 9}{8 \times 9} = 7$

Therefore, B's capital was used for 7 months.

Ex 10 : A, B and C invested capitals in the ratio 2 : 3 : 5; the timing of their investments being in the ratio 4 : 5 : 6. In what ratio would their profit be distributed ?

Soln : We should know that if the three investments be in the ratio a : b : c and the duration for their investments be in the ratio x : y : z, then the profit would be distributed in the ratio ax : by : cz.

Thus, following the same rule, the required ratio

$= 2 \times 4 : 3 \times 5 : 5 \times 6 = 8 : 15 : 30$

Ex 11 : A, B and C invested capitals in the ratio 5 : 6 : 8. At the end of the business term, they received the profits in the ratio 5 : 3 : 12. Find the ratio of time for which they contributed their capitals ?

Soln : Following the same rule:

If investment is in the ratio a : b : c and profit in the ratio p : q : r

Then the ratio of time $= \frac{p}{a} : \frac{q}{b} : \frac{r}{c}$

Therefore, the required ratio $= \frac{5}{5} : \frac{3}{6} : \frac{12}{8} = 1 : \frac{1}{2} : \frac{3}{2}$

$= 2 : 1 : 3$

Ex 12 : A and B enter into a partnership with capitals in the ratio 5 : 6. At the end of 8 months, A withdraws his capital. If they receive profits in the ratio of 5 : 9, find how long B's capital was used.

Soln : This question is similar to the one in Ex. 9. You may solve it by the method used in Ex. 9. But following the rule defined in Ex. 11, we see that the ratio of time of investment

$= \frac{5}{5} : \frac{9}{6} = 1 : \frac{3}{2} = 2 : 3$

Now, we are given that A invested for 8 months.

$\therefore$ B invested for $\frac{8}{2} \times 3 = 12$ months

Note : The validity of the above rules can be checked thus:

Suppose A and B invested Rs 5 and Rs 6 respectively.

A invested for 8 months and B invested for 12 months. Then, the ratio of their profit

$= 5 \times 8 : 6 \times 12 = 10 : 18 = 5 : 9$

Which is the same as given in the question.

Ex. 13: A, B and C are partners. A receives $\frac{2}{5}$ of the profit and B and C share the remaining profit equally. A's income is increased by Rs 220 when the profit rises from 8% to 10%. Find the capitals invested by A, B and C.

Soln: For A's share: (10% - 8%) ≡ Rs 220

$\therefore$ 100% ≡ $\frac{220}{2} \times 100 =$ Rs 11000

$\therefore$ A's capital = Rs 11000

For B's & C's share: $\frac{2}{5} \equiv 11000$

$\therefore \frac{3}{5} \equiv \frac{11000}{2} \times 3 =$ Rs 16500

$\therefore$ B's and C's capitals are Rs 8250 each.

Ex. 14: Two partners invest Rs 125,000 and Rs 85,000 respectively in a business and agree that 60% of the profit should be divided equally between them and the remaining profit is to be treated as interest on capital. If one partner gets Rs 300 more than the other, find the total profit made in the business.

Soln: Detail Method: The difference counts only due to the 40% of the profit which was distributed according to their investments.

Let the total profit be Rs x.

Then 40% of x is distributed in the ratio

125,000 : 85,000 = 25 : 17

Therefore, the share of the first partner = 40% of $x\left(\frac{25}{25+17}\right)$

$= 40\%$ of $x\left(\frac{25}{42}\right) = \frac{40x}{100}\left(\frac{25}{42}\right) = \frac{5x}{21}$

and the share of the second partner = 40% of $x\left(\frac{17}{42}\right) = \frac{17x}{105}$

Now, from the question,

the difference in share $= \frac{5x}{21} - \frac{17x}{105} = 300$

or, $\frac{x(25-17)}{105} = 300$

$\therefore x = \frac{300 \times 105}{8} = \text{Rs } 3937.50$

Direct Method: The ratio of profit = 125,000 : 85,000 = 25 : 17

$\therefore$ total profit $= 300\left(\frac{100}{40}\right)\left(\frac{25+17}{25-17}\right) = \text{Rs } 3937.50$

Ex. 15: A and B entered into a partnership, investing Rs 16,000 and Rs 12,000 respectively. After 3 months, 'A' withdrew Rs 5000 while B invested Rs 5000 more. After 3 months more, C joins the business with a capital of Rs 21,000. After a year, they obtained a profit of Rs 26,400. By what value does the share of B exceed the share of C?

Soln : The above question may be restated as:

A invested Rs 16,000 for 3 months and Rs (16,000 - 5000) for 9 months.

B invested Rs 12,000 for 3 months and Rs (12,000 + 5000) for 9 months.

C invested Rs 21,000 for 6 months.

(These steps should be calculated mentally by you and not in writing.)

Now, A's share : B's share : C's share

$= (16 \times 3 + 11 \times 9) : (12 \times 3 + 17 \times 9) : (21 \times 6)$

$= 147 : 189 : 126 = 7 : 9 : 6$

Therefore, B's share exceeds that of C by

$\frac{26400}{7+9+6} \times (9-6) = \frac{26400 \times 3}{22} = \text{Rs } 3600$

Note: During the calculation, we did not carry the zeroes of thousand because they are of no use in calculating the ratio.

Ex. 16: A, B and C are partners in a business. A, whose money has been used for 4 months, claims $\frac{1}{8}$ of the profit. B, whose money has been used for 6 months, claims $\frac{1}{3}$ of the profit. C had invested Rs 1560 for 8 months. How much money did A and B contribute?

Soln: Ratio of their shares in profit $= \frac{1}{8} : \frac{1}{3} : \left\{1 - \left(\frac{1}{8} + \frac{1}{3}\right)\right\}$

$= \frac{1}{8} : \frac{1}{3} : \frac{13}{24} = 3 : 8 : 13$

Now, for A and C

$A \times 4 : 1560 \times 8 = 3 : 13 \quad \therefore A = \frac{3}{13} \times \frac{1560 \times 8}{4} = \text{Rs } 720$

Now, for B and C

$B \times 6 : 1560 \times 8 = 8 : 13 \quad \therefore B = \frac{8}{13} \times \frac{1560 \times 8}{6} = \text{Rs } 1280$

Ex. 17: Two partners invested Rs 50,000 and Rs 70,000 respectively in a business and agreed that 70% of the profits should be divided equally between them and the remaining profit in the ratio of investment. If one partner gets Rs 90 more than the other, find the total profit made in the business.

Soln : The difference comes only due to the 30% of the profit which was distributed in the ratio of their investments.

Suppose the total profit is Rs x.

Then 30% of x is distributed in the ratio 50,000 : 70,000 = 5 : 7

Therefore, the share of the first partner

$= 30\% \text{ of } \left(\frac{5}{5+7}\right)x = 30\% \text{ of } \frac{5x}{12} = \frac{x}{8}$

and the share of the second partner $= 30\% \text{ of } \left(\frac{7}{5+7}\right)x$

$= 30\% \text{ of } \frac{7x}{12} = \frac{7x}{40}$

Now, the difference in shares $= \frac{7x}{40} - \frac{x}{8} = \text{Rs } 90$

or, $\frac{7x - 5x}{40} = 90 \qquad \therefore x = \frac{90 \times 40}{2} = \text{Rs } 1800$

Quicker Method (Direct Formula) :

Ratio of profits = 50,000 : 70,000 = 5 : 7

$\therefore$ the total profit $= 90\left(\frac{100}{30}\right)\left(\frac{5+7}{7-5}\right) = \text{Rs } 1800$

Ex. 18: A, B and C invested capitals in the ratio 2 : 3 : 4. At the end of the business term, they received the profits in the ratio 3 : 6 : 10. Find the ratio of the periods for which they contributed their capitals.

Soln: If the investment are in the ratio x : y : z and the profits in the ratio P : Q : R, then the ratio of periods $= \frac{P}{x} : \frac{Q}{y} : \frac{R}{z}$

Therefore, the required ratio $= \frac{3}{2} : \frac{6}{3} : \frac{10}{4}$

Multiply each term by the LCM of 2, 3 & 4, i.e., 12.

$\frac{3}{2} \times 12 : \frac{6}{3} \times 12 : \frac{10}{4} \times 12 = 18 : 24 : 30 = 3 : 4 : 5$

Ex. 19. A and B invested in the ratio 3 : 2 in a business. If 5% of the total profit goes to charity and A's share is Rs 855, find the total profit.

Soln : Suppose the total profit is Rs 100.

Then Rs 5 goes to charity.

Now, Rs 95 is divided in the ratio 3 : 2.

$\therefore$ A's share $= \frac{95}{3+2} \times 3 = $ Rs 57

But we see that A's actual share is Rs 855.

$\therefore$ Actual total profit $= 855\left(\frac{100}{57}\right) = $ Rs 1500

Direct Formula : In the above case:

Total profit $= 855\left(\frac{100}{100-5}\right)\left(\frac{3+2}{3}\right)$

$= 855\left(\frac{100}{95}\right)\left(\frac{5}{3}\right) = $ Rs 1500

Ex. 20: In a partnership, A invested $\frac{1}{6}$ of the capital for $\frac{1}{6}$ of the time, B invested $\frac{1}{3}$ of the capital for $\frac{1}{3}$ of the time, and C invested the rest of the capital for the whole period. At the end of the period, they earned a profit of Rs 4600. Find the share of B.

Soln : C invested $1 - \left(\frac{1}{6} + \frac{1}{3}\right) = 1 - \frac{1}{2} = \frac{1}{2}$ part of the capital

Now, ratio of profit = A : B : C $= \frac{1}{6} \times \frac{1}{6} : \frac{1}{3} \times \frac{1}{3} : \frac{1}{2} \times 1$

$= \frac{1}{36} : \frac{1}{9} : \frac{1}{2} = 1 : 4 : 18$

$\therefore$ B's share $= 4600\left(\frac{4}{1+4+18}\right) = 4600\left(\frac{4}{23}\right) = $ Rs 800

EXERCISE

1. How should a profit of Rs 450 be divided between two partners, one of whom has contributed Rs 1200 for 5 months and the other Rs 750 for 4 months ?
2. There are three partners A, B and C in a business. A puts in Rs 2000 for 5 months, B Rs 1200 for 6 months and C Rs 2500 for 3 months; and the profits are Rs 508.82 . How ought it to be divided ?
3. A and B enter into a partnership for a year. A contributes Rs 1500 and B Rs 2000. After 4 months, they admit C, who contributes Rs 2250. If B withdraws his contribution after 9 months, how would they share a profit of Rs 900 at the end of the year ?
4. A, B and C enter into a partnership. A advances one-fourth of the capital for one-fourth of the time; B advances one-fifth of the capital for half of the time; and C, the remainder of the capital for the whole time. How should they divide a profit of Rs 3420 ?
5. Three partners altogether invested Rs 114,000 in a business. At the end of the year, one got Rs 337.50, the second Rs 1125.00 and the third Rs 675 as profit. How much amount did each invest ? What is the percentage of profit ?
6. A and B enter into a speculation ; A puts in Rs 50 and B puts in Rs 45. At the end of 4 months, A withdraws half his capital and at the end of 5 months B withdraws $\frac{1}{2}$ of his; C then enters with a capital of Rs 70; at the end of 12 months, the profits of the concern are Rs 254; how ought it to be divided ?
7. A and B enter into a partnership with capitals as 5:6; and at the end of 8 months, A withdraws. If they receive profits in the ratio of 5:9, find how long B's capital was used.
8. A and B rent a pasture for 10 months; and A puts in 90 oxen for 7 months. How many oxen can B put in for the remaining 3 months, if he pays half as much as A?

Answers

1. The ratio of profit $= 12 \times 5 : 7.5 \times 4 = 60 : 30 = 2 : 1$

   1st partner gets $\frac{450}{3} \times 2 = $ Rs 300

   2nd partner gets $\frac{450}{3} \times 1 = $ Rs 150
2. The ratio of profits $= 20 \times 5 : 12 \times 6 : 25 \times 3 = 100 : 72 : 75$

   Find their shares.

3. A's share : B's share : C's share
$\therefore = 15 \times 12 : 20 \times 9 : 22.5 \times 8 = 180 : 180 : 180 = 1 : 1 : 1$
Find their shares.

4. A's share : B's share : C's share
$= \frac{1}{4} \times \frac{1}{4} : \frac{1}{5} \times \frac{1}{2} : \left\{1 - \left(\frac{1}{4} + \frac{1}{5}\right)\right\} \times 1 = \frac{1}{16} : \frac{1}{10} : \frac{11}{20}$
Multiply each by the LCM of the denominators i.e. 80.
$= 5 : 8 : 44$
Find the shares.

5. The ratio of investments = Ratio of profits
$= 337.5 : 1125 : 675 = 3375 : 11250 : 6750$
Dividing each by 1125, we have the ratio = 3 : 10 : 6
Find the shares.

The reqd. percentage of profit $= \frac{337.5 + 1125 + 675}{11400} \times 100\%$

$= \frac{2137.5}{1140}\% = 1.875\%$

6. A's share : B's share : C's share
$= 50 \times 4 + 25 \times 8 : 45 \times 5 + 22.5 \times 7 : 70 \times 7 = 400 : 382.5 : 490$
Find the shares.

7. The ratio of capitals = 5 : 6
Let the ratio of time = 8 : x

Then $5 \times 8 : 6x = 5 : 9 \quad \therefore \frac{40}{6x} = \frac{5}{9} \quad \therefore x = \frac{40 \times 9}{6 \times 5} = 12$ months.

By Direct Formula : Capitals : 5 : 6; Profit : 5 : 9

$\therefore$ B's time = A's time $\left(\frac{5 \times 9}{6 \times 5}\right) = \frac{8 \times 5 \times 9}{6 \times 5} = 12$ months

8. $\frac{\text{A's share}}{\text{B's share}} = \frac{90 \times 7}{x \times 3} = \frac{2}{1} \quad \therefore x = \frac{90 \times 7}{3 \times 2} = 105$ oxen.

Percentage

The term **per cent** means 'for every hundred'. It can best be defined as:

"A fraction whose denominator is 100 is called a **percentage**, and the numerator of the fraction is called the **rate per cent**."

The following examples illustrate the percents and their fractional values :

1) A student gets 60 per cent marks in Arithmetic means that he obtained 60 marks out of every hundred of full marks. That is, if the full marks be 500, he gets 60+60+60+60+60=300 marks in mathematics. The above five 60s are one 60 for every hundred.

The total marks obtained by the student can be calculated in other ways, like,

$$60\% \text{ of } 500 = \frac{60}{100} \times 500 = 300$$

The above calculations can be made easier by reducing the fractional value to its prime. As, in the above case;

$$60\% = \frac{60}{100} = \frac{3}{5}$$

If we remember that $60\% = \frac{3}{5}$, our calculation becomes easier. In that case, the total marks obtained by the student $= \frac{3}{5} \times 500 = 300$

2) A man invests 5% of his income into shares. It means:
i) he invests Rs 5 out of every Rs 100 of his income into shares.
or, ii) he invests $\frac{5}{100}$ of his income into shares.
or, iii) he invests $\frac{1}{20}$th of his income into shares.

Now, if his income is Rs 1050, how does he invest in shares?

Your quick answer should be $\frac{1050}{20}$ = Rs 52.5

We suggest you not to move with the fraction containg 100, if possible.

3) A tradesman makes a profit of 15 per cent. Means that he makes a profit of Rs 15 when he invests Rs 100. But what does he gain when he invests Rs 900? Which of the applications, mentioned above, is

easier to deal with? Don't you think that the fraction containing hundred is more helpful in this case! (Why?) Because two complete hundreds cancel out easily, giving quick result; like: $\frac{15}{100} \times 900 = 15 \times 9$ $= 135$ (I think you need not write anything to calculate such problems.)

We see that our key operator in this chapter is the prime fraction of per cent value. So, we should collect some of the important (most used) prime fractions:

$3\frac{1}{8} = \frac{1}{32}$	$6\frac{1}{4}\% = \frac{1}{16}$
$5\% = \frac{1}{20}$	$8\% = \frac{2}{25}$
$8\frac{1}{3}\% = \frac{1}{12}$	$10\% = \frac{1}{10}$
$12\% = \frac{3}{25}$	$12\frac{1}{2}\% = \frac{1}{8}$
$13\frac{1}{3}\% = \frac{2}{15}$	$14\frac{2}{7}\% = \frac{1}{7}$
$15\% = \frac{3}{20}$	$16\% = \frac{4}{25}$
$16\frac{2}{3}\% = \frac{1}{6}$	$20\% = \frac{1}{5}$
$25\% = \frac{1}{4}$	$33\frac{1}{3}\% = \frac{1}{3}$
$37\frac{1}{2}\% = \frac{3}{8}$	$40\% = \frac{2}{5}$
$60\% = \frac{3}{5}$	$62\frac{1}{2}\% = \frac{5}{8}$
$66\frac{2}{3}\% = \frac{2}{3}$	$75\% = \frac{3}{4}$
$87\frac{1}{2}\% = \frac{7}{8}$	

Ex. 1 : Find 8 per cent of Rs 625.

Soln : 8% of Rs 625 = $\frac{2}{25} \times 625 =$ Rs 50.

Ex. 2 : What fraction is $12\frac{1}{2}$ per cent?

Soln : $12\frac{1}{2}\% = \frac{12\frac{1}{2}}{100} = \frac{25}{200} = \frac{1}{8}$

Ex. 3 : What percentage is equivalent to $\frac{3}{8}$?

Soln : $\frac{3}{8} \times 100 = \frac{75}{2} = 37\frac{1}{2}\%$

Ex. 4: What percent is equivalent to $\frac{7}{11}$? or express $\frac{7}{11}$ as rate per cent.

Soln: $\frac{7}{11} \times 100 = \frac{700}{11} = 63\frac{7}{11}\%$

Ex. 5: The population of a town has increased from 60,000 to 65,000. Find the increase per cent.

Soln : Increase in population = 65, 000 - 60,000 = 5000

Percentage increase $= \frac{5000}{60,000} \times 100 = \frac{25}{3} = 8\frac{1}{3}\%$

Ex. 6 : Ram's salary is increased from Rs 630 to Rs 700. Find the increase per cent.

Soln : Increase in salary = Rs 700 - Rs 630 = Rs 70.

Percentage increase $= \frac{70}{630} \times 100 = 11\frac{1}{9}\%$

Ex. 7: In an election of two candidates, the candidate who gets 41% is rejected by a majority of 2412 votes. Find the total no. of votes polled.

Soln: (59% - 41% =) 18% = 2412 $\therefore$ $100\% = \frac{2412}{18} \times 100 = 13400$

Ex. 8: If 2 litres of water are evaporated on boiling from 8 litres of sugar solution containing 5% sugar, find the percentage of sugar in the remaining solution.

Soln: As sugar has not been evaporated from the solution, the quantity of sugar in the original 8 litres of solution = the quantity of sugar in the remaining 8 - 2 = 6 litres of solution i.e.,

5% of 8 = x% of 6 $\quad\therefore\ x = \frac{5 \times 8}{6} = 6\frac{2}{3}\%$

Second Method: % of sugar in the original solution
= 5% of 8 litres = 0.4 litres

After evaporation of 2 lt of water, the quantity of the remaining solution = 8 - 2 = 6 litres

$\therefore$ the required percentage of sugar $= \frac{0.4}{6} \times 100\% = 6\frac{2}{3}\%$

Ex. 9: One type of liquid contains 25% of milk, the other contains 3 of milk. A cane is filled with 6 parts of the first liquid and 4 p of the second liquid. Find the percentage of milk in the n mixture.

Soln: The reqd. percentage of milk in the new mixture

$= \frac{\text{Quantity of milk in the new mixture}}{\text{Quantity of the new mixture}} \times 100$

$= \frac{\text{6 parts of 25\% milk + 4 parts of 30\% milk}}{\text{(6 parts + 4 parts) of the liquid}} \times 100$

$= \frac{6 \times \frac{25}{100} + 4 \times \frac{30}{100}}{10} \times 100 = (15 + 12) = 27$

Note: This equation can be solved by the method of Alligation.

25 30
x
6 4

$\frac{30 - x}{x - 25} = \frac{6}{4} = \frac{3}{2}$

or, $60 - 2x = 3x - 75$ or, $5x = 60 + 75$ $\therefore x = 27\%$

Ex. 10: Due to falll in manpower, the production in a factory decrea by 25%. By what per cent should the working hour be increas to restore the original production?

Soln: Decrease in production is only due to decrease in manpow Hence, manpower is decreased by 25%.

Now, suppose that to restore the same production, working ho are increased by x%.

Production = Manpower × Working hours = M × W (say)

Now, M × W = (M - 25% of M) × (W + x% of W)

or, $M \times W = \frac{75}{100} M \times \frac{100 + x}{100} W$

or, $100 \times 100 = 75(100 + x)$

or, $\frac{400}{3} = 100 + x$ $\therefore x = \frac{100}{3} = 33\frac{1}{3}\%$

Method II: To make the calculations easier, suppose

Manpower = 100 units and Working hours = 100 units

Suppose working hours increase by x%.

Then, $(100 - 25)(100 + x) = 100 \times 100$

or, $100 + x = \frac{400}{3}$ $\therefore x = \frac{100}{3} = 33\frac{1}{3}\%$

Direct Formula: Required % increase in working hours

$= \frac{25}{100 - 25} \times 100 = \frac{100}{3} = 33\frac{1}{3}\%$

To find how much per cent one quantity is of another

Ex. 11: Express the fraction which Rs 1.25 is of Rs 10 as a percentage.

Soln : The fraction $= \frac{\text{Rs } 1.25}{\text{Rs } 10} = \frac{125}{1000} = \frac{1}{8}$

Now, $\frac{1}{8} = \frac{\frac{1}{8} \times 100}{100} = \frac{12\frac{1}{2}}{100} = 12\frac{1}{2}\%$

Note : The above question is often put as "what rate per cent is Rs 1.25 of Rs 10?"

Ex. 12: What rate per cent is 6P of Re 1?

The fraction $= \frac{6P}{Re 1} = \frac{6}{100} = \frac{3}{50} = \frac{\frac{3}{50} \times 100}{100} = 6\%$

Ex. 13: 12% of a certain sum of money is Rs 43.5. Find the sum.

Soln : $\frac{12}{100}$ of a sum = Rs $43\frac{1}{2}$ $\therefore$ the sum $= \frac{87}{2} \times \frac{100}{12}$ = Rs 362.50

Theorem : If two values are respectively x% and y% more than a third value, then the first is the $\frac{100 + x}{100 + y} \times 100$ % of the second.

Proof : Let the third value be 100.

Then the first is 100 + x% of 100 = 100 + x

and the second is 100 + y% of 100 = 100 + y

$\therefore$ the first is $\frac{100 + x}{100 + y} \times 100$ % of the second.

Ex 14: Two numbers are respectively 20% and 50% more than a third. What percentage is the first of the second ?

Soln : Following the above theorem, we have the required value

$= \frac{120}{150} \times 100 = 80\%$

Theorem: If A is x% of C and B is y% of C, then A is $\frac{x}{y} \times 100$ % of B.

Proof : Try this and for yourself.

Ex. 15: Two numbers are respectively 20% and 25% of a third number.

What percentage is the first of the second ?

Soln : Following the above theorem, we have the required value

$= \frac{20}{25} \times 100 = 80\%$

Note : The above relationships are very simple. When "What is the first of second" is asked, put the first as the numerator and the second as the denominator and *vice versa*.

Ex. 16: Two numbers are respectively 30% and 40% less than a third number. What per cent is the second of the first ?

Soln : At first, you should find the formula yourself. If you can't find it, go through the following remarks.

(1) Since the two numbers are less than the third; and

(2) we have to find the per cent of the second with respect to the first, our formula should be:

$$\frac{100-40}{100-30} \times 100 = \frac{60}{70} \times 100 = 85\frac{5}{7}\%$$

Ex. 17: A positive number is divided by 5 instead of being multiplied by 5. What % is the result of the required correct value?

Soln: Let the no. be 1, then the correct answer = 5

The incorrect answer that was obtained $= \frac{1}{5}$

$\therefore$ The reqd. % $= \frac{1}{5 \times 5} \times 100\% = 4\%$

Ex. 18: A positive no. is by mistake multiplied by 5 instead of being divided by 5. By what per cent more or less than the correct answer is the result obtained?

Soln: Let the no. be 1, then the correct answer $= \frac{1}{5}$

The incorrect answer that was obtained = 5

$\therefore$ The result is more than the correct answer by $= 5 - \frac{1}{5} = \frac{24}{5}$

$$\therefore \text{ The reqd. \%} = \frac{\frac{24}{5}}{\frac{1}{5}} \times 100\% = 2400\%$$

Percentage Expenditures and Saving

Ex. 19: A man loses $12\frac{1}{2}\%$ of his money and, after spending 70% of the remainder, he is left with Rs 210. How much had he at first ?

Soln : The above question can be solved in many ways. We will discuss a few of them.

I: Let the man be supposed to have Rs x at first.

After losing $12\frac{1}{2}\%$ or $\frac{1}{8}$, he is left with $x - \frac{x}{8} = \text{Rs } \frac{7x}{8}$

After spending 70% of the money, he is left with 30% of the remainder, i.e.,

$$\frac{7x}{8} \times \frac{3}{10} = 210 \quad \therefore x = \frac{210 \times 10 \times 8}{3 \times 7} = \text{Rs } 800$$

II: Suppose he had Rs 100 at first. After losing Rs $12\frac{1}{2}$ he would have Rs $87\frac{1}{2}$ left. He spent 70% of Rs $87\frac{1}{2}$

$\therefore$ he would have $\left(30\% \text{ of Rs } 87\frac{1}{2}\right)$ or Rs $\frac{105}{4}$ left. But he has Rs 210 left. Thus, we have the following proportions :

Rs $\frac{105}{4}$: Rs 210 :: Rs 100 : the required money (By the rule of three)

$\therefore$ the required money = Rs $\frac{4 \times 210 \times 100}{105}$ = Rs 800

III: Quicker Method : It is a very short and fast-calculating method. The only thing is to remember the formula well.

$$\text{His initial money} = \frac{210 \times 100 \times 100}{(100-12.5)(100-70)} = \frac{210 \times 100\ 100}{87.5 \times 30} = \text{Rs } 800$$

Note : As his "initial money" is definitely more than the "left money", there should not be any confusion in putting the larger value (100) in the numerator and the smaller value (100 - 12.5) in the denominator.

Ex. 20: 3.5% of income is taken as tax and 12.5% of the remaining is saved. This leaves Rs 4,053 to spend. What is the income ?

Soln : Quicker Maths gives the solution as :

$$\text{Income} = \frac{4053 \times 100 \times 100}{(100-3.5)(100-12.5)} = \text{Rs } 4{,}800$$

Thus, we derive a general formula in the form of the following theorem :

Theorem : x% of a quantity is taken by the first, y% of the remaining is taken by the second and z% of the remaining is taken by third person. Now, if A is left in the fund, then there was

$$\frac{A \times 100 \times 100 \times 100}{(100-x)(100-y)(100-z)}$$ in the beginning

Proof : The initial amount must be more than 'A'. So, by the rule o fraction, 'A' should be multiplied by the fractions which are m than one. Now, the problem is to find these fractions. Sinc each step the amount is lessened, (100 - x), (100 - y) and (100 - z) should be in our dealing fractions apart from 100.

Thus, the fractions (more than one) by which A is multiplied

$\frac{100}{100-x}, \frac{100}{100-y}$, and $\frac{100}{100-z}$

Therefore, the required initial amount

$$= A\left(\frac{100}{100-x}\right)\left(\frac{100}{100-y}\right)\left(\frac{100}{100-z}\right)$$

Theorem : x% of a quantity is added. Again, y% of the increa quantity is added. Again z% of the increased quantity is added. No it becomes A, then the initial amount is given by

$$\frac{A \times 100 \times 100 \times 100}{(100+x)(100+y)(100+z)}$$

Proof : The initial amount must be less than the final amount 'A'. S by the rule of fraction, A should be multiplied by less-than-o fractions. Since in each step the amount is increased, (100 + (100 + y) and (100 + z) should be our dealing values apart fro 100.

Thus, the fractions (less than one) by which A is multiplied a

$\frac{100}{100+x}, \frac{100}{100+y}$, and $\frac{100}{100+z}$

Therefore, the required initial amount

$$= A\left(\frac{100}{100+x}\right)\left(\frac{100}{100+y}\right)\left(\frac{100}{100+z}\right)$$

Some more examples based on the above theorems :

Ex. 21: After deducting 10% from a certain sum, and then 20% from the remainder, there is Rs 3600 left. Find the original sum.

Soln : The original sum is naturally more than Rs 3600.

Therefore, it should be multiplied by $\frac{100}{(100-10)}$ and $\frac{100}{(100-20}$

$\therefore$ the required sum $= \frac{3600 \times 100 \times 100}{90 \times 80} = 5{,}000$

x. 22: A man had Rs 4800 in his locker two years ago. In the first year, he deposited 20% of the amount in his locker. In the second year, he deposited 25% of the increased amount in his locker. Find the amount at present in his locker.

oln : The amount is certainly more than Rs 4800. And each year, the new amount is added. So, the sum should be multiplied by

$\frac{100+20}{100}$ and $\frac{100+25}{100}$

$\therefore$ the required amount $= \frac{4800 \times 120 \times 125}{100 \times 100} =$ Rs 7200

Note : The above example is different from others. Mark it.

Population Formula

Ex 23: If the original population of a town is P, and the annual increase is r%, what will be the population in n years ?

Soln : Population after one year becomes $P + \frac{Pr}{100} = P\left(1+\frac{r}{100}\right)$

That is, the population P at the beginning of the year is multiplied by $\left(1+\frac{r}{100}\right)$ in the course of the year.

Now, the population at the beginning of the second year is $P\left(1+\frac{r}{100}\right)$

$\therefore$ the population in 2 years $= P\left(1+\frac{r}{100}\right)^2$

$\therefore$ the population in n years $= P\left(1+\frac{r}{100}\right)^n$

Note : If the annual decrease be r%, then the population in n years

$= P\left(1-\frac{r}{100}\right)^n$

Ex. 24: If the annual increase in the population of a town is 4% and the present number of people is 15,625, what will the population be in 3 years?

Soln : The required population $= 15625\left(1+\frac{4}{100}\right)^3$

$= 15625 \times \frac{26}{25} \times \frac{26}{25} \times \frac{26}{25} = 17576$

Ex. 25: If the annual increase in the population of a town be 4% and the

present population be 17576, what was it three years ago ?

Soln : The population 3 years ago $\times \left(\frac{26}{25}\right)^3$ = Present population

$\therefore$ the population 3 years ago $= \frac{17576 \times 25 \times 25 \times 25}{26 \times 26 \times 26} = 15625$

Note : Ex.24 and Ex. 25 are the best examples to look at the game of fractions. In Ex.24, the required population was definitely more, so we had put the higher value (26) in the numerator and the lower value (25) in the denominator. In Ex. 25 the opposite of this can be seen.

When the Rate of Growth is Different for Different Years

Theorem : The population of a town is P. It increases by x% during the first year, increases by y% during the second year and again increases by z% during the third year. The population after 3 years will be $\frac{P \times (100+x)(100+y)(100+z)}{100 \times 100 \times 100}$

Proof : By the rule of fraction, P should be multiplied by more-than-one fractions.

Thus, the population after 3 years is

$$P\left(\frac{100+x}{100}\right)\left(\frac{100+y}{100}\right)\left(\frac{100+z}{100}\right)$$

Note : Mark that the multiplying fraction is $\frac{100+x}{100}$ and not $\frac{100}{100-x}$

Why ?

Because the population is increased by x%, so we should deal with (100 + x) and 100. And since after one year, the population is more than P, so P should be multiplied by $\frac{100+x}{100}$.

We proceed similarly for the succeeding years.

Ex. 26: The population of a town is 8000. It increases by 10% during the first year and by 20% during the second year. What is the population after two years ?

Soln : The required population $= \frac{8000 \times 110 \times 120}{100 \times 100} = 10,560$

When Population Increases for One Year and Then Decreases for the Next Year.

Theorem: In the above theorem, when the population decreases by y% during the second year, while for the first and third years, it follows the same, the population after 3 years will be

$$\frac{P(100+x)(100-y)(100+z)}{100 \times 100 \times 100}$$

Proof : Try to prove it by yourself.

Ex. 27 : The population of a town is 10,000. It increases by 10% during the first year. During the second year, it decreases by 20% and increased by 30% during the third year. What is the population after 3 years?

Soln : The required population $= \frac{10000 \times 110 \times 80 \times 130}{100 \times 100 \times 100} = 11440$

Ex. 28 : During one year, the population of a locality increases by 5% but during the next year, it decreases by 5%. If the population at the end of the second year was 7980, find the population at the beginning of the first year.

Soln : The required population $= 7980 \times \left(\frac{100}{100-5}\right)\left(\frac{100}{100+5}\right)$

$= \frac{7980 \times 100 \times 100}{95 \times 105}$

$= 8000$

Note : In the above example, the population after two years is given and the population in the beginning of the first year is asked. That is why, the fractional values are inversed. Mark that point. The same thing happens to the next example.

Ex. 29: The population of a town increases at the rate of 10% during one year and it decreases at the rate of 10% during the second year. If it has 29,700 inhabitants at present, find the number of inhabitants two year ago.

Soln : The required population $= \frac{29700 \times 100 \times 100}{(100-10) \times (100+10)}$

$= \frac{29700 \times 100 \times 100}{90 \times 110} = 30,000$

Ex. 30: The population of a town is 8000. If the males increase by 6% and the females by 10%, the population will be 8600. Find the number of females in the town.

Soln : Let the population of females be x.

Then 110% of x + 106% of (8000 - x) = 8600

or, $\frac{110x}{100}+\frac{106(8000-x)}{100}=8600$

or, $x(110-106)=8600\times100-8000\times106$

$\therefore x=\frac{8600\times100-8000\times106}{110-106}$

$=\frac{12,000}{4}=3,000$

Note : If we ignore the intermediate steps, we can get the population of females and males directly thus:

The population of females $=\frac{8600\times100-8000(100+6)}{(10-6)}=3,000$

The population of males $=\frac{8600\times100-8000\,(100+10)}{(6-10)}$

$=\frac{20,000}{4}=5,000$

By Method of Alligation

Average % increase $=\frac{600}{8000}\times100=\frac{15}{2}=7.5\%$

Now,

$$\begin{array}{ccc} \text{Male} & & \text{Female} \\ 6\% & & 10\% \\ & 7.5\% & \\ 2.5 & & 1.5 \end{array}$$

$\therefore$ Male : Female = 2.5 : 1.5 = 5 : 3

$\therefore$ the population of females $=\frac{8000}{5+3}\times3=3000$

Reduction in Consumption

Ex. 31: If the price of a commodity be raised by 20%, find by how much per cent must a householder reduce his consumption of that commodity so as not to increase his expenditure.

Soln : I: Let the price and consumption each be 100 units.

Then, his earlier expenditure was = Rs (100 × 100)

Now, the new price = 120 units

To maintain the expenditure, suppose he reduces his consumption by x%, then his total expenditure = Rs [120 × (100 – x)]

From the question, we have,

$100\times100=120\,(100-x)$

or, $120x=120\times100-100\times100$

or, $x=\frac{100(120-100)}{120}=16\frac{2}{3}\%$

II: The raised price $=\frac{120}{100}$ of the former price

$\therefore$ The householder must now consume

$\frac{100}{120}\left(\text{i.e. the reciprocal of }\frac{120}{100}\right)$ of the original amount

$\therefore$ the reduction in consumption $=\left(1-\frac{100}{120}\right)$ of the original consumption

$=\frac{1}{6}$ of the original consumption $=16\frac{2}{3}\%$

III: Quicker Method :

Theorem : If the price of a commodity increases by r%, then the reduction in consumption so as not to increase the expenditure, is

$$\left(\frac{r}{100+r}\times100\right)\%$$

Proof : The formula can be written in the form $\left(r\times\frac{100}{100+r}\right)$. If you watch carefully, you can see the fractional value $\left(\frac{100}{100+r}\right)$ is less than 1, i.e., the numerator is less than the denominator. Why ? Because our required value, in this case, is less than the supplied value (20%). Thus, in this case also we applied the rule of fraction.

Thus, answer $=\frac{20}{100+20}\times100=\frac{50}{3}=16\frac{2}{3}\%$

Ex. 32: If the price of sugar falls down by 10%, by how much per cent must a householder increase its consumption, so as not to decrease expenditure in this item ?

Soln : This question is similar to the previous example. It can also be solved by all the three methods given above. But we will discuss only method III. Try to solve it by the other two methods also on your own.

Quicker Method

Theorem: If the price of a commodity decreases by r%, then increase in consumption, so as not to decrease expenditure on this item, is

$$\left[\frac{r}{(100-r)}\times100\right]\%$$

Proof : If we write the formula in the form $r \times \frac{100}{(100-r)}$, we see that fractional value $\left(\frac{100}{100+r}\right)$ is more than 1. Why ? (Try to de[...] it yourself.)

So, in this case,

Answer $= \frac{10}{(100-10)} \times 100 = 11\frac{1}{9}\%$

Percentage Relationship

If first value is r% more than the second value, then t[...] second is $\left[\frac{r}{100+r} \times 100\right]$% less than the first value.

Proof : By the rule of fraction, r should be multiplied by a fraction whi[...] is less than one. And that fraction should be $\frac{100}{100+r}$

By general mathematical calculation

Let the second value be 100. Then first value is $(100 + r)$.

Now, we see that when the first is $(100 + r)$, the second is mo[...] by r.

Therefore, when the first is 100 the second is $\frac{r}{100+r} \times 100$ mo[...] than the first.

$\therefore$ The second is $\left[\frac{r}{100+r} \times 100\right]$% than the first.

Theorem : If the first value is r% less than the second value then, th[...] second value is $\left(\frac{r}{100-r} \times 100\right)$% more than the first value.

Proof : Try it yourself.

Ex. 33: If A's salary is 25% more than that of B, then how much percen[...] is B's salary less than that of A ?

Soln : I: Suppose B's salary is Rs 100 per month. Then A's salary is Rs 125 per month. We see that B's salary is Rs 25 less than tha[...] of A, when A's salary is Rs 125.

Thus, when A's salary is Rs 100, B's salary is $\frac{25}{125} \times 100 =$ Rs 2[...] less than that of A i.e., B's salary is 20% less than that of A.

II: Quicker Method : If A's income is r% more than B's income, then B's income is less than A's income by $\left[\frac{r}{100+r} \times 100\right]$%

Thus, in this case, answer $= \frac{25}{100+25} \times 100\ \% = 20\%$

[...]ote : Do you get the similarity between this formula and the formula given in Ex. 26 ?

[...]x. 34: If A's salary is 30% less than that of B, then how much per cent is B's salary more than that of A ?

[...]oln : Quicker Method :

If A's salary is r% less than B's, then B's salary is more than A's salary by $\left[\frac{r}{100-r} \times 100\right]$%

Thus, in this case, answer $= \frac{30}{100-30} \times 100 = 42\frac{6}{7}\%$

[...]ote : Do you get the similarity between Ex. 32 and Ex. 34?

[...]x. 35: A number is 50% more than the other. Then how much per cent is the second number less than the first ?

[...]oln. We can apply the above discussed formula in this case also. Then the second number is $\left(\frac{50}{100+50} \times 100\right)\% = 33\frac{1}{3}\%$ less than the first.

[...]x. 36: A number is 20% less than the other; then by how much per cent is the second more than the first ?

[...]oln : Apply the formula given in Ex. 32 or Ex. 34.

One value as a percentage of another

Ex. 37: Express 20 as a percentage of 80.

Soln : If one is 80, the other is 20.

$\therefore$ if one is 100, the other is $\frac{20}{80} \times 100 = 25$

$\therefore$ 20 is 25% of 80.

Thus, without going for details, we may say that "x as a percentage of y" $= \frac{x}{y} \times 100\%$

Ex. 38: Express 250 as a percentage of 50.

Soln : Answer $= \frac{250}{50} \times 100 = 500\%$

Ex. 39: Express 160 as a percentage of 120.

Soln : Answer $= \frac{160}{120} \times 100 = \frac{400}{3} = 133\frac{1}{3}\%$

Ex. 40: Express 20 as a per thousand fraction of 200.

Soln. Answer $= \frac{20}{200} \times 1000 = 100$ per thousand.

First increase and then decrease

Ex. 41: The salary of a worker is first increased by 10% and thereaft... it was reduced by 10%. What was the change in his salary ?

Soln : Let the salary of the worker be Rs 100.

After increase, it becomes Rs 100 + 10% of 100 = Rs 110

After decrease, it becomes Rs 110 - 10% of Rs 110 = Rs 99

$\therefore$ the % reduction = 100 - 99 = 1 %

By Quicker Method :

Theorem : If the value of a number is first increased by x% and lat... decreased by x%, the net change is always a decrease which is equ... to x% of x or $\frac{x^2}{100}$.

Proof : Let the number be A.

When it is increased by x%, it becomes A + x% of A

$$= A + \frac{Ax}{100} = \frac{A(100+x)}{100}$$

Now, when the increased value is decreased by x%, then ... becomes

$$\frac{A(100+x)}{100} - x\% \text{ of } \frac{A(100+x)}{100}$$

$$= \frac{A(100+x)}{100} - \frac{Ax(100+x)}{100 \times 100}$$

$$= \frac{100A(100+x) - Ax(100+x)}{100 \times 100}$$

$$= \frac{[A(100+x)][100-x]}{100 \times 100}$$

$$= \frac{A(100+x)(100-x)}{100 \times 100} = \frac{[A(100)^2 - x^2]}{(100)^2}$$

Now, net change $= \frac{A[(100)^2 - x^2]}{(100)^2} - A = \frac{-Ax^2}{(100)^2}$

And % change $= \frac{-Ax^2 \times 100}{(100)^2 \times A} = \frac{-x^2}{100}\%$

Thus, we see that there is always decrease (because sign is -ve) and is given by $\frac{x^2}{100}\%$. Thus, in the above example,

decrease % $= \frac{(10)^2}{100} = 1\%$

...42: A shopkeeper marks the price of his goods 12% higher than its original price. After that, he allows a discount of 12%. What is his percentage profit or loss ?

...: In this case, there is always a loss. And the % value of loss

$$= \frac{(12)^2}{100} = 1.44\ \%$$

...43: If the population of a town is increased by 15% in the first year and is decreased by 15% in the next year, what effect can be seen in the population of that town ?

...ln : There is a decrease of $\frac{(15)^2}{100}\%$ i.e., 2.25%

...hen both values are different

...eorem : If the value is first increased by x% and then decreased ...y y% then there is $\left(x - y - \frac{xy}{100}\right)\%$ increase or decrease, according ... the +ve or -ve sign respectively.

...roof : Let the value be A.

When it is increased by x%, it becomes

$$A + x\% \text{ of } A = A + \frac{Ax}{100} = \frac{A(x+100)}{100}$$

Now, when the increased value is decreased by y%, it becomes

$$\frac{A(x+100)}{100} - y\% \text{ of } \frac{A(x+100)}{100}$$

$$= \frac{A(x+100)}{100} - \frac{Ay(x+100)}{(100)^2} = \frac{A(x+100)(100-y)}{(100)^2}$$

Now, net change $= \frac{A(x+100)(100-y)}{(100)^2} - A$

$$= \frac{A[(100)^2 + 100x - 100y - xy] - (100)^2 A}{(100)^2}$$

$$= \frac{A\{100x - 100y - xy\}}{(100)^2}$$

$$\% \text{ change} = \frac{A\{100x - 100y - xy\}}{(100)^2} \times \frac{100}{A} = \left[x - y - \frac{xy}{100}\right]\%$$

Theorem : If the order of increase and decrease is changed, the res remains unaffected.

Proof : Try this yourself.

So, combining the above two theorem, we may write as:

$$\text{Effect} = \% \text{ increase} - \% \text{ decrease} - \frac{\% \text{ increase} \times \% \text{ decrease}}{100}$$

The use of the above formula will clear your doubt.

Ex. 44: The salary of a worker was first increased by 10% and thereaf decreased by 5%. What was the change in his salary ?

Soln : Let the salary of the worker be Rs 100

After increase, it becomes Rs 100 + 10% of Rs 100 = Rs 110

After decrease, it becomes Rs 110 - 5% of Rs 110 = Rs 104.5

$\therefore$ The % increase = 104.5 - 100 = 4.5%

Quicker Method : By the above theorem :

If the value firstly increases by x% and then decreased by y% t

there is $\left(x - y - \frac{xy}{100}\right)$% increase or decrease, according to sign +ve or -ve respectively.

Thus, in this case, $10 - 5 - \frac{10 \times 5}{100} = 4.5\%$ increase as the sign is +

Ex. 45: A shopkeeper marks the prices of his goods at 20% higher th the original price. After that, he allows a discount of 10%. Wh profit or loss did he get ?

Soln : By the theorem : $20 - 10 - \frac{20 \times 10}{100} = 8\%$

$\therefore$ he gets 8% profit as the sign obtained is +ve.

Ex. 46: If the salary of a worker is first decreased by 20% and then increased by 10%. What is the percentage effect on his salary

Soln : By Quicker Maths:

$$\% \text{ effect} = \% \text{ Increase} - \% \text{ decrease} - \frac{\% \text{ increase} \times \% \text{ decrease}}{100}$$

$$= 10 - 20 - \frac{10 \times 20}{100} = -12\%$$

$\therefore$ His salary is decreased by 12% (because the sign is -ve).

te : Change of order of increase and decrease means that in the above example, firstly an increase of 10% is performed and then the decrease of 20% is performed. In both the cases, the result remains the same.

. 47: The population of a town was reduced by 12% in the year 1988. In 1989, it was increased by 15%. What is the percentage effect on the population in the beginning of 1990 ?

ln : $\% \text{ effect} = \% \text{ increase} - \% \text{ decrease} - \frac{\% \text{ increase} \times \% \text{ decrease}}{100}$

$$= 15 - 12 - \frac{15 \times 12}{100} = 3 - 1.8 = 1.2$$

Thus, the population was increased by 1.2%.

ccessive increase or decrease

eorem : If the value is increased successively by x% and y% then final increase is given by $\left[x + y + \frac{xy}{100}\right]\%$

oof : If we put -y in place of y in the previous theorem, we get the required result. This is done because we may say that Increase = - Decrease.

. 48: A shopkeeper marks the prices at 15% higher than the original price. Due to increase in demand, he further increases the price by 10%. How much % profit will he get ?

ln : By above theorem : $\% \text{ profit} = 15 + 10 + \frac{15 \times 10}{100} = 26.5\%$

. 49: The population of a town is decreased by 10% and 20% in two successive years. What percent population is decreased after two years ?

ln : Put x = -10 and y = -20 then, $-10 - 20 + \frac{(-10)(-20)}{100} = -28\%$

Therefore, the population decreases by 28%.

ffect on revenue

eorem: (i) If the price of a commodity is diminished by x% and its consumption is increased by y%,

(ii) or, if the price of a commodity is increased by x% and its consumption is decreased by y% then the effect on revenue

$= \text{Inc. \% value} - \text{Dec. \% value} - \frac{\text{Inc. \% value} \times \text{Dec. \% value}}{100}$

the value is increased or decreased according to the +ve or sign obtained.

Note : The above written formula is the general form for both the ca

For Case (i) it becomes: $y - x - \frac{yx}{100}$

Whereas for Case (ii) it becomes : $x - y - \frac{xy}{100}$

Thus, we see that it more easy to remember the general form which works in both the cases equally.

Proof : We are discussing the proof for Case (i).

Let the price of the commodity be Rs A/unit and the consump be B units.

Then total revenue expenses = Rs AB

Now, the new price = A - x% of A $= A - \frac{Ax}{100} = \frac{A(100-x)}{100}$

And new consumption = B + y% of B $= B + \frac{By}{100} = \frac{B(100+}{100}$

$\therefore$ the new revenue expenses $= \frac{A(100-x)}{100} \times \frac{B(100+y)}{100}$

$= \frac{AB(100-x)(100+y)}{100 \times 100}$

Change in revenue expenses $= \frac{AB(100-x)(100+y)}{100 \times 100} - AB$

$= \frac{AB\{100^2 + 100y - 100x - xy\} - 100^2 AB}{100^2}$

$= \frac{AB\{100y - 100x - xy\}}{100^2}$

$\therefore$ the % effect on revenue $= \frac{AB\{100y - 100x - xy\}}{100^2} \times \frac{100}{AB}$

$= y - x - \frac{xy}{100}$

Similarly, we can prove for Case (ii) also.

Ex. 50: The tax on a commodity is diminished by 20% and its consumption increases by 15%. Find the effect on revenue.

Soln : New Revenue = Consumption × Tax

= (115% × 80%) of the original

$= \left(\frac{115}{100} \times \frac{80}{100}\right)$ of the original

$= \left(\frac{115}{100} \times 80\right)$ % of original = 92% of original

Thus, the revenue is decreased by (100 - 92) = 8%

By Theorem:

Effect on revenue

$= \text{Inc. \% value} - \text{Dec. \% value} - \frac{\text{Inc. \%value} \times \text{Dec. \%value}}{100}$

$= 15 - 20 - \frac{15 \times 20}{100} = -8\%$

Therefore, there is a decrease of 8%.

Ex. 51: If the price is increased by 10% and the sale is decreased by 5%, then what will be the effect on income ?

Soln : Let the price be Rs 100 per good and the sale is also of 100 goods.

So, the money obtained after selling all the 100 goods

= 100 × 100 = 10,000

Now, the increased price is Rs 110 per good and the decreased sale is 95 goods.

So, the money obtained after selling all the 95 goods = 110 × 95 = Rs 10,450.

$\therefore$ profit = 10,450 - 10,000 = Rs 450

$\therefore$ % profit $= \frac{450 \times 100}{10000} = 4.5\%$

By Theorem : % effect

$= \text{Inc. \% value} - \text{Dec. \% value} - \frac{\text{Inc. \%value} \times \text{Dec. \%value}}{100}$

$= 10 - 5 - \frac{10 \times 5}{100} = 4.5\%$

$\therefore$ his income increases by 4.5%.

Ex. 52: If the price is decreased by 12% and sale in increased by 10% then what will be the effect on income ?

Soln : By Theorem :

% effect $= 10 - 12 - \frac{12 \times 10}{100} = -3.2\%$

$\therefore$ his income is decreased by 3.2%.

Ex. 53 : The landholding of a person is decreased by 10%. Due to late

monsoon, the production decreases by 8%. Then what is the effect on revenue ?

Soln : Change any of the decreased value in the form of an increase.

Suppose we change as

10% decrease = -10% increase

Now, putting the values in the above formula, we have

% effect = $-10 - 8 - \frac{(-10) \times 8}{100}$

$= -18 + 0.8 = -17.2\%$

Therefore, his revenue is reduced by 17.2%.

Now, suppose we change this as follow

8% decrease = -8% increase

Now, putting the values in the above formula, we have % effect

$= -8 - 10 - \frac{(-8) \times 10}{100}$

$= -18 + 0.8 = -17.2\%$

Thus, we get the same result in both the cases. So, it hardly matters which of the values changes its form.

Ex. 54: The number of seats in a cinema hall is increased by 25%. The price on a ticket is also increased by 10%. What is the effect on the revenue collected ?

Soln: Since there is an increase in the seats as well as in the price, we use:

Decrease = -(Increase)

Thus, the formula becomes :

% effect = $25 - (-10) - \frac{25 \times (-10)}{100} = 35 + 2.5 = 37.5$

Thus, there is an increase of 37.5% in the revenue.

This example can also be solved by changing the form of any increased value to decreased value. Try it yourself by changing the other value.

Note : We see that, $x+y+\frac{xy}{100}$ is our key-formula. All of the above forms can be obtained by only changing the signs. For example:

i) Increase of x% and increase of y%

$= \left(x + y + \frac{xy}{100}\right)$ % increase or decrease according to its sign.

ii) Increase of x% and decrease of y% (put y = -y)

$= \left(x - y - \frac{xy}{100}\right)$ % increase or decrease according to its sign.

iii) Decrease of x% and increase of y% (Put x = -x)

$= \left(-x + y - \frac{xy}{100}\right)$% increase or decrease according to its sign.

iv) Decrease of x% and decrease of y% (Put x = -x and y = -y)

$= \left(-x - y + \frac{xy}{100}\right)$% increase or decrease according to its sign .

Theorem : The pass marks in an examination is x%. If a candidate who secures y marks fails by z marks, then the maximum marks, M $= \frac{100(y+z)}{x}$

Proof: It is very much easy to prove the above theorem. Let the maximum marks be M, then there exists a relation:

x% of M = y + z

or, $M = \frac{y+z}{x\%} = \frac{100\,(y+z)}{x}$

Note : If you have understood the relationship, you don't need to remember the formula. But some of the students get confused in finding the relationship in the examination hall, so we have given the direct formula.

Ex. 55: A student has to secure 40% marks to get through. If he gets 40 marks and fails by 40 marks, find the maximum marks set for the examination.

Soln : By the above theorem, Maximum marks $= \frac{100(40+40)}{40} = 200$

Ex. 56: In an examination, a candidate must get 80% marks to pass. If a andidate who gets 210 marks, fails by 50 marks, find the maximum marks.

Soln : By the above theorem :

Maximum marks $= \frac{100(210+50)}{80} = 325$

Theorem : A candidate scoring x% in an examination fails by 'a' marks, while another candidate who scores y% marks gets 'b' marks more than the minimum required pass marks. Then the maximum marks for that examination are $M = \frac{100(a+b)}{y-x}$

Proof : Let the maximum marks for the examination be M. Thus, marks scored by the first candidate = x% of M

and marks scored by the second candidate = y% of M.
Now, pass marks for both the candidates are equal; so,
x% of M + a = y% of M - b

or, $\frac{Mx}{100} + a = \frac{My}{100} - b$

or, $\frac{M(y-x)}{100} = a + b \quad \therefore M = \frac{100(a+b)}{(y-x)}$

Ex. 57: A candidate scores 25% and fails by 30 marks, while another candidate who scores 50% marks, gets 20 marks more than the minimum required marks to pass the examination. Find the maximum marks for the examination.

Soln : By the theorem :

$$\text{Maximum marks} = \frac{100(30+20)}{50-25} = 200$$

Note : (i) The above formula can be written as

$$\text{Maximum marks} = \frac{100(\text{Diff. of their scores})}{\text{Diff. of their \% marks}}$$

(ii) Difference of their scores = 30 + 20. Because the first candidate gets 30 less than the required pass marks, while the second candidate gets 20 more than the required pass marks.

Ex. 58: A candidate who gets 30% of the marks in a test fails by 50 marks. Another candidate who get 320 marks fails by 30 marks. Find the maximum marks.

Soln : We can't use the above said direct formula in this case. (Why ?) So, we use the fact that pass marks for both the candidates are the same.
If x is the maximum marks, then the
pass marks for the first candidate = 30% of x + 50
and the pass marks for the second candidate = 320 + 30
Therefore, 30% of x + 50 = 320 + 30

or, $\frac{3x}{10} = 300 \quad \therefore x = \frac{300 \times 10}{3} = 1{,}000$

Note : The above method should be understood well, because it works when the form of question is changed.

Theorem : In measuring the sides of a rectangle, one side is taken x% in excess and the other y% in deficit. The error per cent in area calculated from the measurement is $x - y - \frac{xy}{100}$ in excess or deficit, according to the +ve or -ve sign.

Proof : You must be familiar with the above formula. The proof for this is being given below.
Let the sides of the rectangle be a and b.
$\therefore$ area = ab
New sides are : a + x% of a and b - y% of b

or, $\frac{a(100+x)}{100}$ and $\frac{b(100-y)}{100}$

$\therefore$ new area $= \frac{ab(100+x)(100-y)}{(100)^2}$

$$\text{Error} = \frac{ab(100+x)(100-y)}{(100)^2} - ab = \frac{ab[100x - 100y - xy]}{(100)^2}$$

$$\%\text{ error} = \frac{ab[100x - 100y - xy]}{(100)^2} \times \frac{100}{ab} = x - y - \frac{xy}{100}$$

or, % error = % Excess - % Deficit - $\frac{\%\text{ Excess} \times \%\text{ Deficit}}{100}$

Ex. 59: In measuring the sides of a rectangle, one side is taken 5% in excess and the other 4% in deficit. Find the error per cent in area calculated from the measurement.

Soln : By the above theorem :

$\%\text{ error} = 5 - 4 - \frac{5 \times 4}{100} = 1 - \frac{1}{5} = \frac{4}{5}\%$ excess because sign is +ve

Ex. 60: If one of the sides of a rectangle is increased by 20% and the other is increased by 5%, find the per cent value by which the area changes.

Soln : The above theorem will also work in this case. The only change required is to change the form of one of increased % value to the decreased % value, ie.,
5% increase = -5% decrease

Now, % effect = 20 - (-5) - $\frac{20 \times (-5)}{100}$ = 20 + 5 + 1 = 26%

Since the sign is +ve, there is 26% increase in area.

Note : We can also use
20% increase = -20 decrease.
Try to solve by using the above change.

Ex. 61: If the sides of a square are increased by 30%, find the per cent increase in its area.

Soln : We can use the same method that has been used in the previous example.

$$\% \text{ increase in area} = 30 - (-30) - \frac{30 \times (-30)}{100} = 60 + 9 = 69\%$$

Theorem : If the sides of a triangle, rectangle, square, circle, rhombus (or any 2-dimentional figure) are increased by x%, its area is increased by $\frac{x(x+200)}{100}\%$ **or** $\left[2x + \frac{x^2}{100}\right]\%$

Soln : For a triangle:

Suppose a triangle has three sides a, b and c.

Then its area $A = \sqrt{s(s-a)(s-b)(s-c)}$

where, $s = \frac{a+b+c}{2}$

Now, when all the three sides are increased by x%, the sides become :

$$\frac{a(100+x)}{100}, \frac{b(100+x)}{100}, \frac{c(100+x)}{100}$$

$$\text{Now, } s_1 = \frac{(100+x)}{100}\left[\frac{a+b+c}{2}\right] = \frac{(100+x)}{100} s$$

$\therefore$ New area, A_1 =

$$\sqrt{s_1\left(s_1 - \frac{a(100+x)}{100}\right)\left(s_1 - \frac{b(100+x)}{100}\right)\left(s_1 - \frac{c(100+x)}{100}\right)}$$

$$= \sqrt{\left(\frac{100+x}{100}\right)^4 s(s-a)(s-b)(s-c)}$$

$$\therefore \quad A_1 = \left(\frac{100+x}{100}\right)^2 A$$

Now, % increase in area

$$= \frac{A_1 - A}{A} \times 100 = \frac{\left[\left(\frac{100+x}{100}\right)^2 - 1\right]A}{A} \times 100$$

$$= \left[\left(\frac{100+x}{100}\right)^2 - 1\right] \times 100 = \left[\left(1 + \frac{x}{100}\right)^2 - 1\right] \times 100$$

$$= \left[1 + \frac{x^2}{(100)^2} + \frac{2x}{100} - 1\right] 100 = \frac{x(x+200)}{100} = 2x + \frac{x^2}{100}$$

Therefore, we may say that if all the sides of a triangle are increased by x%, then its area increases by $\frac{x(x+200)}{100}\%$ **or** $2x + \frac{x^2}{100}$.

For a Rectangle : Let the sides of a rectangle be a and b.

Then its area; $A = ab$

Now, suppose, its sides become :

$$\frac{a(100+x)}{100} \text{ and } \frac{b(100+x)}{100}$$

$$\text{And its new area; } A_1 = ab\left(\frac{100+x}{100}\right)^2 = \left(\frac{100+x}{100}\right)^2 A$$

$$\therefore \% \text{ increase in area} = \frac{A_1 - A}{A} \times 100$$

$$= \frac{\left[\left(\frac{100+x}{100}\right)^2 - 1\right]A}{A} \times 100$$

$$= \left[\left(\frac{100+x}{100}\right)^2 - 1\right] \times 100 = \frac{x(x+200)}{100} = 2x + \frac{x^2}{100}$$

Therefore, we may say that if sides of a rectangle are increased by x%, then its area is increased by $\frac{x(x+200)}{100}\%$ **or** $\left(2x + \frac{x^2}{100}\right)\%$**. And this is the same as for the triangle.**

Note : Now, we don't need to calculate for square and rhombus. That will give the same result.

For a Rectangle:

Let us have a circle with radius r metres

So, its area = $A = \pi r^2$

When its radius is increased by x%, it becomes : $\frac{r(100+x)}{100}$

$$\text{So, its new area} = A_1 = \pi\left[\frac{r(100+x)}{100}\right]^2$$

$$= \pi r^2\left[\frac{(100+x)}{100}\right]^2 = \left(\frac{100+x}{100}\right)^2 A$$

$\therefore$ % increase in area $= \dfrac{A_1 - A}{A} \times 100$

$$= \left[\left(\frac{100+x}{100}\right)^2 - 1\right] 100 = \frac{x(x+200)}{100}$$

$$= 2x + \frac{x^2}{100}$$

Therefore, we see that it gives the same result for a circle also.

Final conclusion : Now we conclude that this general formula is applicable for any 2-dimensional figure.

Take an example : If the sides of a rectangle are increased by 10%, what is the percentage increase in its area ?

Soln : The required answer $= 2 \times 10 + \dfrac{(10)^2}{100} = 20 + 1 = 21\%$

Note : (1) What will be the effect on the area when the sides are decreased by x% ? (Try this yourself.)

(2) What is the effect on the volume of a three-dimensional object when its sides are increased by x% ? (Try it yourself.)

Theorem : In an examination, x% failed in English and y% failed in Maths. If z% of students failed in both the subjects, the percentage of students who passed in both the subjects is 100 - (x + y + z)

or, (100 - x) + (100 - y) + z

Proof : Percentage of students who failed in English only = (x - z)%

Percentage of students who failed in Maths only = (y - z) %

Percentage of students who failed in both the subjects = z% (given)

$\therefore$ the percentage of students who passed in both the subjects

$= 100 - [(x - z) + (y - z) - z]$

$= 100 - (x + y - z)$

Ex. 62: In an examination, 40% of the students failed in Maths, 30% failed in English and 10% failed in both. Find the percentage of students who passed in both the subjects.

Soln : Following the above theorem :

The required % = 100 - (40 + 30 - 10) = 40%

Note : We should know that the following sets complete the system, i.e.,

100% = % of students who failed in English only

\+ % of students who failed failed in Maths only

\- % of students who failed in both subjects

\+ % of students who passed in both subjects

Ex. 63: A man spends 75% of his income. His income increases by 20% and his expenditure also increses by 10%. Find the percentage increase in his savings.

Soln: Detailed Method: Suppose his monthly income = Rs 100

Thus, he spends Rs 75 and saves Rs 25.

His increased income = 100 + 20% of 100 = Rs 120

His increased expenditure = 75 + 10% of 75 = Rs 82.50

$\therefore$ his new savings = 120 - 82.5 = Rs 37.50

$\therefore$ % increase in his savings $= \dfrac{37.50 - 25}{25} \times 100 = 50\%$

Quicker Method (Direct Formula):

Percentage increase in savings

$$= \frac{20 \times 100 - 10 \times 75}{100 - 75} = \frac{1250}{25} = 50\%$$

Ex. 64: A solution of salt and water contains 15% salt by weight. Of it 30kg water evaporates and the solution now contains 20% of salt. Find the original quantity of salt.

Soln: Suppose there was x kg of solution initially.

The quantity of salt = 15% of $x = \dfrac{15x}{100} = \dfrac{3x}{20}$kg

Now, after evaporation, only (x - 30) kg of mixture contains $\dfrac{3x}{20}$ kg of salt.

or, 20% of $(x - 30) = \dfrac{3x}{20}$; or, $\dfrac{x-30}{5} = \dfrac{3x}{20}$

or, $15x = 20x - 600$; $\therefore x = \dfrac{600}{5} = 120$ kg

Quicker Method (Direct Formula):

Original quantity of solution

= Quantity of evaporated water $\left(\dfrac{\text{Final \% of salt}}{\text{\% Diff. of salt}}\right)$

$= 30\left(\dfrac{20}{20-15}\right) = 120$ kg

Ex. 65: In a library, 20% of the books are in Hindi, 50% of the remaining are in English and 30% of the remaining are in French. The remaining 6300 books are in regional languages. What is the total number of books in the library?

Soln: Suppose there are x books in the library.

Then, the no. of books in Hindi = 20% of $x = \frac{x}{5}$

50% of the remaining, i.e., 50% of $\left(x - \frac{x}{5}\right) = 50\%$ of $\frac{4x}{5}$

$= \frac{2x}{5}$ are in English.

Now, 30% of the remaining, i.e., 30% of $\left\{x - \left(\frac{x}{5} + \frac{2x}{5}\right)\right\}$

$= 30\%$ of $\frac{2x}{5} = \frac{3x}{25}$ books are in French.

Now, $x - \left(\frac{x}{5} + \frac{2x}{5} + \frac{3x}{25}\right) = 6300$

or, $\frac{7x}{25} = 6300 \quad \therefore x = \frac{6300 \times 25}{7} = 22500$

Quicker Method (Direct Formula):

No. of total books

$= 6300\left(\frac{100}{100-20}\right)\left(\frac{100}{100-50}\right)\left(\frac{100}{100-30}\right)$

$= \frac{6300 \times 100 \times 100 \times 100}{80 \times 50 \times 70} = 22{,}500$

Ex. 66: 40 litres of a mixture of milk and water contain 10% of water. How much water must be added to make the water 20% in the new mixture ?

Soln: Quantity of water = 10% of 40 = 4 litres.

Now, suppose x litres of water are added, then

$4 + x = 20\%$ of $(40 + x)$

or, $4 + x = \frac{40 + x}{5}$ or, $20 + 5x = 40 + x$ $\therefore x = \frac{20}{4} = 5$ litres.

Quicker Method (Direct Formula):

The quantity of water to be added $= \frac{40\,(20 - 10)}{(100 - 20)} = 5$ litres.

Ex. 67: The manufacturer of an article makes a profit of 25%, the wholesale dealer makes a profit of 20%, and the retailer makes a profit of 28%. Find the manufacturing price of the article if the retailer sold it for Rs 48.

Soln : By the rule of fraction:

Cost of manufacturing $= 48\left(\frac{100}{100+28}\right)\left(\frac{100}{100+20}\right)\left(\frac{100}{100+25}\right)$

$= 48\left(\frac{100}{128}\right)\left(\frac{100}{120}\right)\left(\frac{100}{125}\right) =$ Rs 25

[...]x. 68: What quantity of water should be added to reduce 9 litres of 50% acidic liquid to 30% acidic liquid?

[...]oln: Detailed Method:

Acid in 9 litres = 50% of 9 = 4.5 litres.

Suppose x litres of water are added. Then, there are 4.5 litres of acid in (9 + x) litres of diluted liquid.

Now, according to the question,

30% of (9 + x) = 4.5

or, $\frac{3}{10}(9 + x) = 4.5$

or, $27 + 3x = 45$ or, $3x = 18$

$\therefore x = \frac{18}{3} = 6$ litres.

Quicker Method: The quantity of water to be added

$= \frac{9\,(50 - 30)}{30} = 6$ litres.

Ex. 69: In an examination the percentage of students qualified to the number of students appeared from school 'A' is 70%. In school 'B' the number of students appeared is 20% more than the students appeared from school 'A' and the number of students qualified from school 'B' is 50% more than the students qualified from school 'A'. What is the percentage of students qualified to the number of students appeared from school 'B'?

1) 30% 2) 70% 3) 87.5%
4) 78.5% 5) None of these

Soln: Detailed method:

Suppose 100 students appeared from school A. Then we have

	Appeared	Passed
A →	100	70
B →	120	70 + 50% of 70 = 105

Required % $= \frac{105}{120} \times 100 = 87.5\%$

Direct Formula (Quicker Method):

$$\text{Required \%} = \frac{70 \times (100 + 50)\%}{100 \times (100 + 20)\%} \times 100 = \frac{70 \times 150}{100 \times 120} \times 100$$
$$= 87.5\%$$

Ex. 70: In 1 kg mixture of sand and iron, 20% is iron. How much sa should be added so that the proportion of iron becomes 10%?

1) 1 kg 2) 200 gms 3) 800 gms 4) 1.8 kg 5) None of these

Soln: In 1 kg mixture, iron = 20% of 1000 gm = 200 gm

and sand = 800 gm

Suppose x gm send is added to the mixture

Then, total mixture = (1000 + x) gm

Now, % of iron $= \frac{200}{(1000 + x)} \times 100 = 10$ (given)

or, $1000 + x = 2000$

$\therefore x = 1000$ gm = 1 kg

Quicker Method: When a certain quantity of goods B is added to chang the percentage of goods A in a mixture of A and B then the quantity o B to be added is

$$\left[\frac{\text{Previous \% value of A}}{\text{Changed \% value of A}} \times \text{Mixture quantity}\right] - \text{Mixture Quantit}$$

$\therefore$ In above question, required quantity of sand to be added

$$= \frac{20}{10} \times 1 - 1 = 2 - 1 = 1 \text{ kg}$$

Ex. 71: Weights of two friends Ram and Shyam are in the ratio of 4 : 5 Ram's weight increases by 10% and the total weight of Ram and Shyam together becomes 82.8 kg, with an increase of 15%. By what per cent did the weight of Shyam increase?

1) 12.5% 2) 17.5% 3) 19% 4) 21% 5) None of these

Soln: Let the weights of Ram and Shyam be 4x and 5x.

Now, accoding to question,

$$\frac{4x \times 110}{100} + \text{Shyam's new wt} = 82.8 \ ... \text{(i)}$$

and $\frac{(4x + 5x =)\ 9x \times 115}{100} = 82.8$ ... (ii)

From (ii), x = 8

Putting in (i), we get

Shyam's new wt = (82.8 - 35.2 =) 47.6

% increase in Shyam's wt $= \left(\frac{47.6 - 40}{40} \times 100\right) = 19\%$

uicker Method: If Shyam's weight increase by x% then there exist a relationship

$4(100 + 10) + 5(100 + x) = (4 + 5)[100 + 15]$ .... (*)

or, $440 + 5(100 + x) = 1035$

or, $100 + x = 119$ $\therefore x = 19$

OR

By method of Alligation:

Ram Shyam

10% x%

15%

4 5 (given)

By the rule of alligation

$$\frac{x - 15}{15 - 10} = \frac{4}{5}$$

or, $x - 15 = 4$ $\therefore x = 19$

Average

An average, or more accurately an arithmetic mean is, in crude terms, the sum of n different data divided by n.

For example, if a batsman scores 35, 45 and 37 runs in first, second and third innings respectively, then his average runs in 3 innings is equal to $\frac{35+45+37}{3} = 39$ runs.

Therefore, the two mostly used formula in this chapter are:

$$Average = \frac{Total\ of\ data}{No.\ of\ data}$$

And, $Total = Average \times No.\ of\ data$

Ex. 1 : The average age of 30 boys of a class is equal to 14 yrs. When the age of the class teacher is included the average becomes 15 yrs. Find the age of the class teacher.

Soln : Total ages of 30 boys $= 14 \times 30 = 420$ yrs.

Total ages when class teacher is included

$= 15 \times 31 = 465$ yrs

$\therefore$ Age of class teacher $= 465 - 420 = 45$ yrs.

Direct Formula :

Age of new entrant = New average + No. of old members × Increase in average $= 15 + 30(15 - 14) = 45$ yrs

Ex. 2 : The average weight of 4 men is increased by 3 kg when one of them who weighs 120 kg is replaced by another man. What is the weight of the new man?

Soln : Quicker Approach : If the average is increased by 3 kg, then the sum of weights increases by $3 \times 4 = 12$ kg.

And this increase in weight is due to the extra weight included due to the inclusion of new person.

$\therefore$ Weight of new man $= 120 + 12 = 132$ kg

Direct Formula : Weight of new person = weight of removed person + No. of persons × increase in average $= 120 + 4 \times 3 = 132$ kg

Ex. 3 : The average of marks obtained by 120 candidates in a certain examination is 35. If the average marks of passed candidates is 39 and that of the failed candidates is 15, what is the number of candidates who passed the examination?

Soln: Let the number of passed condidates be x.

Then total marks = $120 \times 35 = 39x + (120 - x) \times 15$
or, $4200 = 39x + 1800 - 15x$ or, $24x = 2400$
$\therefore x = 100$
$\therefore$ number of passed candidates = 100.

Direct Formula :

Number of passed candidates

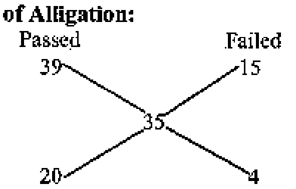
$$= \frac{\textit{Total candidates (Total average - Failed average)}}{\textit{Passed average - Failed average}}$$

and Number of failed candidates

$$= \frac{\textit{Total candidates (Passed average - Total average}}{\textit{Passed average - Failed average}}$$

$\therefore$ No. of passed candidates $= \frac{120(35 - 15)}{39 - 15} = 100$

By Method of Alligation:



$\therefore$ No. of passed candidates : No. of failed candidates = 20:4 = 5:1

$\therefore$ No. of passed candidates $= \frac{120}{5+1} \times 5 = 100$

Ex. 4 : The average of 11 results is 50. If the average of first six resu[lts] is 49 and that of last six is 52, find the sixth result.

Soln : The total of 11 results = $11 \times 50 = 550$
The total of first 6 results = $6 \times 49 = 294$
The total of last 6 results = $6 \times 52 = 312$
The 6th result is common to both;
$\therefore$ Sixth result = $294 + 312 - 550 = 56$

Direct Formula :
6th result = $50 + 6\{(52 - 50) + (49 - 50)\} = 50 + 6\{2 - 1\} = 5$6

Ex. 5 : The average age of 8 persons in a committee is increased by 2 years when two men aged 35 yrs and 45 yrs are substituted b[y] two women. Find the average age of these two women.

Soln : By the direct formula used in Ex. 2,
the total age of two women = $2 \times 8 + (35 + 45)$
$= 16 + 80 = 96$ yrs.

$\therefore$ average age of two women $= \frac{96}{2} = 48$ yrs.

Ex. 6 : The average age of a family of 6 members is 22 yrs. If the age of the youngest member be 7 yrs, then what was the average age of the family at the birth of the youngest member?

Soln : Total ages of all members = $6 \times 22 = 132$ yrs.
7 yrs ago, total sum of ages = $132 - (6 \times 7) = 90$ yrs.
But at that time there were 5 members in the family.
$\therefore$ Average at that time = $90 \div 5 = 18$ yrs.

Ex. 7 : A man bought 13 shirts of Rs 50 each, 15 pants of Rs 60 each and 12 pairs of shoes at Rs 65 a pair. Find the average value of each article.

Soln : Direct Method :

Average $= \frac{13 \times 50 + 15 \times 60 + 12 \times 65}{13 + 15 + 12} =$ Rs 58.25

Ex. 8 : The average score of a cricketer in two matches is 27 and in three other matches is 32. Then find the average score in all the five matches.

Soln : Direct Method :

Average in 5 matches $= \frac{2 \times 27 + 3 \times 32}{2 + 3} = \frac{54 + 96}{5} = 30$

Ex. 9 : The average of 11 results is 30, that of the first five is 25 and that of the last five is 28. Find the value of the 6th number.

Soln : Direct Formula :
6th number = Total of 11 results - (Total of first five + Total of last five results)
$= 11 \times 30 - (5 \times 25 + 5 \times 28)$
$= 330 - 265 = 65$

Note : Ex 4 and Ex 9 are different. In Ex 4 the 6th result is common to both the groups but in Ex 9 the 6th result is excluded in both the results.

Ex. 10 : In a class, there are 20 boys whose average age is decreased by 2 months, when one boy aged 18 years is replaced by a new boy. Find the age of the new boy.

Soln : This example is similar to Ex. 2. The only difference is that in Ex 2 the average increases after replacement whereas in this case the average decreases. Thus you can see the difference in direct formula.

Direct Formula :

Age of new person = Age of removed person - No. of persons × Decrease in average.age

$= 18 - 20 \times \frac{2}{12}$

$= 18 - \frac{10}{3} = \frac{44}{3} = 14\frac{2}{3}$ yrs = 14 yrs 8 months.

Ex. 11 : A batsman in his 17th innings makes a score of 85, and thereby increases his average by 3. What is his average after 17 innings?

Soln : Let the average after 16th innings be x, then $16x + 85$
$= 17(x + 3) =$ Total score after 17th innings.
$\therefore x = 85 - 51 = 34$
$\therefore$ average after 17 innings $= x + 3 = 34 + 3 = 37$.

Direct Formula : Average after 16 innings = 85 - 3 × 17 = 34
Average after 17 innings = 85 - 3(17 - 1) = 37

Ex. 12 : A cricketer has completed 10 innings and his average is 21.5 runs. How many runs must he make in his next innings so as to raise his average to 24?

Soln : Total of 10 innings = 21.5 × 10 = 215
Suppose he needs a score of x in 11th innings; then average in 11 innings $= \frac{215 + x}{11} = 24$
or, $x = 264 - 215 = 49$

Direct Formula : Required score = 11 × 24 - 21.5 × 10 = 49

Note : The above formula is based on the theory that the difference is counted due to the score in last innings.

Average related to speed

Theorem : *If a person travels a distance at a speed of x km/hr and the same distance at a speed of y km/hr, then the average speed during the whole journey is given by* $\frac{2xy}{x+y}$ *km/hr.*

or,

If half of the journey is travelled at a speed of x km/hr and the next half at a speed of y km/hr, then average speed during the whole journey is $\frac{2xy}{x+y}$ *km/hr*

or,

If a man goes to a certain place at a speed of x km/hr and returns to the original place at a speed of y km/hr, then the average speed during up-and-down journey is $\frac{2xy}{x+y}$ *km/hr.*

Note : In all the above three cases the two parts of the journey are equal; hence the last two may be considered as a special case of the first. That's why all the three lead to the same result.

Proof : Proof for this is given in "Time and Distance" Chapter.

Theorem : *If a person travels three equal distances at a speed of x km/hr, y km/hr and z km/hr respectively, then the average speed during the whole journey is* $\frac{3xyz}{xy + yz + xz}$ *km/hr.*

Proof : Let the three equal distances be A km.

Time taken at the speed of x km/hr $= \frac{A}{x}$ hrs.

Time taken at the speed of y km/hr $= \frac{A}{y}$ hrs.

Time taken at the speed of z km/hr $= \frac{A}{z}$ hrs.

Total distance travelled in time $\frac{A}{x} + \frac{A}{y} + \frac{A}{z} = 3A$ km

$\therefore$ Average speed during the whole journey

$$= \frac{3A}{\frac{A}{x} + \frac{A}{y} + \frac{A}{z}} = \frac{3xyzA}{Ayz + Axz + Axy} = \frac{3xyz}{xy + yz + xz} \text{ km/hr}$$

Ex. 13 : A train travels from A to B at the rate of 20 km per hour and from B to A at the rate of 30 km/hr. What is the average rate for the whole journey?

Soln : By the formula: Average speed $= \frac{2 \times 20 \times 30}{20 + 30} = 24$ km/hr.

Ex. 14 : A person divides his total route of journey into three equal parts and decides to travel the three parts with speeds of 40, 30 and 15 km/hr respectively. Find his average speed during the whole journey.

Soln: By the theorem: Average speed $= \frac{3 \times 40 \times 30 \times 15}{40 \times 30 + 30 \times 15 + 40 \times 15}$

$= \frac{3 \times 40 \times 30 \times 15}{2250} = 24$ km/hr.

Ex. 15 : One-third of a certain journey is covered at the rate of 25 km/h one-fourth at the rate of 30 km/hr and the rest at 50 km/hr. Fi the average speed for the whole journey.

Soln : Let the total journey be x km. Then $\frac{x}{3}$ km at the speed of 25 km/

and $\frac{x}{4}$ km at 30 km/hr and the rest distance $\left(x - \frac{x}{3} - \frac{x}{4} = \right)\frac{5}{12}$

at the speed of 50 km/hr.

Total time taken during the journey of x km

$$= \frac{x}{3 \times 25} \text{ hrs} + \frac{x}{4 \times 30} \text{ hrs} + \frac{5x}{12 \times 50} \text{ hrs} = \frac{18x}{600} \text{ hrs} = \frac{3x}{100} \text{ hr}$$

$$\therefore \text{ average speed} = \frac{x}{\frac{18x}{600}} = \frac{100}{3} = 33\frac{1}{3} \text{ km/hr.}$$

Note : In this example the three parts of the journey are not equal; so we didn't apply the theorem.

Ex. 16: The average salary of the entire staff in a office is Rs 120 p month. The average salary of officers is Rs 460 and that o non-officers is Rs 110. If the number of officers is 15, then fin the number of non-officers in the office.

Soln: Let the required number of non-officers = x

Then, $110x + 460 \times 15 = 120(15 + x)$

or, $120x - 110x = 460 \times 15 - 120 \times 15 = 15(460 - 120)$

or, $10x = 15 \times 340$; $\therefore x = 15 \times 34 = 510$

Quicker Method: (Method of alligation):

Officers Non-officers

460 110

120

10 340

Therefore ratio of officers to non-officers = 10 : 340 = 1 : 34.

$\therefore$ number of non-officers $= 15 \times \frac{34}{1} = 510$

OR, Direct Formula: No. of non-officers

$$= \text{No. of officers} \times \left(\frac{\text{Av. salary of officers} - \text{Mean average}}{\text{Mean average} - \text{Av. salary of non-officers}}\right)$$

$$= 15\left(\frac{460 - 120}{120 - 110}\right) = 510.$$

Ex. 17: There were 35 students in a hostel. If the number of students increases by 7, the expenses of the mess increase by Rs 42 per day while the average expenditure per head diminishes by Re 1. Find the original expenditure of the mess.

Soln : Suppose the average expenditure was Rs x. Then total expenditure = 35x.

When 7 more students join the mess, total expenditure = 35x + 42

Now, the average expenditure $= \frac{35x + 42}{35 + 7} = \frac{35x + 42}{42}$

Now, we have $\frac{35x + 42}{42} = x - 1$

or, $35x + 42 = 42x - 42$

or, $7x = 84$ $\therefore x = 12$

Thus the original expenditure of the mess = 35 × 12 = Rs 420 .

Direct formula :

If decrease in average = x

increase in expenditure = y

increase in no. of students = z

and number of students (originally) = N, then

the original expenditure $= N\left[\frac{x(N + z) + y}{z}\right]$

In this case, $35\left[\frac{1(35 + 7) + 42}{7}\right] = 35 \times 12 = \text{Rs } 420$

Note : This formula may be used in different cases of such examples. Try it.

EXERCISES

1. In an examination, a batch of 60 students made an average score of 55 and another batch of 40 made it only 45. What is the overall average score?
   1) 52 2) 40 3) 51 4) 56 5) None of these
2. The average marks of a student in four subjects is 75. If the student obtained 80 marks in the 5th subject then the new average is
   1) 80 2) 76 3) 92 4) 95 5) None of these
3. The average of first 61 natural numbers is
   1) 30 2) 30.5 3) 31 4) 32 5) None of these
4. In an exam, the average was found to be 50 marks. After deducting computational errors the marks of the 100 candidates had to be changed from 90 to 60 each and the average came down to 45 marks. The total

1) 300 2) 600 3) 200 4) 150 5) None of these

5. The average age of a group of 16 persons is 28 yrs and 3 months. Two persons each 58 yrs old left the group. The average age of the remaining persons is
   1) 26 2) 24 3) 22 4) 20 5) None of these
6. The average weight of 50 balls is 5 gm. If the weight of the bag be included the average weight increases by 0.05 gm. What is the weight of the bag?
   1) 5.05 2) 6.05 3) 7.05 4) 7.55 5) None of these
7. The average age of a group of 10 students is 15 yrs. When 5 more students joined the group the average age rose by 1 yr. The average age (in years) of the new students is
   1) 18 yrs 2) 17 yrs 3) 16 yrs 4) 12 yrs 5) None of these
8. The average weight of 8 persons is increased by 2.5 kg when one of them who weighs 56 kg is replaced by a new man. The weight of the new man is
   1) 73 kg 2) 72 kg 3) 75 kg
   4) 80 kg 5) None of these
9. The average weight of A, B and C is 84 kg. If D joins the group, the average weight of the group becomes 80 kg. If another man E who weighs 3 kg more than D, replaces A, then the average of B, C, D and E becomes 79 kg. What is the weight of A?
   1) 64 kg 2) 72 kg 3) 75 kg 4) 80 kg 5) None of these
10. The average of 11 results is 50. If the average of first 6 results is 49 and the that of last 6 is 52, find the 6th result.
    1) 50 2) 52 3) 56 4) 60 5) None of these
11. A man drives to his office at 60 km/hr and returns home along the same route at 30 km/hr. Find the average speed.
    1) 50 km/hr 2) 45 km/hr 3) 40 km/hr
    4) 55 km/hr 5) None of these
12. Find the average of five consecutive even numbers a, b, c, d and e.
13. A man covers $\frac{1}{3}$ of his journey by train at 60 km/hr, next $\frac{1}{3}$ by bus at 30 km/hr and the rest by cycle at 10 km/hr. Find his average speed during whole journey.
    1) 30 km/hr 2) $33\frac{1}{3}$ km/hr 3) 20 km/hr
    4) [illegible] 5) None of these

ANSWERS

1. 3; Average of combined group $= \frac{60 \times 55 + 40 \times 45}{60 + 40} = 51$

2. 2; $\frac{4 \times 75 + 80}{5} = 76$

3. 3; Sum of first 61 natural numbers $= \frac{61(61+1)}{2}$

   $\therefore$ Average $= \frac{61(62)}{2 \times 61} = 31$

By Direct Formula :

The average of first 'n' natural numbers is $\frac{n+1}{2}$.

Hence in this case, average $= \frac{61+1}{2} = 31$

4. 2; Let the total number of candidates $= x$

   $\therefore \frac{50x - 100(90-60)}{x} = 45 \qquad \therefore x = 600$

5. 2; $\frac{16 \times 28\frac{1}{4} - 2 \times 58}{14} = 24$

6. 4; $51 \times 5.05 - 50 \times 5 = 7.55$ gm.

By Direct Formula :

Wt of bag = Old average + Increase in average (Total no. of objects)
$= 5 + 0.05\,(51) = 5 + 2.55 = 7.55$ gm.

7. 1; Total age of 10 students $= 15 \times 10 = 150$ yrs
   Total age of 15 students $= 15 \times 16 = 240$ yrs
   $\therefore$ Average of new students $= \frac{240 - 150}{5} = 18$ yrs.

8. 5; $56 + 8 \times 2.5 = 76$ yrs

9. 3; $A + B + C = 3 \times 84 = 252$ kg
   $A + B + C + D = 4 \times 80 = 320$ kg
   $\therefore D = 320 - 252 = 68$ kg
   $\therefore E = 68 + 3 = 71$ kg
   Now, $\frac{320 - A + 71}{4} = 79$
   $\therefore A = 75$ kg

10. 3; $6 \times 49 + 6 \times 52 - 11 \times 50 = 294 + 312 - 550 = 56$

11. 3; **By Direct Formula :**

$$\text{Average} = \frac{2 \times 60 \times 30}{60 + 30} = \frac{2 \times 60 \times 30}{90} = 40 \text{ km/hr}$$

12. Average of five consecutive even numbers or odd numbers is the middle term. In this case the average is c.

13. 3; $$\text{Average} = \frac{3 \times 60 \times 30 \times 10}{60 \times 30 + 60 \times 10 + 30 \times 10}$$

$$= \frac{3 \times 60 \times 30 \times 10}{2700} = 20 \text{ km/hr}$$

Problem Based on Ages

To solve the problems based on ages, students are required the knowledge of linear equations. This method needs some basic concepts as well as some more time than it deserves. Sometimes it is easier to solve the problems by taking the given choices in account. But this hit-and-trial method proves costly sometimes, when we reach our solution much later. We have tried to evaluate some easier as well as quicker methods to solve this type of questions. Although we are not able to cover each type of questions in this section, our attempt is to minimise your difficulties.

Have a look at the following questions

Ex. 1 : The age of the father 3 years ago was 7 times the age of his son. At present the father's age is five times that of his son . What are the present ages of the father and the son?

Ex. 2 : At present the age of the father is five times that of the age of his son. Three years hence, the father's age would be four times that of his son. Find the present ages of the father and the son.

Ex. 3 : Three years earlier the father was 7 times as old as his son. Three years hence the father's age would be four times that of his son. What are the present ages of the father and the son?

By the conventional method :

Soln : 1. Let the present age of son = x yrs.

Then, the present age of father = 5x yrs.

3 years ago,

$7(x - 3) = 5x - 3$

or, $7x - 21 = 5x - 3$

or, $2x = 18 \quad \therefore x = 9$ yrs.

Therefore, son's age = 9yrs.

Father's age = 45 yrs.

Soln : 2. Let the present age of son = x yrs.

Then, the present age of father = 5x yrs.

3 yrs hence,

$4(x + 3) = 5x + 3$

or, $4x + 12 = 5x + 3$

$\therefore x = 9$ yrs. Therefore, son's age = 9 yrs and father's age = 45 yrs.

Soln. 3. Let the present age of son = x yrs.
and the present age of father = y yrs.
3 yrs earlier, $7(x - 3) = y - 3$ or, $7x - y = 18$ ---- (1)
3 yrs hence, $4(x + 3) = y + 3$
or, $4x + 12 = y + 3$ or, $4x - y = -9$ ------ (2)
Solving (1) & (2) we get, x = 9 yrs. & y = 45 yrs.

Quicker Method :

Soln. 1 : Son's age $= \dfrac{3 \times (7-1)}{7-5} = 9$ yrs

and father's age $= 9 \times 5 = 45$ yrs.

Undoubtably you get confused with the above method, but is very easy to understand and remember. See the following form question:

Q: *t_1 yrs earlier the father's age was x times that of his son. At presen the father's age is y times that of his son. What are the present ages of the son and the father?*

$$\boxed{\textit{Son's age} = \frac{t_1(x-1)}{x-y}}$$

Soln. 2: Son's age $= \dfrac{(4-1) \times 3}{5-4} = 9$ yrs

and father's age $= 9 \times 5 = 45$ yrs

To make more clear, see the following form :

Q: *The present age of the father is y times the age of his son. t_2 yrs hence, the father's age become z times the age of his son. What are the present ages of the father and his son?*

$$\boxed{\textit{Son's age} = \frac{(z-1)t_2}{y-z}}$$

Soln. 3: Son's age $= \dfrac{3(4-1) + 3(7-1)}{7-4} = \dfrac{9+18}{3} = 9$ yrs.

To make the above formula clear see the following form o question:

Q : *t_1 yrs earlier the age of the father was x times the age of his son. t_2 yrs hence, the age of the father becomes z times the age of his son. What are the present ages of the son and the father?*

$$\boxed{\text{Son's age} = \frac{t_2(z-1) + t_1(x-1)}{(x-z)}}$$

In the tabular form the above three types can be arranged as:

	t_1 yrs earlier	Present	t_2 yrs hence
Father's age that of Son's	x times	y times	z times

$$\text{Son's age} = \frac{t_1(x-1)}{x-y} \qquad \text{Son's age} = \frac{(z-1)t_2}{(y-z)}$$

$$\boxed{\text{Son's age} = \frac{t_2(z-1) + t_1(x-1)}{(x-z)}}$$

Ex. 4 : The age of a man is 4 times that of his son. 5 yrs ago, the man was nine times as old as his son was at that time. What is the present age of the man?

Soln : By the table, we see that formula (1) will be used.

Son's age $= \dfrac{5(9-1)}{(9-4)} = 8$ yrs. $\therefore$ Father's age $= 4 \times 8 = 32$ yrs.

Note : The relation between **'earlier'** and **'present'** ages are given; so we look for the formula derived from the two corresponding columns of the table. That gives the formula (1).

Ex. 5 : After 5 yrs the age of a father will be thrice the age of his son, whereas five years ago, he was 7 times as old as his son was. What are their present ages?

Soln : Formula (3) will be used in this case. So

Son's age $= \dfrac{5(7-1) + 5(3-1)}{7-3} = 10$ yrs.

From the first relationship of ages, if F is the age of the father then $F + 5 = 3(10 + 5)$ $\therefore F = 40$ yrs.

Ex. 6 : 10 yrs ago, Sita's mother was 4 times older than her daughter. After 10 yrs, the mother will be twice older than the daughter. What is the present age of Sita?

Soln : In this case also, formula (3) will be used.

Daughter's age $= \dfrac{10(4-1) + 10(2-1)}{4-2} = 20$ yrs.

Ex. 7 : One year ago the ratio between Samir's and Ashok's age was 4 : 3. One year hence the ratio of their ages will be 5 : 4. What is the sum of their present ages in years ?

Soln : One yr ago Samir's age was $\frac{4}{3}$ of Ashok's age.

One yr hence Samir's age will be $\frac{5}{4}$ of Ashok's age.

$\therefore$ Ashok's age (by formula(3));

$$A = \frac{1\left(\frac{4}{3}-1\right)+1\left(\frac{5}{4}-1\right)}{\frac{4}{3}-\frac{5}{4}} = \frac{\frac{1}{3}+\frac{1}{4}}{\frac{1}{12}} = 7 \text{ yrs.}$$

Now, by the first relation:

$$\frac{(S-1)}{7-1} = \frac{4}{3}$$

$\therefore S = 8 + 1 = 9$ yrs.

$\therefore$ Total ages = A + S = 9 + 7 = 16 yrs.

where A = Ashok's present age

and S = Samir's present age

Ex. 8 : Ten yrs ago A was half of B in age. If the ratio of their present ages is 3 : 4, what will be the total of their present ages ?

Soln : 10 yrs ago A was $\frac{1}{2}$ of B's age.

At present A is $\frac{3}{4}$ of B's age.

$$\therefore \text{B's age [use formula (1)]} = \frac{10\left(\frac{1}{2}-1\right)}{\frac{1}{2}-\frac{3}{4}} = 20 \text{ yrs.}$$

A's age = $\frac{3}{4}$ of 20 = 15 yrs.

Ex. 9 : The sum of the ages of a mother and her daughter is 50 yrs. Al 5 yrs ago, the mother's age was 7 times the age of the daught What are the present ages of the mother and the daughter?

Soln : Let the age of the daughter be x yrs.

Then, the age of the mother is (50 - x) yrs.

5 yrs ago, 7 (x - 5) = 50 - x - 5

or, 8x = 50 - 5 + 35 = 80

$\therefore$ x = 10

Therefore, daughter's age = 10 yrs.

and mother's age = 40 yrs.

Quicker Method (Direct Formula) :

$$\text{Daughter's age} = \frac{\text{Total ages} + \text{No. of yrs ago (Times} - 1)}{\text{Times} + 1}$$

$$= \frac{50 + 5(7-1)}{7+1} = 10 \text{ yrs.}$$

Thus, daughter's age = 10 yrs and mother's age = 40 yrs.

Ex. 10 : The sum of the ages of a son and father is 56 yrs. After 4 yrs, the age of the father will be three times that of the son. What is the age of the son?

Soln : Let the age of the son be x yrs.

Then, the age of the father is (56 - x) yrs.

After 4 yrs, 3(x + 4) = 56 - x + 4

or, 4x = 56 + 4 - 12 = 48

$\therefore$ x = 12 yrs.

Thus, son's age = 12 yrs.

Quicker Method (Direct Formula) :

$$\text{Son's age} = \frac{\text{Total ages} - \text{No. of yrs after (Times} - 1)}{\text{Times} + 1}$$

$$= \frac{56 - 4(3-1)}{3+1} = \frac{48}{4} = 12 \text{ yrs.}$$

Note : Do you get the similarities between the above two direct methods? They differ only in signs in the numerator. When the question deals with '*ago*' a +ve sign exists and when it deals with '*after*' a -ve sign exists in the numerator.

Ex. 11 : The sum of the present ages of the father and the son is 56 yrs. 4 yrs hence the son's age will be $\frac{1}{3}$ that of the father. What are the present ages of the father and the son?

Soln : Son's age is $\frac{1}{3}$ that of father.

$\Rightarrow$ Father's age is 3 times that of son.

Now we use the formula as in Ex. 10. Try it.

Important Note : We can solve the above question without changing the form of 'times'. When the question remains in its original form, we find the age of father directly as:

$$\text{Father's age} = \frac{56 - 4\left(\frac{1}{3} - 1\right)}{\left(\frac{1}{3} + 1\right)} = \frac{56 + \frac{8}{3}}{\frac{4}{3}} = \frac{176}{4} = 44 \text{ yrs.}$$

And hence we may get the age of son. Thus this reveal an important fact that in each of the examples from 1 to 11, w may get the second age directly by inverting the ratio. Fo example, for 4 times we may use $\frac{1}{4}$, for $\frac{4}{3}$ we may use $\frac{3}{4}$, and s on. Try to solve each of the above examples by inverting the ratio

Ex. 12 : The ratio of the father's age to the son's age is 4 : 1. The produc of their ages is 196. What will be the ratio of their ages after years?

Soln : Let the ratio of proportionality be x, then

$4x \times x = 196$ or, $4x^2 = 196$ or, $x = 7$

Thus, Father's age = 28 yrs, Son's age = 7 yrs.

After 5 yrs, Father's age = 33 yrs.

Son's age = 12 yrs.

$\therefore$ Ratio = 33 : 12 = 11 : 4

Ex. 13 : The ratio of Rita's age to the age of her mother is 3 : 11. The difference of their ages is 24 yrs. What will be the ratio of their ages after 3 yrs?

Soln : Difference in ratios = 8

Then $8 \equiv 24 \therefore 1 \equiv 3$

ie, value of 1 in ratio is equivalent to 3 yrs

Thus Rita's age = 3 × 3 = 9 yrs.

Mother's age = 11 × 3 = 33 yrs.

After 3 years, the ratio = 12 : 36 = 1 : 3

Ex. 14 : The ratio of the ages of the father and the son at present is 6:1 After 5 years the ratio will become 7 : 2. What is the present age of the son?

Soln : Father : Son

Present age = 6 : 1

After 5 yrs = 7 : 2

$$\text{Son's age} = 1 \times \frac{5(7-2)}{6 \times 2 - 7 \times 1} = 5 \text{ yrs.}$$

$$\text{Father's age} = 6 \times \frac{5(7-2)}{6 \times 2 - 7 \times 1} = 30 \text{ yrs.}$$

Then what direct formula comes?

Father : Son

Present age = x : y

After T yrs = a : b

$$\text{Then Son's age} = y \times \frac{T(a-b)}{\text{difference of cross product}}$$

$$\text{and, Father's age} = x \times \frac{T(a-b)}{\text{difference of cross product}}$$

Ex. 15 : The ratio of the ages of the father and the son at present is 3:1. 4 years earlier, the ratio was 4 : 1. What are the present ages of the son and the father?

Soln : Father : Son

Present age = 3 : 1

4 yrs before = 4 : 1

$$\text{Son's age} = 1 \times \frac{4(4-1)}{4 \times 1 - 3 \times 1} = 12 \text{ yrs.}$$

$$\text{Father's age} = 3 \times \frac{4(4-1)}{4 \times 1 - 3 \times 1} = 36 \text{ yrs.}$$

Now the direct formula comes as :

Father : Son

Present age = x : y

T yrs before = a : b

$$\text{Then Son's age} = y \times \frac{T(a-b)}{\text{difference of cross product}}$$

$$\text{and, Father's age} = x \times \frac{T(a-b)}{\text{difference of cross product}}$$

Note : 1. While evaluating the difference of cross-product always take the +ve sign.

2. Both the above direct formulas look similar. The only difference you can find is in the denominators. But it has been simplified as "difference of cross-products" to make it easier to remember. So with the help of one formula only you can solve both the questions.

3. The above two questions (Q. 14 and Q. 15) are similar to the questions discussed earlier (Q. 1 & Q. 2). Do you get the point? If you change 'ratio' in 'times' you will get the same thing. Now,

you have two simple methods. Use the method which you feel easier to remember.

Thus, you may solve

Q. 14 as Q. 2 and vice versa.

Q. 15 as Q. 1 and vice versa.

4. We suggest you to go through both the methods and choose the better of the two.

Ex. 16: A man's age is 125% of what it was 10 years ago, but $83\frac{1}{3}\%$ of what it will be after 10 years. What is his present age?

Soln: Detail Method: Let the present age be x yrs. Then

125% of $(x - 10) = x$; and $83\frac{1}{3}\%$ of $(x + 10) = x$

$\therefore 125\%$ of $(x - 10) = 83\frac{1}{3}\%$ of $(x + 10)$

or, $\frac{5}{4}(x - 10) = \frac{5}{6}(x + 10)$

or, $\frac{5}{4}x - \frac{5}{6}x = \frac{50}{6} + \frac{50}{4}$ or, $\frac{5x}{12} = \frac{250}{12}$ $\therefore x = 50$ yrs

Direct Method: With the help of the above detail method, we can define a general formula as: Present age

$$= \frac{125 \times \text{No. of yrs ago} + 83\frac{1}{3} \times \text{No. of yrs after}}{125 - 83\frac{1}{3}}$$

$$= \frac{125 \times 10 + 83\frac{1}{3} \times 10}{125 - 83\frac{1}{3}} = \frac{10\left(\frac{375 + 250}{3}\right)}{\frac{375 - 250}{3}} = \frac{10 \times 625}{125} = 50 \text{ yrs}$$

Try Yourself

1. The ratio of the ages of Geeta and her mother is 1 : 5. After 7 years the ratio becomes 3 : 8. What are the present ages of Geeta and her mother? (Ans : 5 yrs, 25 yrs)
2. The ratio of the ages of A and B is 3 : 11. After 3 years the ratio becomes 1 : 3. What are the ages of A and B? (Ans. : 9 yrs, 33 yrs)
3. The ratio of the ages of Mohan and Meera is 3 : 4. Four years earlier the ratio was 5 : 7. Find the present ages of Mohan and Meera. (Ans. : 24 yrs, 32 yrs)

Profit and Loss

In this chapter, the use of "Rule of Fraction" is dominant. We should understand this rule very well because it is going to be used in almost all the questions.

The Rule of Fraction says

If our required value is greater than the supplied value we should multiply the supplied value with a fraction which is more than one. And if our required value is less than the supplied value we should multiply the supplied value with a fraction which is less than one.

(1) If there is a gain of x%, the calculating figures would be 100 and (100 + x).

(2) If there is a loss of y%, the calculating figures would be 100 and (100 - y).

(3) If the required value is more than the supplied value, our multiplying fractions should be $\frac{100 + x}{100}$ or $\frac{100}{100 - y}$ (both are greater than 1).

(4) If the required value is less than the supplied value, our multiplying fractions should be $\frac{100}{100 + x}$ or $\frac{100 - y}{100}$ (both are less than 1).

Profit = Selling Price (SP) - Cost Price (CP)

Loss = Cost Price (CP) - Selling Price (SP)

To find the gain or loss per cent

The profit or loss is generally reckoned as so much per cent **on the cost.**

$$\boxed{\text{Gain or loss per cent} = \frac{\text{Loss or gain}}{\text{CP}} \times 100}$$

Ex. 1. A man buys a toy for Rs 25 and sells it for Rs 30. Find his gain per cent.

Soln: $\%\text{ Gain} = \frac{\text{Gain}}{\text{CP}} \times 100 = \frac{5}{25} \times 100 = 20\%$

Ex. 2. A boy buys a pen for Rs 25 and sells it for Rs 20. Find his loss per cent.

Soln: $\%\text{ Loss} = \frac{\text{Loss}}{\text{CP}} \times 100 = \frac{5}{25} \times 100$

Ex.3: If a man purchases 11 oranges for Rs 10 and sells 10 oranges for

Rs 11. How much profit or loss does he make?

Soln: Suppose that the person bought $11 \times 10 = 110$ oranges.

$$\text{CP of 110 oranges} = \frac{10}{11} \times 110 = \text{Rs } 100$$

$$\text{SP of 110 oranges} = \frac{11}{10} \times 110 = \text{Rs } 121$$

$$\therefore \text{ Profit} = \text{Rs } 121 - \text{Rs } 100 = \text{Rs } 21$$

$$\text{and \% profit} = \frac{\text{Profit}}{\text{CP}} \times 100 = \frac{21}{100} \times 100 = 21\%$$

Quicker Maths: Rewrite the statements as follows:

Purchases 11 oranges for Rs 10

Sells 10 oranges for Rs 11

Now, percentage profit or loss is given by:

$$\frac{11 \times 11 - 10 \times 10}{10 \times 10} \times 100 = 21\%$$

Since the sign is +ve, there is a gain of 21%.

Note : The above form of structural adjustment should be remembered. The first line deals with purchase whereas the second line deals with sales. Once you get familiar with the form, you need to write only the figures and not the letters.

Ex.4: A man purchases 8 pens for Rs. 9 and sells 9 pens for rupees 8. How much profit or loss does he make ?

Soln: Quicker Maths:

Purchases 8 pens for Rs 9

Sells 9 pens for Rs 8

$$\text{\% profit or loss} = \frac{8 \times 8 - 9 \times 9}{9 \times 9} \times 100 = \frac{-1700}{81} = -20.98\%$$

Since the sign is -ve, there is a loss of 20.98%.

Ex.5: A boy buys oranges at 9 for Rs 16 and sells them at 11 for Rs 20. What does he gain or lose per cent ?

Soln: Quicker Maths:

9 ↘ ↗ 16

11 ↙ ↘ 20

$$\text{\% profit or loss} = \frac{9 \times 20 - 16 \times 11}{16 \times 11} \times 100 = 2\frac{3}{11}\%$$

Since the sign is +ve, there is a gain of $2\frac{3}{11}\%$.

Dishonest dealer using false weight

Ex. 6: A dishonest dealer professes to sell his goods at cost price, but he uses a weight of 960 gm for the kg weight. Find his gain per cent.

Soln: Suppose goods cost the dealer Re 1 per kg. He sells for Re 1 what cost him Re 0.96.

$$\therefore \text{ Gain on Re } 0.96 = \text{Re } 1 - \text{Re } 0.96 = \text{Re } 0.04$$

$$\therefore \text{ Gain on Rs } 100 = \frac{0.04}{0.96} \times 100 = \text{Rs } 4\frac{1}{6}$$

$$\therefore \text{ Gain \%} = 4\frac{1}{6}\%$$

Direct formula:

$$\boxed{\begin{aligned} \text{\% gain} &= \frac{\text{Error}}{\text{True value} - \text{Error}} \times 100 \\ \text{or, \% gain} &= \frac{\text{True weight} - \text{False weight}}{\text{False weight}} \times 100 \end{aligned}}$$

$$= \frac{40}{1000 - 40} \times 100 = 4\frac{1}{6}\%$$

Ex.7: A dishonest dealer professes to sell his goods at cost price, but he uses a weight of 950 gm for the kg weight. Find his gain per cent.

Soln: Direct formula:

$$\text{\% gain} = \frac{\text{Error}}{\text{True value} - \text{Error}} \times 100$$

$$= \frac{50}{950} \times 100 = 5.26\%$$

Ex. 8: A grocer sells rice at a profit of 10% and uses a weight which is 20% less. Find his total percentage gain.

Soln : Detail method :

Suppose he bought at Rs x/kg.

Then he sells at Rs $\left(\frac{110x}{100}\right)$ per $\frac{80}{100}$ kg

or, at Rs $\frac{110x}{100} \times \frac{100}{80}$ per kg or, at Rs $\frac{11x}{8}$ per kg

$$\text{Now, \% profit} = \frac{\frac{11x}{8} - x}{x} \times 100 = \frac{300}{8} = 37.5\%$$

Quicker Method :

$$\text{Total percentage profit} = \frac{\%\text{ profit} + \%\text{ less in wt}}{100 - \%\text{ less in wt}} \times 100$$

$$= \frac{10+20}{100-20} \times 100 = \frac{30 \times 100}{80} = 37.5\%$$

Ex. 9: A dishonest dealer sells goods at $6\frac{1}{4}\%$ loss on cost price but uses 14 gms instead of 16 gms. What is his percentage profit or loss?

Soln : Detail method : Suppose the cost price is Rs x per kg.

Then he sells the goods for Rs $x\left(\frac{100 - \frac{25}{4}}{100}\right) = \text{Rs } \frac{15x}{16}$ per kg

Now, suppose he bought y kg of goods.

Then, his total investment = Rs xy

and his total return = Rs $\frac{15x}{16} \times y\left(\frac{16}{14}\right) = \text{Rs } \frac{15}{14}xy$

$\therefore$ his % profit $= \frac{\frac{15}{14}xy - xy}{xy} \times 100 = \frac{50}{7} = 7\frac{1}{7}\%$

Direct Formula : If the shopkeeper sells his goods at x% loss on cost price but uses y gm instead of z gm, then his % profit or loss is

$[100 - x]\frac{z}{y} - 100$ as the sign is +ve or -ve.

In the above case,

$$\%\text{ profit or loss} = \left[100 - 6\frac{1}{4}\right]\left[\frac{16}{14}\right] - 100$$

$$= \frac{375}{4} \times \frac{16}{14} - 100 = \frac{1500 - 1400}{14} = \frac{100}{14} = \frac{50}{7} = 7\frac{1}{7}\%$$

Since the sign is +ve, there is a profit of $7\frac{1}{7}\%$.

Note : See another form of the above question in Ex. 10.

Ex. 10: A dishonest dealer sells the goods at $6\frac{1}{4}\%$ loss on cost price but uses $12\frac{1}{2}\%$ less weight. What is his percentage profit or loss?

Soln : In this case, we use the direct formula as:

Profit or loss percentage

$$= \frac{100 - 6\frac{1}{4}}{100 - 12\frac{1}{2}} \times 100 - 100 = \frac{100 - \frac{25}{4}}{100 - \frac{25}{2}} \times 100 - 100$$

$$= \frac{\frac{375}{4}}{\frac{175}{2}} \times 100 - 100 = \frac{15}{14} \times 100 - 100 = \frac{100}{14} = 7\frac{1}{7}\%.$$

Since sign is +ve, there is a profit of $7\frac{1}{7}\%$

Note : Ex. 9 and Ex. 10 are the same question. In Ex. 9, he uses 14 gms for 16 gms. This implies that he uses $\frac{16-14}{16} \times 100 = \frac{25}{2} = 12\frac{1}{2}\%$ less weight. Thus, we see that any of the direct formula can be used in both the cases.

Ex. 11: A seller uses 840 gm in place of one kg to sell his goods. Find his actual % profit or loss

a) when he sells his article on 4% loss on cost price.

b) when he sells his article on 4% gain on cost price.

Soln: Detail Method: Suppose the cost price of 1000 gm is Rs 100.

$\therefore$ Cost price of 840 gm $= \frac{100}{1000}(840) = \text{Rs } 84$

For **(a)**, selling price of 840 gm = Rs (100 - 4) = Rs 96

$\therefore$ Profit = SP - CP = 96 - 84 = Rs 12

$\therefore$ % profit $= \frac{12 \times 100}{84} = \frac{100}{7} = 14\frac{2}{7}\%$.

For **(b)**, selling price of 840 gm = Rs (100 + 4) = Rs 104

$\therefore$ Profit = SP - CP = 104 - 84 = Rs 20

$\therefore$ % profit $= \frac{20 \times 100}{84} = 23\frac{17}{21}\%$

Quicker Method: There is a general formula for such type of questions. See the two cases separately:

Case I: If a seller uses 'X' gm in place of one kg (1000 gm) to sell his goods and bears a loss of x% on cost price then his actual gain or loss percentage is $(100 - x)\left[\frac{100}{X}\right] - 100$ according as the sign is +ve or -ve.

Case II: If a seller uses 'X' gm in place of one kg (1000 gm) to sell his goods and gains a profit of x% on cost price, then his actual gain or loss percentage is $(100 + x)\left[\frac{100}{X}\right] - 100$ according as the sign is +ve or -ve.

Combining the two cases, we have

Gain or loss% $= (100 \pm x)\left[\frac{1000}{X}\right] - 100$

according as the sign is +ve or -ve

In the above case x = 4 and X = 840 gm. Therefore,

(a) % loss or gain $= (100 - 4)\left(\frac{1000}{840}\right) - 100$

$$= \frac{96 \times 1000}{840} - 100 = \frac{800}{7} - 100 = \frac{100}{7} = 14\frac{2}{7}\%$$

Since the sign is +ve, there is a gain of $14\frac{2}{7}\%$

(b) % gain $= (100 + 4)\left(\frac{1000}{840}\right) - 100$

$$= \frac{104 \times 1000}{840} - 100 = \frac{2600}{21} - 100 = \frac{500}{21} = 23\frac{17}{21}\%$$

Another Example

A seller used 990 gm in place of one kg to sell the rice. Find his actual profit or loss percentage when he sells

(a) On 10% loss on cost price. (b) On 10% profit on cost price.

Using the above general formula:

(a) % loss or gain $= (100 - 10)\left(\frac{1000}{990}\right) - 100$

$$= \frac{1000}{11} - 100 = -\frac{100}{11} = -9\frac{1}{11}\%$$

$\Rightarrow$ there is a loss of $9\frac{1}{11}\%$.

(b)% gain $= (100 + 10)\left(\frac{1000}{990}\right) - 100 = \frac{1000}{9} - 100 = \frac{100}{9} = 11\frac{1}{9}\%$

To find the selling price

Ex. 12: A man bought a cycle for Rs 250. For how much should he sell it so as to gain 10% ?

Soln: If CP is Rs 100, the SP is Rs 110.

If CP is Re 1, the SP is Rs $\frac{110}{100}$.

If CP is Rs 250, the SP is Rs $\frac{110}{100} \times 250 =$ Rs 275.

Another suggested method (By Rule of Fraction)

If he wanted to sell the bicycle at a gain of 10%, the selling price (required value) must be greater than the cost price (supplied value), so we should multiply Rs 250 with a more-than-one value fraction. Since there is a gain, our calculating figures should be 100 and (100 + 10) and the fraction should be $\frac{110}{100}$.

Thus, selling price $= 250 \times \frac{110}{100} =$ Rs 275.

OR, As there is a gain, SP must be greater than CP.

so, SP = (100 + 10) % of CP

$$= \frac{110}{100} \times 250 = \text{Rs } 275$$

Ex. 13: A man bought a cycle for Rs 560. For how much shall he sell it so as to lose 10% ?

Soln: As there is a loss, the SP must be less than CP.

So, SP = (100 - 10) % of CP

$$= \frac{90}{100} \times 560 = \text{Rs } 504.$$

By Rule of Fraction :

Calculating figures are 100 and (100 - 10)

Since the required value is less than 1,

the multiplying fraction $= \frac{90}{100}$.

Thus, selling price $= 560 \times \frac{90}{100} =$ Rs 504

Note : Once you understand the theorem well, you need to write only the figures and hence you may save a lot of time.

To find the Cost Price

Ex. 14: If by selling an article for Rs 390 a shopkeeper gains 20%, find his cost.

Soln: If the SP be Rs 120, the CP is Rs 100

If the SP be Rs 390, the CP is Rs $\frac{100}{120} \times 390 =$ Rs 325.

By Rule of Fraction :

Required value is less than the supplied value; therefore Rs

should be multiplied by $\frac{100}{100+20}$.

$\therefore$ CP = 390 × $\frac{100}{120}$ = Rs 325.

Ex. 15: By selling goods for Rs 352.88, I lost 12 %; find the cost pr

Soln: CP should be more than SP; so we multiply SP by

$\frac{100}{100-12} = \frac{100}{88}$ (a fraction which is more than 1)

$\therefore$ CP = 352.88 × $\frac{100}{88}$ = Rs 401.

Goods passing through successive hands

Ex. 16: A sells a good to B at a profit of 20% and B sells it to C profit of 25% . If C pays Rs 225 for it, what was the cost pr for A?

Soln: During both the transactions there are profits. So our calculat figures would be 120, 125 and 100. A's cost price is certainly than C's selling price.

$\therefore$ Required price = 225 × $\frac{100}{120}$ × $\frac{100}{125}$ = Rs 150

Remark: Since we need a value which is less than the given value, our multiplying fractions should be less than one. That is why multiplied 225 with $\frac{100}{120}$ and $\frac{100}{125}$.

Ex.17: A sells a bicycle to B at a profit of 30% and B sells it to C at loss of 20%. If C pays Rs 520 for it, at what price did A buy

Soln: In the whole transaction there is a gain of 30% and a loss of 20 so our calculating figures would be 130, 80 and 100.

B's cost of price = 520 × $\frac{100}{80}$

A's cost price = 520 × $\frac{100}{80}$ × $\frac{100}{130}$ = Rs 500.

Alternative method for Ex. 16 & Ex. 17

(1) When there are two successive profits of x% and y% , th the resultant profit per cent is given by

$$\left(x + y + \frac{xy}{100}\right)$$

Thus for Ex.16, the resultant profit

$= 20 + 25 + \frac{20 \times 25}{100} = 50\ \%$

Thus CP = 225 × $\frac{100}{150}$ = Rs 150

(2) When there is a profit of x% and loss of y% in a transaction, then the resultant profit or loss per cent is given by

$$\left(x - y - \frac{xy}{100}\right)$$

according to the + ve and the -ve signs respectively.

Thus for Ex.17, the resultant profit or loss

$= 30 - 20 - \frac{30 \times 20}{100} =$ 4% profit, because sign is +ve.

$\therefore$ required price = $\frac{520 \times 100}{104}$ = Rs 500.

Note: The second formula $\left(x - y - \frac{xy}{100}\right)$ is obtained from the first $\left(x + y + \frac{xy}{100}\right)$ by putting -y for y. Can you find the reason ?

Ex.18: By selling a horse for Rs 570, a tradesman would lose 5% . At what price must he sell it to gain 5%?

Soln: (100 - 5)% of the CP = Rs 570

$\therefore$ (100 + 5)% of the CP = $\frac{570}{95}$ × 105 = Rs 630

If you don't want to go into details of the method, you may follow the **method of fraction.**

Our calculating figures are (100 - 5), (100 + 5) and 100.

Cost price of horse = 570 × $\frac{100}{95}$

Thus, SP = 570 × $\frac{100}{95}$ × $\frac{105}{100}$ = Rs 630

Ex.19: A machine is sold for Rs 5060 at a gain of 10% . What would have been the gain or loss per cent if it had been sold for Rs 4370?

Soln: Calculating figures are 110 and 100.

$\therefore$ CP = 5060 × $\frac{100}{110}$ = Rs 4600

2nd SP = Rs 4370 $\therefore$ loss % = $\frac{230 \times 100}{4600}$ = 5%

Ex.20: I sold a book at a profit of 12% . Had I sold it for Rs 18 more,

18% would have been gained. Find the cost price.

Soln: Here,

118% of cost - 112% of cost = Rs 18

$\therefore$ 6% of cost = Rs 18

$\therefore \text{cost} = \dfrac{18 \times 100}{6} = \text{Rs } 300$

Quicker Maths: Ignoring the intermediate steps, we have a **dire** **formula** for such questions.

$$\boxed{\text{Cost} = \frac{\text{More gain} \times 100}{\text{Difference in percentage profit}}}$$

$\therefore \text{cost} = \dfrac{18 \times 100}{18 - 12} = \text{Rs } 300$

Ex.21: A man sold a horse at a loss of 7% . Had he been able to sell i at a gain of 9% , it would have fetched Rs 64 more than it di What was the cost price?

Soln: Here 109 % of cost - 93% of cost = Rs 64

$\therefore$ 16 % of cost = Rs 64 $\quad \therefore \text{cost} = \dfrac{64 \times 100}{16} = \text{Rs } 400$

By direct formula:

$$\frac{64 \times 100}{9 - (-7)} = \frac{64 \times 100}{16} = \text{Rs } 400$$

Note: Since 7% loss = (-7) % profit

Ex.22: A person sells an article at a profit of 10%. If he had bought it at 10% less and sold it for Rs 3 more, he would have gained 25%. Find the cost price.

Soln: Let the actual cost price = Rs 100

Actual selling price at 10% profit = Rs 110

Supposed cost price at 10% less = Rs 90

Supposed selling price at 25% gain

$= \text{Rs } 90 \times \dfrac{125}{100} = \text{Rs } 112.5$

$\therefore$ the difference in the selling prices

= Rs 112.5 − Rs 110 = Rs 2.5

If the difference is Rs 2.5, the CP = Rs 100

If the difference is Rs 3, the $\text{CP} = \dfrac{100}{2.5} \times 3 = \text{Rs } 120$

By Rule of Fraction :

Let the cost of the article be A.

$\text{Actual selling price} = A\left(\dfrac{110}{100}\right)$

$\text{Supposed cost price} = A\left(\dfrac{90}{100}\right)$

$\text{Supposed selling price} = A\left(\dfrac{90}{100}\right)\left(\dfrac{125}{100}\right)$

Therefore, we find a relationship :

$$A\left(\frac{110}{100}\right) + 3 = A\left(\frac{90}{100}\right)\left(\frac{125}{100}\right) \text{ ------ } (*)$$

$$\text{or, } A\left\{\frac{90 \times 125 - 110 \times 100}{100 \times 100}\right\} = 3$$

$$\therefore A = \frac{3 \times 100 \times 100}{90 \times 125 - 110 \times 100} \text{ ---- } (*)\,(*)$$

$$= \frac{3 \times 100 \times 100}{250} = \text{Rs } 120$$

Note : In the above example, the relationship given in (*) should be clear. Both sides of the equation are the supposed selling price of the article. With help of that equation we get cost price in (*) (*). If you remember (*) (*), you can save much time. But since the type of question varies frequently, you are suggested to proceed after finding the relationship given in (*).

Ex. 23: A person bought an article and sold it at a loss of 10%. If he had bought it for 20% less and sold it for Rs 55 more he would have had a profit of 40%. Find the cost price of the article.

Soln : If the cost price is A, the supposed selling price

$$= A\left(\frac{90}{100}\right) + 55 = A\left(\frac{80}{100}\right)\left(\frac{140}{100}\right)$$

$$\text{or, } A\left[\frac{80 \times 140 - 100 \times 90}{100 \times 100}\right] = 55 \Rightarrow A = \frac{55 \times 100 \times 100}{11200 - 9000}$$

$$= \frac{55 \times 100 \times 100}{2200} = \text{Rs } 250$$

Note : If we write the direct formula, we will have to keep one thing in mind that for x% loss and y% gain our calculating figures will be (100 - x) and (100 + y).

Ex.24: A man buys an article and sells it at a profit of 20% . If he bought

it at 20% less and sold it for Rs 75 less, he would have gained 25%. What is the cost price ?

Soln: Let the actual cost price = Rs 100

Actual selling price at 20% profit = Rs 120

Supposed cost price at 20% less = Rs 80

Supposed selling price at 25% gain = Rs 80 $\times \frac{125}{100}$ = Rs 100

$\therefore$ the difference in selling price = Rs 120 - Rs 100 = Rs 20

If the difference is Rs 20, the CP = Rs 100

If the difference is Rs 75, the CP = $\frac{100}{20} \times 75$ = Rs 375

By the rule of fraction :

Let the cost price be A, then

$$A\left(\frac{120}{100}\right) - 75 = A\left(\frac{80}{100}\right)\left(\frac{125}{100}\right)$$

$$\text{or, } A\left[\frac{120 \times 100 - 80 \times 125}{100 \times 100}\right] = 75$$

$$\text{or, } A = \frac{75 \times 100 \times 100}{120 \times 100 - 80 \times 125} = \frac{75 \times 100 \times 100}{2000} = \text{Rs } 375$$

Ex. 25: A dealer sold a radio at a loss of 2.5%. Had he sold it for Rs 100 more, he would have gained 7.5%. For what value should he sell it in order to gain $12\frac{1}{2}$%?

Soln : Suppose he bought the radio for Rs x.

Then selling price at 2.5% loss = Rs x $\left(\frac{100-2.5}{100}\right) = \frac{97.5x}{100}$

and selling price at 7.5% gain = Rs x $\left(\frac{100+7.5}{100}\right)$ = Rs $\frac{107.5x}{100}$

From the question, $\frac{107.5x}{100} - \frac{97.5x}{100}$ = Rs100

or, 10x = 100 × 100 $\quad \therefore$ x = Rs 1000

Therefore to gain 12.5%, he should sell for

Rs 1000$\left(\frac{100+12.5}{100}\right)$ = Rs 1125

Quicker Method :

$$\text{Selling price} = \frac{\text{More rupees } (100 + \% \text{ final gain})}{\% \text{ gain} + \% \text{ loss}}$$

$$= \frac{100(112.5)}{7.5 + 2.5} = \text{Rs } 1125$$

Ex. 26: An article is sold at a profit of 20%. If both the cost price and selling price are Rs 100 less, the profit would be 4% more. Find the cost price.

Soln: Suppose the cost price of that article is Rs x.

The selling price = Rs x $\left(\frac{120}{100}\right)$

New cost price and selling price is Rs (x - 100)

and Rs $\left[x\left(\frac{120}{100}\right) - 100\right]$ respectively.

$$\text{New profit} = \text{Rs}\left[x\left(\frac{120}{100} - 100\right) - (x - 100)\right]$$

$$= \text{Rs}\left[x\left\{\frac{120}{100} - 1\right\}\right] = \text{Rs } x\left(\frac{20}{100}\right)$$

$$\therefore \text{ New percentage profit} = \frac{x\left(\frac{20}{100}\right)}{x - 100} \times 100 = \frac{20x}{x - 100}\%$$

We are also given that the new percentage of profit = 20 + 4 = 24%

or, $\frac{20x}{x-100} = 24 \quad$ or, 4x = 2400 $\quad \therefore$ x = 600

Thus cost of the article = Rs 600

Direct Formula: When cost price and selling price are reduced by the same amount (say A) then

$$\text{Cost price} = \frac{[\text{Initial profit \%} + \text{Increase in profit \%}] \times A}{\text{Increase in profit \%}}$$

In this case, Cost price = Rs $\frac{(20+4) \times 100}{4}$ = Rs 600

Note: What happens when the cost price and selling price are reduced by different amounts?

For that case we have derived a general formula: Take the following form of questions:

"An article is sold at P% profit. If its CP is lowered by Rs c and at the same time its SP is also lowered by Rs s, then percentage of profit increases by p%. Find the cost price of that article."

$$\text{Cost Price} = \frac{c\,(P+p) - 100\,(s-c)}{p}$$

Ex: (a) An article is sold at 20% profit. If its CP and SP are less by Rs 10 and Rs 5 respectively the percentage profit increases by 10%. Find the cost price.

Soln: Using the above formula:

$$\frac{10\,(20+10) - 100\,(5-10)}{10} = \frac{800}{10} = \text{Rs } 80$$

Ex: (b) An article is sold at 25% profit. If its CP and SP are more by Rs 20 and Rs 4 respectively, the percentage of profit decreases by 15%. Find the cost price.

Soln: We may use the above formula in this question if we put the +ve and -ve signs correctly.

For example, in this case,

CP and SP are decreased by Rs (-20) and Rs (-4) respectively whereas % profit increases by (-15)%.

$$\text{Now, CP} = \frac{-20\,(25-15) - 100\,\{-4-(-20)\}}{-15}$$

$$= \frac{-200-1600}{-15} = \frac{-1800}{-15} = \frac{1800}{15} = \text{Rs } 120$$

Thus, we see that the above question can be asked in so many ways by changing "increase" into "decrease" and "decrease" into "increase". If you understand the signs used in the above formula, you can solve all these types of questions very easily.

Ex. 27: A person sells his table at a profit of $12\frac{1}{2}\%$ and the chair at a loss of $8\frac{1}{3}\%$ but on the whole he gains Rs 25. On the other hand if he sells the table at a loss of $8\frac{1}{3}\%$ and the chair at a profit of $12\frac{1}{2}\%$ then he neither gains nor loses. Find the cost price of the table.

Soln: Suppose the cost price of a table = Rs T and cost price of a chair = Rs C. Then; $12\frac{1}{2}\%$ of T + $\left(-8\frac{1}{3}\%\right)$ of C = 25

and $\left(-8\frac{1}{3}\%\right)$ of T + $12\frac{1}{2}\%$ of C = 0

or, $\frac{25}{2}T - \frac{25}{3}C = 2500$ ------(1)

$-\frac{25}{3}T + \frac{25}{2}C = 0$ -------(2)

(1) ÷ 2 + (2) ÷ 3 gives $\frac{25}{4}T - \frac{25}{9}T = 1250$

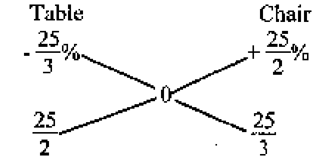
or, $T\left[\frac{225-100}{36}\right] = 1250$

$\therefore$ T = 360 $\quad\therefore$ Price of a table = Rs 360

OR,

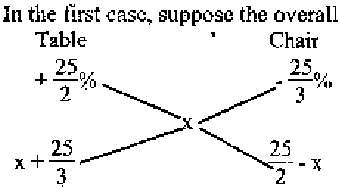
By Rule of Alligation:

In second case:



$\therefore$ Ratio of cost of table to chair $= \frac{25}{2} : \frac{25}{3} = \frac{1}{2} : \frac{1}{3} = 3:2$

In the first case, suppose the overall profit% is x then



$\therefore$ ratio of cost price of table and chair $= \left(x+\frac{25}{3}\right) : \left(\frac{25}{2} - x\right) = 3:2$

or, $\dfrac{x+\frac{25}{3}}{\frac{25}{2}-x} = \frac{3}{2}$ $\quad$ or, $2x + \frac{50}{3} = \frac{75}{2} - 3x$

or, $5x = \frac{75}{2} - \frac{50}{3} = \frac{125}{6}$ $\therefore x = \frac{125}{30} = \frac{25}{6}\%$

Now, $\frac{25}{6}\% \equiv$ Rs 25

$\therefore$ 100% ≡ Rs 600

$\therefore$ cost of a table $= \frac{600}{3+2} \times 3 =$ Rs 360.

Note: We see that the method of alligation becomes lengthy because w are not in a condition to use it directly. If we had the % value profit in the first case, the method of alligation would have be easier.

Ex. 28: An article is sold at 20% profit. If its cost price is increased Rs 50 and at the same time if its selling price is also increased Rs 30, the percentage of profit decreases by $3\frac{1}{3}\%$. Find the co price.

Soln: Suppose the cost price = Rs x

Then SP = Rs $\frac{120}{100}x$ = Rs $\frac{6}{5}x$

Now, new CP = Rs (x + 50)

new SP = Rs $\left[\frac{120}{100}x + 30\right]$ = Rs $\left[\frac{6}{5}x + 30\right]$

Now, the new % profit $= 20 - 3\frac{1}{3} = 16\frac{2}{3}\% = \frac{50}{3}\%$

Thus, $\left(100 + \frac{50}{3}\right)\%$ of $(x + 50) = \frac{6}{5}x + 30$

or, $\frac{350}{300}(x + 50) = \frac{6}{5}x + 30$ or, $\frac{7}{6}x + \frac{175}{3} = \frac{6}{5}x + 30$

or, $\left(\frac{6}{5} - \frac{7}{6}\right)x = \frac{175}{3} - 30$ or, $\frac{1}{30}x = \frac{85}{3}$ $\therefore$ x = Rs 850.

Quicker Method: If we think about the changes only, we find that $3\frac{1}{3}\%$ o cost price

$= \left(100 + 16\frac{2}{3}\right)\%$ of increase in CP - Increase in SP.

or, $\frac{10}{300} \times CP = \frac{350}{300} \times 50 - 30$

$\therefore CP = \frac{350 - 180}{6} \times \frac{300}{10}$ = Rs 850.

Theorem : *If cost price of x articles is equal to the selling price of y articles, then profit percentage* $= \frac{x - y}{y} \times 100\ \%$

Ex.29: The cost price of 10 articles is equal to the selling price of 9 articles. Find the profit per cent.

Soln: Let the cost price of 1 article be Re 1

$\therefore$ Cost of 10 articles = Rs 10

$\therefore$ Selling price of 9 articles = Rs 10

$\therefore$ Selling price of 10 articles = Rs $\frac{10 \times 10}{9}$ = Rs $\frac{100}{9}$

$\therefore$ gain on Rs 10 = Rs $\frac{100}{9}$ - Rs 10 = Rs $\frac{10}{9}$

$\therefore$ gain on Rs 100 = Rs $\frac{100}{9}$ = Rs $11\frac{1}{9}$

$\therefore$ profit per cent is $11\frac{1}{9}\%$.

Another Method :

To avoid much calculation we should suppose that the total investment = 10 × 9 = Rs 90

Then cost price of 1 article $= \frac{90}{10}$ = Rs 9

and selling price of 1 article $= \frac{90}{9}$ = Rs 10

$\therefore$ % profit $= \frac{10 - 9}{9} \times 100 = \frac{1}{9} \times 100 = 11\frac{1}{9}\ \%$

By Direct Formula (given in theorem):

% profit $= \frac{10 - 9}{9} \times 100 = 11\frac{1}{9}\ \%$

Ex. 30: I sell 16 articles for the same money as I paid for 20. What is my gain per cent?

Soln: I got a profit of 4 articles at the cost of 16 articles.

$\therefore$ % profit $= \frac{4}{16} \times 100 = 25\%$

or,

Let the total investment be 16 × 20 = Rs 320

$$CP = \frac{320}{20} = \text{Rs } 16$$

$$SP = \frac{320}{16} = \text{Rs } 20$$

$$\% \text{ profit} = \frac{20-16}{16} \times 100 = 25\%$$

By Direct Formula :

$$\% \text{ profit} = \frac{20-16}{16} \times 100 = 25\%$$

Ex. 31: A wholeseller sells 30 pens for the price of 27 pens to a retail The retailer sells the pens at the marked price. Find the perc profit of the retailer.

Soln : $\% \text{ profit} = \frac{100}{9} = 11\frac{1}{9}\%$

Ex. 32: By selling 66 metres of cloth, a person gains the cost of 22 metr Find his gain % .

Soln: $\% \text{ gain} = \frac{22}{66} \times 100 = 33\frac{1}{3}\%$

Dealing in two or more parts

Ex. 33: If goods be purchased for Rs 450, and one-third be sold at a lo of 10%, at what gain per cent should the remainder be sold so to gain 20% on the whole transaction ?

Soln: Cost of $\frac{1}{3}$ of goods $= \frac{450}{3} = \text{Rs } 150$

The selling price of one-third of goods

$$= \text{Rs } 150 \times \frac{90}{100} = \text{Rs } 135$$

The total selling price is to be Rs 450 $\times \frac{120}{100} = \text{Rs } 540$

Hence the selling price of the remaining two-thirds of the goo must be (Rs 540 - Rs 135) or Rs 405.

But the cost price of this two-thirds = Rs 300

$$\therefore \text{ gain } \% = \frac{105}{300} \times 100 = 35\%$$

Short-cut suggested method:

If we ignore all the intermediate steps we would reach at th following:

Let x be the required per cent, then

$$\frac{1}{3} \text{ of } (100 - 10) + \frac{2}{3} \text{ of } (100 + x) = \text{whole of } (100 + 20)$$

$$\text{or, } \frac{90}{3} + \frac{2(100+x)}{3} = 120$$

$$\text{or, } 290 + 2x = 360 \quad \therefore\ x = \frac{70}{2} = 35\%$$

By method of alligation : This question can be solved by the method of alligation very quickly. See Ex 33 in Chapter **ALLIGATION.**

Ex.34: If goods be purchased for Rs 840, and one-fourth be sold at a loss of 20%, at what gain per cent should the remainder be sold so as to gain 20% on the whole transaction ?

Soln: Let x be the required per cent, then

$$\frac{1}{4}(100 - 20) + \frac{3}{4}(100 + x) = (100 + 20)$$

$$\text{or, } 20 + 75 + \frac{3x}{4} = 120$$

$$\text{or, } \frac{3x}{4} = 25$$

$$\therefore\ x = \frac{100}{3} = 33\frac{1}{3}\%$$

By method of alligation : See Ex 34 in chapter **ALLIGATION**

Ex. 35: A man purchases 5 horses and 10 cows for Rs 10000. He sells the horses at 15% profit and the cows at 10% loss. Thus he gets Rs 375 as profit. Find the cost of 1 horse and 1 cow separately.

Soln : Detail Method:

Let the cost of 1 horse be Rs x, then total selling price

$$= 5x\left(\frac{115}{100}\right) + (10000 - 5x)\left(\frac{90}{100}\right) = 10375$$

$$\text{or, } 575x + 90 \times 10000 - 450x = 10375 \times 100$$

$$\text{or, } 125x = 137500 \quad \therefore\ x = \frac{137500}{125} = \text{Rs } 1100$$

Therefore, Cost of one horse = Rs 1100 and cost of one cow

$$= \frac{10000 - 5 \times 1100}{10} = \text{Rs } 450$$

Short-cut Method (By method of alligation):

$$\text{Overall \% of profit} = \frac{375}{10000} \times 100 = 3.75\%$$

Now, Horse — Cow

15% profit — -10% profit

3.75% profit

13.75 — 11.25

$\therefore$ ratio between amount invested for the horses and that for the cows = 11:9

$\therefore$ cost price of 5 horses $= \frac{10000}{11+9} \times 11 =$ Rs 5500

$\therefore$ cost price of 1 horse = Rs 1100

and cost price of 1 cow = Rs $\frac{10000 \times 9}{20} \div 10 =$ Rs 450

Ex. 36: Two-thirds of a consignment was sold at a profit of 6% and the rest at a loss of 3%. If there was an overall profit of Rs 540, find the value of the consignment.

Soln: Detail Method: Suppose the value of consignment was Rs x. Then $\frac{2}{3}$x was sold at 6% profit, i.e., for Rs $\frac{2}{3}x\left(\frac{106}{100}\right)$.

And $\frac{1}{3}$ part is sold at Rs $\frac{x}{3}\left(\frac{97}{100}\right)$.

Now, Profit $= \left\{\frac{2x}{3}\left(\frac{106}{100}\right) + \frac{x}{3}\left(\frac{97}{100}\right)\right\} - x$

$= \frac{212x + 97x}{300} - x = \frac{309x - 300x}{300} = \frac{9x}{300}$

Since, $\frac{9x}{300} = 540$

$\therefore$ x = Rs $\frac{540 \times 300}{9} =$ Rs 18,000

Quicker Method:

Value of Consignment $= \frac{\text{Total Profit} \times 100}{\%\text{ profit} \times \frac{2}{3} - \%\text{ loss} \times \frac{1}{3}}$

$= \frac{540 \times 100}{6 \times \frac{2}{3} - 3 \times \frac{1}{3}} =$ Rs 18,000

Note: 1. The above formula can be written in the form (General form):

If x part is sold at m% profit, y part is sold at n% profit, z part is sold at p% profit and Rs P is earned as overall profit then the value of total consignment $= \frac{P \times 100}{xm + ny + pz}$

In Ex. 36, we used profit = -(loss) in the denominator.

Ex. 37: A person bought two watches for Rs 480. He sold one at a loss of 15% and the other at a gain of 19% and he found that each watch was sold at the same price. Find the cost prices of the two watches.

Soln : We are not going to discuss the detailed method. Only some hint regarding it is enough for you. If the CP of the first watch is x, then $x\left(\frac{100-15}{100}\right) = (480 - x)\left(\frac{100+19}{100}\right)$

Now solve for x and get the two prices.

Direct Formula : CP of watch sold at loss $= \frac{480 \times (100 + \%\text{ profit})}{(100-15) + (100+19)}$

$= \frac{480 \times 119}{204} =$ Rs 280

$\therefore$ CP of watch sold at gain = 480 - 280 = Rs 200.

Note : (1) CP of watch sold at gain

$= \frac{480 \times (100 - \%\text{ loss})}{(100-15) + (100-19)} =$ Rs 200.

(2) The direct formula has been derived from the detailed method. Try to find it yourself.

Ex. 38: $\frac{1}{3}$ of a commodity is sold at 15% profit, $\frac{1}{4}$ is sold at 20% profit and the rest at 24% profit. If a total profit of Rs 62 is earned, then find the value of the commodity.

Soln : Suppose the value of the commodity was Rs x. Then $\frac{x}{3}$ was sold at 15% profit, $\frac{x}{4}$ was sold at 20% profit and $x - \left(\frac{x}{3} + \frac{x}{4}\right) = \frac{5x}{12}$ was sold at 24% profit.

Now, profit $\frac{x}{3}\left(\frac{15}{100}\right) + \frac{x}{4}\left(\frac{20}{100}\right) + \frac{5x}{12}\left(\frac{24}{100}\right) = 62$

or, $\frac{x}{20} + \frac{x}{20} + \frac{x}{10} = 62$ or, $\frac{4x}{20} = 62$ $\therefore x = \frac{62 \times 20}{4} =$ Rs 310

Quicker Method (Direct Formula) :

$$\text{Value of commodity} = \frac{62 \times 100}{\frac{1}{3} \times 15 + \frac{1}{4} \times 20 + \frac{5}{12} \times 24}$$

$$= \frac{62 \times 100}{5 + 5 + 10} = \text{Rs } 310$$

Note : Use -ve sign when some part is sold at loss. For example, see the Ex. 39.

Ex 39: $\frac{2}{3}$ of a consignment was sold at 6% profit and the rest at a loss of 3%. If there was an overall profit of Rs 540, find the value of the consignment.

Soln : $\text{Value of consignment} = \frac{540 \times 100}{\frac{2}{3} \times 6 + \frac{1}{3}(-3)} = \frac{540 \times 100}{4 - 1} = \text{Rs } 18{,}000$

Ex. 40: Nandlal purchased 20 dozen notebooks at Rs 48 per dozen. He sold 8 dozen at 10% profit and the remaining 12 dozen at 20% profit. What is his profit percentage in this transaction?

Soln : Cost price of 20 dozen notebooks = 20 × 48 = Rs 960

$$\text{Selling price of 8 dozen notebooks} = \text{Rs } 8 \times 48\left(\frac{110}{100}\right)$$

$$\text{Selling price of 12 dozen notebooks} = \text{Rs } 12 \times 48\left(\frac{120}{100}\right)$$

$$\therefore \text{ total selling price} = \text{Rs } \frac{2112}{5} + \text{Rs } \frac{3456}{5} = \text{Rs } \frac{5568}{5}$$

$$\text{Profit} = \frac{5568}{5} - 960 = \text{Rs } \frac{768}{5} \qquad \therefore \text{ profit} = \frac{768 \times 100}{5 \times 960} = 16\%$$

Quicker Method : (Direct formula) :

$$\text{Percentage profit} = \frac{(\text{First part} \times \%\text{ profit on first part} + \text{second part} \times \%\text{ profit on second part})}{\text{Total of two parts}}$$

Here, total = 20 dozens are sold in two parts;
first part = 8 dozens and second part = 12 dozens.

$$\therefore \%\text{ profit} = \frac{8 \times 10 + 12 \times 20}{20} = \frac{320}{20} = 16\%$$

Reduction in price

Ex. 41: A reduction of 10% in the price of sugar enables a person to obtain 25 kg more for Rs 225. What is the reduction price per kg? Find also the original price per kg.

Soln: Owing to the fall in price, there is a saving of 10% on Rs 225, i.e., Rs $\frac{45}{2}$.

For this Rs $\frac{45}{2}$, a person purchases 25 kg of sugar.

Hence reduced price per kg = Rs 22.5 ÷ 25 = Re 0.90 = 90 P

$$\text{Original price} = 90\text{P} \times \frac{100}{100 - 10} = \text{Re } 1$$

Sale of Mixture

Ex. 42: A grocer mixes 26 kg of sugar which cost Rs 2 a kg with 30 kg of sugar which cost Rs 3.60 a kg and sells the mixture at Rs 3 a kg. What is his total gain and the profit per cent ?

Soln: Profit = (26 kg + 30 kg) × Rs 3/kg - [26 kg × Rs 2/kg + 30 kg × Rs 3.60/kg] = Rs 168 - Rs (52 + 108) = Rs 168 - Rs 160 = Rs 8

$$\%\text{ profit} = \frac{8}{160} \times 100 = 5\%$$

Theorem : *A man purchases a certain number of articles at x a rupee and the same number at y a rupee. He mixes them together and sells them at z a rupee. Then his gain or loss %*

$$= \left[\frac{2xy}{z(x+y)} - 1\right] \times 100 \text{ according as the sign is +ve or -ve.}$$

Proof : Let the man purchase A number of articles.

$$\text{At the rate of x articles per rupee, CP of Articles} = \text{Rs } \frac{A}{x}$$

$$\text{At the rate of y articles per rupee, CP of A articles} = \text{Rs } \frac{A}{y}$$

$$\text{Total cost price} = \frac{A}{x} + \frac{A}{y}$$

Now, selling price of 2A articles at the rate of z articles per rupee

$$= \frac{2A}{z}$$

$$\%\text{ profit or loss} = \frac{SP - CP}{CP} \times 100 \text{ (According to +ve or -ve sign respectively)}$$

$$= \frac{\frac{2A}{z} - \left(\frac{A}{x} + \frac{A}{y}\right)}{\frac{A}{x} + \frac{A}{y}} \times 100$$

$$= \frac{\frac{2A}{z} - A\left(\frac{x+y}{xy}\right)}{A\left(\frac{x+y}{xy}\right)} \times 100 = \left[\frac{2xy}{z(x+y)} - 1\right] \times 100$$

Ex. 43: A man purchases a certain number of mangoes at 3 per rupee and the same number at 4 per rupee. He mixes them together and sells them at 3 per rupee. What is his gain or loss per cent?

Soln : By the theorem :

$$\text{Profit or loss per cent} = \left[\frac{2 \times 3 \times 4}{3(3+4)} - 1\right] \times 100$$

$$= \left[\frac{24}{21} - 1\right] \times 100 = \frac{100}{7} = 14\frac{2}{7}\%$$

Since the sign is +ve, there is a gain of $14\frac{2}{7}\%$

Ex. 44: A man purchases a certain number of toffees at 25 a rupee and the same number at 20 a rupee. He mixes them together and sells them at 45 for 2 rupees. What does he gain or lose per cent in the transaction ?

Soln: Suppose the man bought x number of toffees at 25 a rupee. Then,

$$\text{Profit} = \frac{2x}{45} \times 2 - \left(\frac{x}{25} \times 1 + \frac{x}{20} \times 1\right)$$

$$= \frac{4x}{45} - \frac{9x}{100} = -\frac{x}{900}$$

-ve sign shows that there is a loss.

$$\therefore \% \text{ loss} = \frac{\frac{x}{900}}{\frac{9x}{100}} \times 100 = \frac{x}{900} \times \frac{100}{9x} \times 100 = \frac{100}{81} = 1\frac{19}{81}\%$$

By the theorem:

$$x = 25,\ y = 20 \text{ and } z = \frac{45}{2} = 22.5$$

$$\therefore \% \text{ profit or loss} = \left[\frac{2xy}{z(x+y)} - 1\right] \times 100$$

$$= \left[\frac{2 \times 25 \times 20}{22.5\ (25+20)} - 1\right] \times 100$$

$$= \frac{1000 - 1012.5}{1012.5} \times 100 = -\frac{12.5}{1012.5} \times 100$$

$$= -\frac{100}{81} = -1\frac{19}{81}\%$$

Since the sign is -ve there is a loss of $1\frac{19}{81}\%$

Ex. 45: Oranges are bought at 11 for a rupee and an equal number more at 9 for a rupee. If these are sold at 10 for a rupee, find the loss or gain per cent.

Soln : By the theorem :

$$\% \text{ profit or loss} = \left[\frac{2 \times 11 \times 9}{10(11+9)} - 1\right] \times 100$$

$$= \left[\frac{198}{200} - 1\right] \times 100 = \frac{-2}{200} \times 100 = -1\%$$

Since the sign is -ve there is a loss of 1%.

Note : (1) From the above two examples, we find that when $z = \frac{x+y}{2}$, there is always loss.

(2) If there is $x = y = z$, there is neither gain nor loss. Do you agree?

Ex. 46: If toffees are bought at the rate of 25 for a rupee, how many must be sold for a rupee so as to gain 25% ?

Soln: SP of 25 toffees = Re 1 $\times \frac{125}{100}$ = Rs $\frac{5}{4}$

$\therefore$ No. of toffees sold for Rs $\frac{5}{4}$ = 25

$\therefore$ No. of toffees sold for Re 1 = $\frac{25 \times 4}{5} = 20$

Short-cut Method (Method of Fraction):

As there is 25% gain so our calculating figure would be 125 and 100. Now, to gain a profit the number of articles sold for one rupee

must be less than the number bought for one rupee. Thus the multiplying fraction is $\frac{100}{125}$.

$\therefore$ required no. of toffees $= 25 \times \frac{100}{125} = 20$

Ex.47: A man buys 5 horses and 7 oxen for Rs 5850. He sells the horses at a profit of 10% and oxen at a profit of 16% and his whole gain is Rs 711. What price does he pay for a horse?

Soln: Suppose the man pays Rs x for a horse. Then we reach at the equation:

10% of $5x + 16\%$ of $(5850 - 5x) = 711$

or, $\frac{5x}{10} + \frac{16}{100}(5850 - 5x) = 711$

or, $\frac{x}{2} + \frac{4}{25}(5850 - 5x) = 711$

or, $25x + 8(5850 - 5x) = 711 \times 50 = 35550$

or, $15x = 46,800 - 35,550 = 11,250 \quad \therefore x = \text{Rs } 750$

Ex. 48: A person bought some oranges at the rate of 5 per rupee. He bought the same number of oranges at the rate of 4 per rupee. He mixes both the types and sells at 9 for rupees 2. In this business he bears a loss of Rs 3. Find out how many oranges he bought in all.

Soln: Detail Method: Suppose he bought x oranges of each quality.

Then his total investment $= \frac{x}{5} + \frac{x}{4} = \text{Rs } \frac{9x}{20}$

Total selling price $= \text{Rs } \frac{2x \times 2}{9} = \text{Rs} \frac{4x}{9}$

$\therefore$ total loss $= \frac{9x}{20} - \frac{4x}{9} = \frac{81x - 80x}{180} = \frac{x}{180}$.

then Rs $\frac{x}{180} = \text{Rs } 3 \quad \therefore x = 180 \times 3 = 540$

Therefore he bought $2 \times 540 = 1080$ oranges in total.

Quicker Method : In the above question:

If x oranges/rupee and y oranges/rupee are mixed in same numbers and sold at z oranges/rupee then

Number of total apples bought $= \frac{\text{loss rupees} \times 2xyz}{z(x + y) - 2xy}$

$= \frac{3 \times 2 \times 5 \times 4 \times 4.5}{4.5(5 + 4) - 2 \times 5 \times 4} = \frac{120 \times 4.5}{40.5 - 40} = 1080$ oranges

Tradesman's discount for cash payment

Ex. 49: A tradesman marks his goods at 25% above his cost price and allows purchasers a discount of $12\frac{1}{2}\%$ for cash. What profit % does he make ?

Soln: Let the cost price = Rs 100

Marked price = Rs 125

Discount $= 12\frac{1}{2}\%$ of Rs $125 = \text{Rs } 15\frac{5}{8}$

$\therefore$ reduced price $= \text{Rs } 125 - \text{Rs } 15\frac{5}{8} = \text{Rs } 109\frac{3}{8}$

$\therefore$ gain per cent $= 109\frac{3}{8} - 100 = 9\frac{3}{8}\%$

Theorem: *If a trademan marks his goods at x % above his cost price* and allows purchasers a discount of y % for cash, then there is $\left(x - y - \frac{xy}{100}\right)\%$ profit or loss according to + ve or - ve sign respectively.

Here, $x = 25\%$, $y = 12\frac{1}{2}\%$

$\therefore \left(x - y - \frac{xy}{100}\right)\% = \left(25 - 12\frac{1}{2} - \frac{25 \times 25}{200}\right)\%$

$= 9\frac{3}{8}\%$ profit.

Note : Thus we see that if

x = marked percentage above CP

y = discount in percent

z = profit in percent

Then there exists a relationship;

$$\boxed{z = x - y - \frac{xy}{100}}$$

Ex. 50: A trader allows a discount of 5% for cash payment. How much % above cost price must he mark his goods to make a profit of 10% ?

Soln: If we use the relationship discussed above, we have

$10 = x - 5 - \frac{5x}{100}$

or, $\frac{19x}{20} = 15 \quad \therefore x = \frac{15 \times 20}{19} = 15\frac{15}{19}\%$

Ex. 51: A man buys two horses for Rs 1350. He sells one so as to lose 6% and the other so as to gain 7.5%. On the whole he neither gains nor loses. What does each horse cost ?

Soln: Loss on one horse = gain on the other

$\therefore$ 6 % of the cost of first horse

= 7.5 % of the cost of the second.

$$\therefore \frac{\text{Cost of first horse}}{\text{Cost of second horse}} = \frac{7.5\,\%}{6\,\%} = \frac{15}{12} = \frac{5}{4}$$

Dividing Rs 1350 in the ratio of 5:4,

Cost of first horse = Rs 750

Cost of the second = Rs 600

By the method of Alligation : (See Ex 35 in Chapter **ALLIGATION**)

Direct Formula :

Cost of first horse

$$= \frac{\text{CP of both} \times \text{\% loss or gain on 2nd}}{\text{\% loss or gain on 1st + \%loss or gain on 2nd}}$$

Cost of second horse

$$= \frac{\text{CP of both} \times \text{\% loss or gain on 1st}}{\text{\% loss or gain on first + \% loss or gain on 2nd}}$$

In this case :

$$\text{Cost of 1st horse} = \frac{1350 \times 7.5}{6 + 7.5} = \text{Rs } 750$$

$$\text{Cost of 2nd horse} = \frac{1350 \times 6}{6 + 7.5} = \text{Rs } 600$$

Some More Uses of Rule of Fraction

Ex. 52: Manju Sells an article to Anju at a profit of 25%. Anju sells it to Sonia at a gain of 10% and Sonia sells to Bobby at a profit of 5%. If Sonia sells it for Rs 231, find the cost price at which Manju bought the article.

Soln: Sonia bought for Rs $231\left(\frac{100}{100+5}\right)$

Anju bought for Rs $231\left(\frac{100}{105}\right)\left(\frac{100}{100+10}\right)$

Manju bought for Rs $231\left(\frac{100}{105}\right)\left(\frac{100}{110}\right)\left(\frac{100}{100+25}\right)$ = Rs 160

Ex. 53: Satish marks his goods 25% above cost price but allows 12.5% discount for cash payment. If he sells the article for Rs 875, find his cost price.

Soln: Marked price = Rs $875\left(\frac{100}{100-12.5}\right)$ = Rs $875\left(\frac{100}{87.5}\right)$

Cost price = Rs $875\left(\frac{100}{87.5}\right)\left(\frac{100}{100+25}\right)$ = Rs 800

Ex. 54: If oranges are bought at the rate of 30 for a rupee, how many must be sold for a rupee in order to gain 25%?

Soln: He must sell less than 30 oranges in order to gain.

Hence required number of oranges = $30\left(\frac{100}{100+25}\right) = 24$

Ex. 55: By selling oranges at 32 a rupee, a man loses 40%. How many for a rupee should he sell in order to gain 20%?

Soln: The man bought less than 32 oranges for a rupee.

Therefore, he bought $32\left(\frac{100-40}{100}\right) = 32\left(\frac{60}{100}\right)$ oranges for a rupee.

He must sell less than $32\left(\frac{60}{100}\right)$ oranges for a rupee.

So, the required number of oranges = $32\left(\frac{60}{100}\right)\left(\frac{100}{100+20}\right)$

$$= 32\left(\frac{60}{100}\right)\left(\frac{100}{120}\right) = 16$$

Ex. 56: If a man sells two horses for Rs 3910 each, gaining 15% on one and losing 15% on the other, find his total gain or loss.

Soln: By the theorem, there is always loss in this case

and the per cent value is given by $\frac{(15)^2}{100} = 2.25\%$

Now, we see that for SP Rs (100 - 2.25) there is loss of Rs 2.25

$\therefore$ When SP is Rs 3910, loss = $\frac{2.25}{97.75} \times 3910$ = Rs 90

$\therefore$ Total loss over two horses = 2 × 90 = Rs 180

Ex. 57: By selling an article for Rs 19.50 a dealer makes a profit of 30%. By how much should he increase his selling price so as to make a profit of 40%?

Soln: Cost price = Rs $19.50\left(\frac{100}{100+30}\right)$ = Rs $19.50\left(\frac{100}{130}\right)$

New selling price = Rs $19.50\left(\frac{100}{130}\right)\left(\frac{100+40}{100}\right)$

= Rs $19.50\left(\frac{100}{130}\right)\left(\frac{140}{100}\right)$ = Rs 21

$\therefore$ increase in SP = 21 - 19.5 = Rs 1.5

Ex. 58: A man bought a certain quantity of rice at the rate of Rs 150 per quintal. 10% of the rice was spoiled. At what price should he sell the remaining to gain 20% of his outlay?

Soln: "10% of the rice is spoiled" may be considered as if he bought the rice at 10% loss.

$\therefore$ CP per quintal = Rs $150\left(\frac{100}{100-10}\right)$ = Rs $150\left(\frac{100}{90}\right)$

Then SP = Rs $150\left(\frac{100}{90}\right)\left(\frac{100+20}{100}\right)$ = Rs $150\left(\frac{120}{90}\right)$ = Rs 200.

Ex. 59: A person sold his watch for Rs 144, and got a percentage of profit equal to the cost price. Find the cost of the watch.

Soln: Let the cost of the watch = Rs x

Then, $x\left(\frac{100+x}{100}\right) = 144$

or, $x^2 + 100x - 14400 = 0$

or, $(x + 180)(x - 80) = 0$ $\therefore$ x = -180 or 80

The only +ve value should be our answer, so cost of watch = Rs 80.

Note: In such questions, you are suggested to move from the given choices.

Ex. 60: What profit per cent is made by selling an article at a certain price, if by selling at 2/3 of that price there would be a loss of 20%?

Soln: $\frac{2}{3}$ of the selling price = (100 - 20)% of the cost price

or, Selling price = $\frac{80 \times 3}{2}$% of the C.P. = 120% of the C.P.

$\therefore$ 20% profit.

Ex. 61: A sells a pen to B at a gain of 20% and B sells it to C at gain of 10% and C sells it to D at a gain of 12.5%. If D pays Rs 14.85, what did it cost A?

Soln: By the rule of fraction:

A's cost = $14.85\left(\frac{100}{112.5}\right)\left(\frac{100}{110}\right)\left(\frac{100}{120}\right)$ = Rs 10.

Ex. 62: Suresh purchased a horse at $\frac{9}{10}$ of its selling price and sold it at 8% more than its selling price. Find his gain per cent.

Soln: Cost price = $\frac{9}{10}$

Selling price = $\frac{108}{100} = \frac{27}{25}$

% profit = $\frac{\frac{27}{25}-\frac{9}{10}}{\frac{9}{10}} \times 100 = \frac{270-225}{250 \times 9} \times 10 \times 100 = 20$

$\therefore$ 20% profit.

Ex. 63: The marked price of a radio is Rs 480. The shopkeeper allows a discount of 10% and gains 8%. If no discount is allowed, find his gain per cent.

Soln: Selling price = $480\left(\frac{100-10}{100}\right)$ = Rs 432

Cost price = $432\left(\frac{100}{100+8}\right)$ = Rs 400

If there is no discount, SP = Rs 480

$\therefore$ % profit = $\frac{480-400}{400} \times 100 = 20\%$

OR

If we recall the relationship

$z = x - y - \frac{xy}{100}$

Where, z = % profit = 8%

x = % higher mark

$\therefore$ y = % discount = 10%

We have $8 = x - 10 - \frac{10x}{100}$ or, $\frac{9x}{10} = 18$ $\therefore$ x = 20%

Hence, the shopkeeper marks 20% higher. If he gives no discount his gain is the same as he marks higher. Therefore, % gain = 20%.

Ex. 64: A dealer bought a horse at 20% discount on its original price. He sold it at a 40% increase on the original price. What percentage of profit did he get?

Soln: Let the original CP = Rs 100

Dealer's CP = 100 - 20% of 100 = Rs 80

Dealer's S.P. = 100 + 40% of 100 = Rs 140

Dealer's profit % = $\frac{140-80}{80} \times 100 = 75\%$

If we ignore the intermadiate steps we have a **direct formula** as

$$\frac{(100+40)-(100-20)}{(100-20)} \times 100 = 75\%$$

Other form of the above example:

Ex. 65: A trader bought a car at 20% discount on its original price. He sold it at a 40% increase on the price he bought it. What percentage of profit did he make on the original price?

Soln: Let the original price be Rs 100

$\therefore$ C.P. = $100\left(\frac{80}{100}\right)$ = Rs 80

S.P. = 80 + 40% of 80 = Rs 112

% profit on original price = $\frac{112-100}{100} \times 100 = 12\%$

OR

Using the direct formula:

% profit = $40 - 20 - \frac{40 \times 20}{100} = 12\%$

Ex. 66: There would be 10% loss if rice is sold at Rs 5.40 per kg. At what price per kg should it be sold to earn a profit of 20%?

Soln: By the rule of fraction :

$$S.P. = 5.4\left(\frac{100}{100-10}\right)\left(\frac{100+20}{100}\right) = 5.4\left(\frac{120}{90}\right) = \text{Rs } 7.2/\text{kg.}$$

Ex. 67: A horse worth Rs 9000 is sold by A to B at 10% loss. B sells the horse back to A at 10% gain. Who gains and who loses? Find also the values.

Soln: A sells to B for Rs $9000\left(\frac{90}{100}\right)$ = Rs 8100

Again, B sells to A for Rs $8100\left(\frac{110}{100}\right)$ = Rs 8910

Thus, A loses Rs (8910 - 8100) = Rs 810.

In this whole transaction, A's investment is only Rs 9000 (the cost of the horse) because the horse returned to his hand.

$\therefore$ A's % loss = $\frac{810}{9000} \times 100 = 9\%$

B gains Rs 810 (the same as A loses) and his investment in this transaction is Rs 8100.

$\therefore$ B's % gain = $\frac{810}{8100} \times 100 = 10\%$

Quicker Maths (direct formula): In such case, the first buyer bears loss and his % of loss is given by $\frac{\%\text{ gain }(100 - \%\text{ loss})}{100}$.

In this case, A's loss% = $\frac{10\ (100-10)}{100} = 9\%$

and loss amount = $9000\left(\frac{9}{100}\right)$ = Rs 810

Ex. 68: A man purchased two cows for Rs 500. He sells the first at 12% loss and the second at 8% gain. In this bargain he neither gains nor loses. Find the selling price of each cow.

Soln: If you recall, you will find that

CP of first cow = $\frac{500 \times 8}{12+8}$ = Rs 200

$\therefore$ SP of first cow = $200\left(\frac{100-12}{100}\right)$ = Rs 176

And CP of second cow = $\frac{500 \times 12}{12+8}$ = Rs 300

$\therefore$ SP of second cow = $300\left(\frac{108}{100}\right)$ = Rs 324

Ex. 69: A milkman buys some milk. If he sells it at Rs 5 a litre, he loses

Rs 200, but when he sells it at Rs 6 a litre, he gains Rs 150. How much milk did he purchase?

Soln: Difference in selling price = Rs 6/litre - Rs 5/litre = Re 1/litre.
If he increases the SP by Re 1/litre, he gets Rs 200 + Rs 150 = Rs 350 more.

$\therefore$ he purchased $\frac{\text{Rs } 350}{\text{Re } 1/\text{litre}} = 350$ litres milk.

Quicker Maths (direct formula):

$$\text{Quantity of milk} = \frac{\text{Difference of amount}}{\text{Difference of rate}} = \frac{150-(-200)}{6-5}$$

$$= \frac{350}{1} = 350 \text{ litres.}$$

Ex. 70: An article is marked for sale at Rs 275. The shopkeeper allows a discount of 5% on the marked price. His net profit is 4.5%. What did the shopkeeper pay for the article?

Soln: We know that if the shopkeeper marked x% higher then

$$4.5 = x - 5 - \frac{5x}{100} \Rightarrow x = 10\%$$

Therefore, cost price $= 275\left(\frac{100}{100+10}\right) = \text{Rs } 250$

Ex. 71: 9 kg of rice cost as much as 4 kg of sugar; 14 kg of sugar cost as much as 1.5 kg of tea; 2 kg of tea cost as much as 5 kg of coffee; find the cost of 11kg of coffee, if 2.5 kg of rice cost Rs 12.50.

Soln: 2.5 kg of rice cost Rs 12.50

$\therefore$ 9 kg of rice cost Rs $\frac{12.50}{2.5} \times 9 = \text{Rs } 45$

Cost of 9 kg of rice = Cost of 4 kg of sugar = Rs 45

$\therefore$ Cost of 14 kg of sugar = Rs $\frac{45}{4} \times 14$ = Cost of 1.5 kg of tea.

$\therefore$ Cost of 2 kg of tea $= \frac{45 \times 14 \times 2}{4 \times 1.5} = \text{Rs } 210$ = Cost of 5 kg of coffee

$\therefore$ Cost of 11 kg of coffee $= \frac{210}{5} \times 11 = \text{Rs } 462$

By the Rule of Column

This type of question creates confusion and leads to unsuccessful attempt. A simple method has been derived which is easy to understand and apply.

As per its name, the whole information is arranged in columns. Once you learn the method of arrangement, your problem will be solved within seconds. The following two points should be taken care of while arranging the information in columns. (It is easy to understand the method with the help of an example.)

Take an example

x kg of milk costs as much as y kg of rice;
z kg of rice costs as much as p kg of pulse;
w kg of pulse costs as much as t kg of wheat;
u kg of wheat costs as much as v kg of edible oil.
Find the cost of m kg of edible oil if n kg of milk costs Rs A.

Step I: Arrange the informations like;

Rs A	= n kg milk
x kg milk	= y kg rice
z kg rice	= p kg pulse
w kg pulse	= t kg wheat
u kg wheat	= v kg edible oil
m kg edible oil = ?	

Note: While arranging the data, the first point to be marked is that the first commodity in the right-side column should be the same as the second commodity in the left-side column. Similarly, the second commodity in the right-side column should be the same as the third commodity in the left-side column. And so on. That is why the last information (n kg of milks cost Rs A) is written at the top.

Step II: Mark the side of question mark (?). It is in the right-side column. So, the figures in the left-side column will go in the numerator and the figures in the right side column will go in the denominator.

$$\boxed{? = \frac{A \times x \times z \times w \times u \times m}{n \times y \times p \times t \times v}}$$

Note: The second remarkable point is the position of question mark (?). Our numerator and denominator depend on it, and hence also the required answer.

Suppose the above example is changed. Instead of the last given sentence, we are given

"Find the cost of n kg of milk if k kg of edible oil costs Rs B". Then our arrangement will be (taking the first point into consideration)

? = n kg milk
x kg milk = y kg rice
z kg rice = p kg pulse
w kg pulse = t kg wheat
u kg wheat = v kg edible oil
m kg edible oil = Rs B.

We see that the question mark (?) is in the left-side column, so the right side is our numerator and the left side is our denominator.

$\therefore$ required answer = $\boxed{? = \frac{B \times v \times t \times p \times y \times n}{m \times u \times w \times z \times x}}$

Hope you have understood the method. Now apply it to the above example.

Soln (Ex. 71): Rs 12.5 = 2.5 kg rice
9 kg rice = 4 kg sugar
14 kg sugar = 1.5 kg tea
2 kg tea = 5 kg coffee
11 kg coffee = ?

$\therefore ? = \frac{12.5 \times 9 \times 14 \times 2 \times 11}{2.5 \times 4 \times 1.5 \times 5}$ = Rs 462

Note: Solve the same question if the last sentence is changed to "Find the cost of 2.5 kg of rice if the cost of 11 kg of coffee is Rs 462".

Ex. 72: A fruit merchant makes a profit of 25% by selling mangoes at a certain price. If he charges Re 1 more on each mango, he would gain 50%. Find what price per mango did he sell at first. Also find the cost price per mango.

Soln: Suppose the cost price of a mango be Rs x.

Then, first selling price = Rs $x\left(\frac{100+25}{100}\right)$ = Rs $\frac{5x}{4}$

If he charges Re 1 more and gets 50% profit then there exists a relationship:

$\frac{5x}{4} + 1 = x\left(\frac{100+50}{100}\right) = \frac{3x}{2}$

or, $\frac{3x}{2} - \frac{5x}{4} = 1$

$\therefore$ x = Rs 4

$\therefore$ Cost price/mango = Rs 4

and first selling price = $4\left(\frac{125}{100}\right)$ = Rs 5

Quicker Maths (direct formula):

$$\boxed{\begin{aligned}&\text{Cost price} = \frac{100 \times \text{More charge}}{\%\text{ Difference in profit}}\\ &\text{and Selling price} = \frac{\text{More charge }(100 + \%\text{ first profit})}{\%\text{ Difference in profit}}\end{aligned}}$$

Thus in this case

C.P. $= \frac{100 \times 1}{50 - 25}$ = Rs 4, S.P. $= \frac{1 \times 125}{50 - 25}$ = Rs 5

Ex. 73: A fruit merchant makes a profit of 20% by selling a commodity at a certain price. If he charges Rs 3 more on each commodity, he would gain 50%. Find the cost price and first selling price of that commodity.

Soln: By Direct Formula:

C.P. $= \frac{100 \times 3}{50 - 20}$ = Rs 10, S.P. $= \frac{3\,(120)}{50 - 20}$ = Rs 12

Ex. 74: A salaried employee sticks to save 10% of his income every year. If his salary increases by 25% and he still sticks to his decision of his saving habit of 10%, by what per cent has his saving increased?

Soln: There should be no hesitation in saying that his saving will be increased by as many per cent as his salary is increased by. So the required answer is 25%.

But what happens when his saving % is changed? See the following example:

Ex. 75: A person saves 10% of his income. If his income increases by 20% and he decides to save 15% of his income, by what per cent has his saving increased?

Soln: By Quicker Maths (direct formula):

% increase in saving $= \frac{(100 + 20)\,15 - 10 \times 100}{10} = 80\%$

Note : If he sticks to his previous saving habit of 10% then by the direct formula :

% increase in saving $= \frac{120 \times 10 - 10 \times 100}{10} = 20\%$, which is the same as % increase in income.

Theorem : *When each of the two commodities is sold at the same price and a profit of P% in made on the first and a loss of L% is made on the second, then the percentage gain or loss*

$= \frac{100(P-L)-2PL}{(100+P)+(100-L)}$ *according to the +ve or -ve sign.*

Proof: Let each commodity be sold at Rs A

A profit of P% is made on the first, then cost price of the first commodity = Rs $A\left(\frac{100}{100+P}\right)$

A loss of L% is made on the second, then cost price of the second commodity = Rs $A\left(\frac{100}{100-L}\right)$

$$\text{Total C.P.} = A\left(\frac{100}{100+P}\right)+A\left(\frac{100}{100-L}\right)$$

$$= A\left[\frac{100(100-L)+100(100+P)}{(100+P)(100-L)}\right]$$

$$= \frac{100A[(100-L)+(100+P)]}{(100+P)(100-L)}$$

Total S.P. = 2A

$$\therefore \ \% \text{ profit or loss} = \frac{SP-CP}{CP}\times 100$$

$$= \frac{2A - \frac{100A[(100-L)+(100+P)]}{(100+P)(100-L)}}{\frac{100A[(100-L)+(100+P)]}{(100+P)(100-L)}}\times 100$$

$$= \frac{2(100+P)(100-L)-100[(100-L)+(100+P)]}{100[(100-L)+(100+P)]}\times 100$$

$$= \frac{2\times 100^2+200P-200L-2PL-2\times 100^2+100L-100P}{(100+P)+(100-L)}$$

$$= \frac{100P-100L-2PL}{(100+P)+(100-L)} = \frac{100(P-L)-2PL}{(100+P)+(100-L)}$$

Note: In the special case when P = L, we have

$$\frac{100\times 0-2P^2}{200} = -\frac{P^2}{100}$$

Since the sign is -ve, there is always loss and the value is given as $\frac{(\%\text{ value})^2}{100}$.

Ex.76: Each of the two horses is sold for Rs 720. The first one is sold at 25% profit and the other one at 25% loss. What is the % loss or gain in this deal ?

Soln: Total selling price of two horses = 2 × 720 = Rs 1,440

The CP of first horse = $720 \times \frac{100}{125}$ = Rs 576

The CP of second horse = $720 \times \frac{100}{75}$ = Rs 960

Total CP of two horses = 576 + 960 = Rs 1,536

Therefore, loss = Rs 1,536 - Rs. 1,440 = Rs 96

$\therefore \ \%\text{ loss} = \frac{96\times 100}{1536} = 6.25\ \%$

Direct Formula: (See theorem; note)

In this type of question where SP is given and profit and loss percentage are same, **there is always loss** and the

$\%\text{ loss} = \frac{(25)^2}{100} = \frac{625}{100} = 6.25\%$

Note: The above example is a special case when percentage values of loss and gain are the same. But what happens when they are different? See in the following example.

Ex.77: Each of the two cars is sold at the same price. A profit of 10% is made on the first and a loss of 7% is made on the second. What is the combined loss or gain ?

Soln: By the theorem

$\frac{100(10-7)-2\times 10\times 7}{200+10-7} = \frac{160}{203}$ % gain as the sign is +ve.

Note: You may notice that Ex.77 is a special case of Ex.76.

Ex. 78: A man sells two horses for Rs 1710. The cost price of the first is equal to the selling price of the second. If the first is sold at 10% loss and the second at 25% gain, what is his total gain or loss (in rupees)?

Soln : We suppose that the cost price of the first horse is Rs 100. Then we arrange our values in a tabular form:

	1st horse	2nd horse	Total
CP	100	$100\left(\frac{100}{125}\right)=80$	180
SP	$100\left(\frac{90}{100}\right)=90$	100	190

$\therefore$ **CP : SP** = 180 : 190 = 18 : 19

$\therefore$ **Profit** $= \frac{19-18}{19} \times 1710 =$ Rs 90

Note : We suggest you to solve such lengthy questions by making a tabular arrangement like the above one. This gives a quick solution without any confusion.

Direct Formula : If you need the direct formula for this question, see the following:

$$\text{Profit} = \frac{(100-10) - 100\left(\frac{100}{100+25}\right)}{100+(100-10)} \times 1710$$

$$= \frac{90-80}{190} \times 1710 = \text{Rs } 90.$$

Note : In the above formula, 10% loss is represented as (100 - 10) and 25% profit is represented as (100 + 25). Also, if we find the value -ve, we may conclude that there is a loss.

Ex. 79: A dealer sells a table for Rs 400, making a profit of 25%. He sells another table at a loss of 10%, and on the whole he makes neither profit nor loss. What did the second table cost him?

Soln: Profit on the first table $= 400\left(\frac{25}{125}\right) =$ Rs 80

$\Rightarrow$ he loses Rs 80 on the second table (Since there is neither profit nor loss)

$\therefore$ Cost price of second table $= \frac{80}{10} \times 100 =$ Rs = 800

Direct Formula: In the case, when there is neither profit nor loss and selling price of first is given, then

cost price of second $= 400\left[\frac{100}{125}\right]\left[\frac{25}{10}\right] =$ Rs 800

Ex. 80: Rakesh calculates his profit percentage on the selling price whereas Ramesh calculates his on the cost price. They find that the difference of their profits is Rs 100. If the selling price of both of them are the same and both of them get 25% profit, find their selling price.

Soln: Suppose the selling price for both of them is Rs x.

Now, cost price of Rakesh $= x\left(\frac{100-25}{100}\right) = \frac{3}{4}x.$

and cost price of Ramesh $= x\left(\frac{100}{100+25}\right) = \frac{4}{5}x$

Rakesh's profit $= x - \frac{3}{4}x = \frac{x}{4}$

Ramesh's profit $= x - \frac{4}{5}x = \frac{x}{5}$

Now, difference of their profits $= \frac{x}{4} - \frac{x}{5} =$ Rs 100 (given)

or, $\frac{x}{20} = 100$

$\therefore$ x = Rs 2000

Thus selling price = Rs 2000.

Quicker Method (Direct Formula)

$$\text{Selling price} = \frac{\text{Diff in profit} \times 100 \times (100+25)}{(25)^2}$$

$$= \frac{100 \times 100 \times 125}{25 \times 25} = \text{Rs } 2000$$

Note: What happens when % profits are different? In that case use the following formula:

If Rakesh gets x% profit and Ramesh gets y% profit then ,

$$\text{Selling price} = \frac{\text{Diff in profit} \times 100 \times (100+y)}{(100)^2 - (100+y)(100-x)}$$

Please note that when % profit is calculated over cost price we use (100 + y) and when % profit is calculated over selling price we use (100 - x).

If we put x = y = 25 in the above general formula, we can get the previously-used formula.

Ex. 81. If a discount of 10% is given on the marked price of an article, the shopkeeper gets a profit of 20%. Find his % profit if he offers a discount of 20% on the same article.

Soln : Detail Method :

Suppose the marked price = Rs 100

Then selling price at 10% discount = Rs (100 - 10) = Rs 90

Since he gets 20% profit, his cost price $= 90\left(\frac{100}{120}\right) =$ Rs 75

Now, at 20% discount, the selling price

$= \text{Rs } (100 - 20) = \text{Rs } 80$

Thus his % profit $= \frac{80 - 75}{75} \times 100 = \frac{500}{75} = \frac{20}{3} = 6\frac{2}{3}\%$

Quicker Method (Direct Formula):

Required % profit

$$= (100 + \%\text{ first profit})\left[\frac{100 - \%\text{ 2nd discount}}{100 - \%\text{ 1st discount}}\right] - 100$$

$$= (100 + 20)\left[\frac{100 - 20}{100 - 10}\right] - 100$$

$$= 120\left(\frac{80}{90}\right) - 100 = \frac{320}{3} - 100 = \frac{20}{3} = 6\frac{2}{3}\%$$

Ex. 82. A farmer sold a cow and a calf for Rs 760 and got a profit of 10% on the cow and 25% on the calf. If he sells the cow and the calf for Rs 767.50 and gets a profit of 25% on the cow and 10% on the calf, find the individual cost price of the cow and the calf.

Soln : Quicker Method : (i) For cost of cow:

	Cow		Calf	
(1)	110%	+	125%	= 760
(2)	125%	+	110%	= 767.5

$$\text{Cost of cow} = \frac{125\%\text{ of }767.5 - 110\%\text{ of }760}{(125\%)^2 - (110\%)^2}$$

$$= \frac{\frac{5}{4} \times 767.5 - \frac{11}{10} \times 760}{(1.25)^2 - (1.1)^2}$$

$$= \frac{959.375 - 836}{(1.25 + 1.1)(1.25 - 1.1)} = \frac{123.375}{2.35 \times 0.15} = \text{Rs } 350$$

(ii) For cost of calf:

	Cow		Calf
(1) SP	110%	+	125% = 760
(2) SP	125%	+	110% = 767.5

$$\text{Cost of calf} = \frac{125\%\text{ of }760 - 110\%\text{ of }767.5}{(125\%)^2 - (110)^2}$$

$$= \frac{950 - 844.25}{2.35 \times 0.15} = \text{Rs } 300$$

Ex. 83. A profit of 20% is made on goods when a discount of 10% is given on the marked price. What profit per cent will be made when a discount of 20% is given on the marked price?

Soln: Detail Method: Suppose the cost price of the goods is Rs 100.

Then selling price in the first case $= 100\left(\frac{120}{100}\right) =$ Rs 120

Therefore, marked price $=$ Rs $120\left(\frac{100}{100 - 10}\right) =$ Rs $\frac{400}{3}$

Now, selling price in the second case $= \frac{400}{3}\left(\frac{100 - 20}{100}\right) =$ Rs $\frac{320}{3}$

Therefore, % profit $= \frac{320}{3} - 100 \quad (\because CP = 100)$

$$= \frac{20}{3} = 6\frac{2}{3}\%$$

Quicker Method: In such cases:

$$\%\text{ profit} = (100 + \%\text{ profit})\left[\frac{100 - \%\text{ II discount}}{100 - \%\text{ I discount}}\right] - 100$$

$$= 120 \times \frac{80}{90} - 100 = \frac{320}{3} - 100 = \frac{20}{3} = 6\frac{2}{3}\%$$

Ex. 84. What will be the percentage profit after selling an article at a certain price if there is a loss of $12\frac{1}{2}\%$ when the article is sold at half of the previous selling price?

Soln: Detailed Method: Suppose the previous selling price = Rs x

Now, the later selling price = Rs $\frac{x}{2}$

There is a loss of $12\frac{1}{2}\%$ when selling price $= \frac{x}{2}$

$$\therefore \text{ cost price} = \frac{x}{2}\left(\frac{100}{100 - 12.5}\right) = \frac{100x}{175} = \frac{4x}{7}$$

Now, when selling price is Rs x, % profit

$$= \frac{x - \frac{4x}{7}}{\frac{4x}{7}} \times 100 = \frac{7x - 4x}{4x} \times 100 = \frac{3}{4} \times 100 = 75\%$$

Quicker Method:

Required % profit = 100 - 2 × % loss

$$= 100 - 2 \times 12\frac{1}{2} = 100 + 25 = 75\%$$

Ex. 85: What will be the percentage profit after selling an article a certain price if there is a loss of 45% when the article is sold half of the previous selling price?

Soln: Quicker Method:

% profit = 100 - 2 × % loss = 100 - 2 × 45 = 10%

Note: The more general formula for the Ex. 85 may be like:

When the second selling price is $\frac{1}{x}$ of the original selling pri then

% profit = x (100 - % loss) - 100

This formula is the same as used in Ex. 84 and Ex. 85. In the two cases

% profit = 2(100 - % loss) - 100

= 200 - 2 × % loss - 100 = [100 - 2 × % loss]

which are the same as used in Ex. 84 & Ex. 85

Ex. 86: What will be % profit after selling an article at a certain price there is loss of 45% when the article is sold at $\frac{1}{3}$ of previous sellin price?

Soln: By the general formula given in **note** of Ex. 85

% profit = 3(100 - % loss) - 100

= 3 × (100 - 45) - 100 = 3 × 55 - 100 = 65%

Ex. 87: A horse and a cow were sold for Rs 540, making a profit of 25% on the horse and 20% on the cow. By selling for Rs 538, the prof would be 20% on the horse and 25% on the cow. Find the cost o each.

Soln: Detailed Method:

Suppose the cost price of a cow and a horse are Rs C and Rs H respectively.

Then selling price of both = 125% of H + 120% of C = Rs 540

or, $\frac{5}{4}H + \frac{6}{5}C = 540$ or, 25 H + 24 C = 540 × 20 .... (i)

Total selling price in the second case = 120% of H + 125% of C = 538

or, $\frac{6}{5}H + \frac{5}{4}C = 538$ or, 24 H + 25 C = 538 × 20 .... (ii)

Performing (1) × 25 - (2) × 24, we have

$(25)^2 H - (24)^2 H - (540 \times 20 \times 25) - (538 \times 20 \times 24)$

or, 49 H = 11760 ∴ H = Rs 240

If we put the value of H in (1) we get;

24 C = 540 × 20 - 25 × 240 = 4800

∴ C = Rs 200.

∴ Cost of a horse is Rs 240 and that of a cow is Rs 200.

Quicker Method: In the above case, when the % of profit interchange in the two cases:

$$H + C = \frac{540 + 538}{125\% + 120\%} = \frac{540 + 538}{1.25 + 1.20} = \frac{1078}{2.45} = 440$$

$$\text{and } H - C = \frac{540 - 538}{125\% - 120\%} = \frac{2}{1.25 - 1.20} = \frac{2}{0.05} = \frac{200}{5} = 40$$

Now, solve the above two easier equations by adding and subtracting and get H = Rs $\frac{480}{2}$ = Rs 240 and

C = Rs $\frac{400}{2}$ = Rs 200.

Ex. 88: 5% more is gained by selling a cow for Rs 1010 than by selling it for Rs 1000. Find the cost price of the cow.

Soln: Suppose the cost price = Rs x

$$\text{Then } \frac{1000 - x}{x} \times 100 + 5 = \frac{1010 - x}{x} \times 100$$

$$\text{or, } \frac{100}{x}[1010 - x - 1000 + x] = 5$$

$$\text{or, } \frac{100}{x}(10) = 5$$

∴ x = Rs 200

Quicker Method:

5% of cost price = Rs (1010 - 1000) = Rs 10

$$\therefore CP = \frac{10 \times 100}{5} = \text{Rs } 200.$$

Direct Formula: Cost price $= \frac{100 \times \text{Diff. in SP}}{\% \text{ diff. in profit}} = \frac{100 \times 10}{5}$ = Rs 200.

Ex. 89: I bought two calculators for Rs. 480. I sold one at a loss of 15% and the other at a gain of 19% and then I found that each calculator was sold at the same price. Find the cost of the calculator sold at a loss.

Soln: Let the CP of the calculator which was sold at 15% loss be Rs x

$$\text{then } x\left(\frac{100 - 15}{100}\right) = (480 - x)\left(\frac{100 + 19}{100}\right)$$

or, $85x = 480 \times 119 - 119x$

or, $204x = 480 \times 119$

$\therefore x = \dfrac{480 \times 119}{204} = \text{Rs } 280$

Quicker Method (Direct Formula) :

Cost of the calculator sold at 15% loss

$= \dfrac{480(100 + \%\ \text{profit})}{(100 - \%\ \text{loss}) + (100 + \%\ \text{profit})}$

$= \dfrac{480 \times 119}{(100 - 15)(100 + 19)} = \dfrac{480 \times 119}{204} = \text{Rs } 280$

and cost of the calculator sold at 19% profit

$= \dfrac{480\ (100 - \%\ \text{loss})}{(100 - \%\ \text{loss}) + (100 + \%\ \text{profit})} = \dfrac{480 \times 85}{204} = \text{Rs } 200$

Ex. 90: I buy two tables for Rs. 1,350. I sell one so as to lose 6% and the other so as to gain $7\frac{1}{2}$ %. On the whole I neither lose nor gain. What did each table cost?

Soln: Let the first table costs Rs x

Then $x\left(\dfrac{94}{100}\right) + (1350 - x)\left(\dfrac{107.5}{100}\right) = 1350$

or, $x\left(\dfrac{107.5 - 94}{100}\right) = 1350\left(\dfrac{107.5 - 100}{100}\right)$

$\therefore x = \dfrac{1350 \times 7.5}{107.5 - 94} = \dfrac{1350 \times 7.5}{13.5} = \text{Rs } 750$

Thus, the prices of tables are Rs 750 and Rs (1300 - 750 =) Rs 600

Quicker Method (Direct Formula) :

Price of tables = Rs $\dfrac{1350 \times 7.5}{7.5 + 6}$ and Rs $\dfrac{1350 \times 6}{7.5 + 6}$

= Rs 750 and Rs 600.

Note : This can also be solved by the method of Alligation.

We have overall profit = 0%

Then I II

-6% 7.5%

0

7.5% 6%

Therefore costs of the tables are in the ratio 7.5 : 6 , 5 : 4

$\therefore$ Cost of tables $= \dfrac{1350}{5 + 4} \times 5 = \text{Rs } 750$

and $\dfrac{1350}{9} \times 4 = \text{Rs } 600$

EXERCISES

1. A book costing 15 P was sold for 18 P. What is the gain or loss per cent?
2. If oranges are bought at 11 for 10 P and sold at 10 for 11 P, what is the gain or loss per cent ?
3. A dishonest dealer professes to sell his goods at cost price but uses a weight of 875 grams for the kilogram weight. His gain per cent is ___.
4. A man buys milk at 60 P per litre, adds one-third of water to it and sells the mixture at 72 P per litre. The profit per cent is _________.
5. A watch costing Rs 120 was sold at a loss of 15%. The selling price is _______.
6. If mangoes are bought at 15 a rupee, how many must be sold for a rupee to gain 25% ?
7. Find the cost price if, by selling goods for Rs. 279, a merchant loses 7 per cent.
8. A man sells two watches for Rs. 99 each. On one he gained 10% and on the other he lost 10%. His gain or loss per cent is _________.
9. By selling goods for Rs. 153, a man loses 10% . For how much should he sell them to gain 20%?
10. By selling goods for Rs. 240, a merchant gains 25%. How much per cent would he gain by selling it for Rs. 216 ?
11. What profit per cent is made by selling an article at a certain price, if by selling at two-third of that price there would be a loss of 20% ?
12. By selling oranges at 32 a rupee, a man loses 40%. How many a rupee must he sell to gain 20 p.c. ?
13. The cost price of 16 articles is equal to the selling price of 12 articles. The gain or loss per cent is __________.
14. By selling 33 metres of cloth, I gain the selling price of 11 metres. The gain per cent is ______.
15. 5% more is gained by selling a cow for Rs. 350 than by selling it for Rs. 340. The cost price of the cow is ________.
16. A man buys apples at a certain price per dozen and sells them at times that price per hundred. His gain or loss per cent is ______.
17. A shopkeeper marks his goods 20 per cent above cost price, allows 10% discount for cash. The net profit per cent is ______.

18. A shopkeeper bought a table marked at Rs. 200 at successive discounts of 10% and 15% respectively. He spent Rs. 7 on transport and sold the table for Rs. 208. Find his profit per cent.
19. A merchant sold his goods for Rs. 75 at a profit per cent equal to th cost price. His cost price is__________.
20. I purchased a box full of pencils at the rate of 7 for Rs. 5 and sold the whole box at the rate of 9 for Rs. 8. In this process I gained Rs. 44. How many pencils were contained in the box ?
21. A dishonest dealer professes to sell his goods at a profit of 20% and also weighs 800 grams in place of a kg. Find his actual gain % .
22. A merchant professes to sell his goods at a loss of 10% , but weigh 750 gms in place of a kg. Find his real loss or gain per cent.
23. By selling salt at Re. 1 a kg, a man gains 10%. By how much must he raise the price so as to gain 21% ?
24. A milkman buys some milk contained in 10 vessels of equal size. If he sells his milk at Rs. 5 a litre, he loses Rs. 200; while selling it at Rs. 6 a litre, he would gain Rs. 150 on the whole. Find the number of litres contained in each cask.
25. A watch passes through three hands and each gains 25% . If the third sells it for Rs. 250, what did the first pay for it ?
26. If by selling an article for Rs. 60, a person loses $\frac{1}{7}$ of his outlay (cost), what would he have gained or lost per cent by selling it for Rs. 77?
27. I sold a book at a profit of 7%. Had I sold it for Rs. 7.50 more, 22% would have been gained. Find the cost price .
28. A reduction of 40 per cent in the price of bananas would enable a man to obtain 64 more for Rs. 40. What is the reduced price per dozen?
29. A man purchased [illegible] $\frac{3}{4}$ of the list price and [illegible] at half more than the list price. [illegible] his gain per cent ?
30. I lose 9 per cent by selling pencils at the rate of 15 a rupee. How many for a rupee must I sell them to gain 5 per cent?
31. Goods are sold so that when 4 per cent is taken off the list price, a profit of 20% is made. How much per cent is the list price more than the cost price ?
32. A watch was sold at a loss of 10 per cent. If it were sold for Rs. 70 more, there would have been a gain of 4 per cent. What is the C.P. of the watch ?
33. A man sells an article at 5% profit. If he had bought it at 5% less and sold it for Re. 1 less, he would have gained 10%. Find the cost price.

Answers

1. % gain $= \frac{18-15}{15} \times 100 = 20\%$

2. 11 10
   10 11

$\frac{11 \times 11 - 10 \times 10}{10 \times 10} \times 100\%$

= 21% profit, since sign is positive.

3. % gain $= \frac{\text{True wt} - \text{False wt}}{\text{False wt}} \times 100$

$= \frac{1000 - 875}{875} \times 100 = \frac{100}{7} = 14\frac{2}{7}\%$

4. When he added $\frac{1}{3}$ of water, the cost of one litre of impure milk

$= 60P\left(\frac{3}{4}\right) = 45P$ ------(*)

$\therefore$ % profit $= \frac{72-45}{45} \times 100 = 60\%$

Note (*) Quantity of milk $= 1 + \frac{1}{3} = \frac{4}{3}$ [illegible]

$\frac{4}{3}$ litre cost 60 P. By rule of fraction 1 litre will cost less than 60; hence we multiplied 60 by less-than-one fraction i.e. $\frac{3}{4}$

5. By rule of fraction :

SP $= 120\left(\frac{85}{100}\right) =$ Rs 102.

6. By rule of fraction: $15\left(\frac{100}{125}\right) = 12$

7. By rule of fraction:

Cost price $= 279\left(\frac{100}{100-7}\right) =$ Rs 300

8. There is always loss in such case and the loss % $= \frac{(10)^2}{100}\% = 1\%$

9. By rule of fraction: $153\left(\frac{100}{90}\right)\left(\frac{120}{100}\right) =$ Rs 204

10. CP $= 240\left(\frac{100}{125}\right) =$ Rs 192.

$\therefore$ % profit $= \frac{216-192}{192} \times 100 = \frac{25}{2} = 12\frac{1}{2}\%$

11. Let the cost price be Rs 100.

$\frac{2}{3}$ of original SP = 100 - 20% of 100 = Rs 80

$\therefore$ Original SP $= \frac{80 \times 3}{2} =$ Rs 120

$\therefore$ % profit $= \frac{120-100}{100} \times 100 = 20\%$

Quicker Formula :

% profit or loss $= \left[\frac{100-20}{\frac{2}{3}} - 100\right]$% profit or loss according to +ve or negative sign.

$= \frac{80}{\frac{2}{3}} - 100 = 20\%$ profit.

12. **By the rule of fraction :**

He must have purchased less number of oranges for a rupee, as he bears a loss. Therefore, no. of oranges purchased for a rupee

$= 32\left(\frac{60}{100}\right)$

Now, to gain 20%, he must sell less number of oranges for a rupee.

And that number is $= 32\left(\frac{60}{100}\right)\left(\frac{100}{120}\right) = 16$

13. Suppose he invested Rs 16 × 12.

Then CP of 1 article = Rs $\frac{16 \times 12}{16} =$ Rs 12

and SP of 1 article = Rs $\frac{16 \times 12}{12} =$ Rs 16

$\therefore$ % profit $= \frac{16-12}{12} \times 100 = \frac{100}{3} = 33\frac{1}{3}\%$

Quicker Method :

% profit $= \left(\frac{\text{No. of purchased good} - \text{No. of sold goods}}{\text{No. of sold goods}}\right) \times 100$

In this case, $\frac{16-12}{12} \times 100 = 33\frac{1}{3}\%$

14. Suppose the S. P. per metre = Re 1

Then S. P. of 33 metres = Rs 33

Profit = Rs 11

$\therefore$ C. P. of 33 metres = 33 - 11 = Rs 22

$\therefore$ % Profit $= \frac{11}{22} \times 100 = 50\%$

Quicker Method : % profit $= \frac{11}{33-11} \times 100 = 50\%$

Note : For the above two questions never use the detailed method. Remember the direct formula and its usage.

15. Difference in 5% profit = Diff. in Rs 10 profit

$\therefore$ 100% $= \frac{10}{5} \times 100 =$ Rs 200

Quicker Method (direct formula):

Cost price $= \frac{100(\text{Diff in S.P.})}{\text{Diff in profit \%}} = \frac{100(350-340)}{5} =$ Rs 200

16. Let C. P. = Rs x/dozen = Rs $\frac{100x}{12}$ per hundred and SP 8x/hundred

$\therefore$ % profit $= \frac{8x - \frac{100x}{12}}{\frac{100x}{12}} \times 100 = \frac{96x - 100x}{12 \times 100x} \times 100 \times 12 = -4\%$

-ve sign shows that there is loss of 4%

Quicker Method (direct formula) :

% profit or loss = 8 × dozen - Hundred = 96 - 100 = -4%

Since sign is -ve, there is loss of 4%

17. Use the **Direct Formula** :

% profit $= 20 - 10 - \frac{20 \times 10}{100} = 20 - 10 - 2 = 8\%$

18. Single equivalent discount $= 10 + 15 - \frac{10 \times 15}{100} = 23.5\%$

$\therefore$ CP for the shopkeeper = 200 - 23.5% of 200 = Rs 153

Total cost = 153 + 7 = Rs 160

Profit % $= \frac{208-160}{160} \times 100 = 30\%$

Quicker Method :

$$C.P. = 200\left(\frac{90}{100}\right)\left(\frac{85}{100}\right) = Rs\ 153$$

, Total cost = 153 + 7 = Rs 160

$$\%\ profit = \frac{208-160}{160}\times 100 = 30\%$$

19. $x + x\%$ of $x = 75$

or, $x^2 + 100x - 7500 = 0$

or, $(x - 50)(x + 150) = 0$

$\therefore x = 50$ or -150

neglecting the -ve value, the CP = Rs 50.

20. CP of 7 pencils is Rs 5

SP of 7 pencils is Rs $\frac{8}{9}\times 7 = Rs\ \frac{56}{9}$

$\therefore$ Profit on 7 pencils $= \frac{56-45}{9} = Rs\ \frac{11}{9}$

$\therefore$ Total pencils $= \frac{7\times 9}{11}\times 44 = 252$

21. Let CP of 1000 gm = Rs 100

SP of 800 gm = 100 + 20% of 100 = Rs 120

or, SP of 1000 gm $= \frac{120}{800}\times 1000 = Rs\ 150$

$\therefore$ % profit $= \frac{150-100}{100}\times 100 = 50\%$

Quicker Method :

$$\%\ profit = (100 + \%\ profit)\left(\frac{True\ weight}{False\ weight}\right) - 100$$

$$= 120\left(\frac{1000}{800}\right) - 100 = 50\%$$

22. Let CP of 1000 gm = Rs 100

SP of 750 gm = Rs 90 (as there is 10% loss)

or, SP of 1000 gm $= \frac{90}{750}\times 1000 = Rs\ 120$

$\therefore$ % profit $= \frac{120-100}{100}\times 100 = 20\%$

Quicker Method :

$$\%\ profit\ or\ loss = (100 - 10)\left(\frac{1000}{750}\right) - 100 = 20\%$$

Since sign is +ve, there is profit of 20%.

23. **By Rule of fraction:** $SP = 1\left(\frac{100}{110}\right)\left(\frac{121}{100}\right) = Rs\ 1.10$

$\therefore$ he must raise the price by 0.1 rupee or 10 paise.

24. Suppose he has x litre of milk in total.

Thus we have $5x + 200 = 6x - 150$

or, $x(6 - 5) = 200 + 150$

$\therefore x = 350$ litres.

$\therefore$ each vessel contains $\frac{350}{10} = 35$ litres.

Quicker Method :

$$\text{Total quantity of milk} = \frac{\text{Difference in Amount}}{\text{Difference in Rates}}$$

$$= \frac{150-(-200)}{6-5} = 350\ \text{litres.}$$

Note : Difference in amount = Gain + loss = 150 + 200 = 350

25. **By Rule of fraction :**

First Purchased for $250\left(\frac{100}{125}\right)\left(\frac{100}{125}\right)\left(\frac{100}{125}\right)$

$= 250\left(\frac{4}{5}\right)\left(\frac{4}{5}\right)\left(\frac{4}{5}\right) = Rs\ 128$

26. $\text{Cost Price} = \frac{\text{Selling price}}{1-\frac{1}{7}} = \frac{60\times 7}{6} = Rs\ 70$

$\therefore$ % profit $= \frac{77-70}{70}\times 100 = 10\%$

27. **Quicker Method :**

$$\text{Cost Price} = \frac{7.5\times 100}{\text{Difference in \% profit}} = \frac{7.5\times 100}{22-7} = Rs\ 50$$

28. He purchases 64 more bananas for 40% of Rs 40 or, Rs 16.

$\therefore$ Reduced price per dozen $= \frac{16}{64}\times 12 = Rs\ 3.$

29. Let the listed price = Rs 100

Then, CP $= \frac{3}{4}\times 100 = Rs\ 75$ and SP $= \frac{3}{2}\times 100 = Rs\ 150$

$\therefore$ % profit $= \frac{150-75}{75}\times 100 = 100\%$

Direct Method : % profit $= \dfrac{\left(1+\frac{1}{2}\right)-\frac{3}{4}}{\frac{3}{4}} \times 100 = 100\%$

30. **By the rule of fraction :**

He purchased $15\left(\dfrac{100-9}{100}\right)$ for a rupee.

Now to gain 5%, he must sell $15\left(\dfrac{91}{100}\right)\left(\dfrac{100}{105}\right) = 13$ for a rupee

31. Let the CP = Rs 100

Actual SP = 100 + 20% of 100 = Rs 120

$\therefore$ Marked Price $= 120 \times \dfrac{100}{100-4} = \dfrac{120 \times 100}{96} =$ Rs 125

$\therefore$ Marked price is $\dfrac{125-100}{100} \times 100 = 25\%$ more than the cost price.

Quicker Maths (Direct Formula) :

We may use the formula : $z = x - y - \dfrac{xy}{100}$

Where, z = % profit = 20%

x = marked % above CP = ?

y = discount = 4%

or, $20 = x - 4 - \dfrac{4x}{100}$

or, $x = \dfrac{24 \times 100}{96} = 25\%$

32. Try it. Follow the rule as in Q. 27

33. Let cost price = x

Then we have, $x\left(\dfrac{95}{100}\right)\left(\dfrac{110}{100}\right) = x\left(\dfrac{105}{100}\right) - 1$

or, $x = \dfrac{100 \times 100}{105 \times 100 - 95 \times 100} = 200$

$\therefore$ Cost price = Rs 200

Simple Interest

Interest is the money paid by the borrower to the lender for the use of money lent.

The sum lent is called the **principal. Interest** is usually calculated at the rate of so many rupees for every Rs 100 of the money lent for a year. This is called the **rate per cent per annum.**

'Per annum' means for a year. The words 'per annum' are sometimes omitted. Thus 6 p.c. means that Rs 6 is the interest on Rs 100 in one year.

The sum of the principal and interest is called the **amount.**

The interest is usually paid **yearly, half-yearly** or **quarterly** as agreed upon.

Interest is of two kinds, **Simple** and **Compound.** When interest is calculated on the original principal for any length of time it is called **simple interest**. Compound interest is defined in the next chapter.

To find Simple Interest multiply the principal by the number of years and by the rate per cent and divide the result by 100.

This may be remembered in the symbolic form

$$SI = \frac{p \times t \times r}{100}$$

Where I = interest, p = principal, t = number of years, r = rate %

Ex.1. Find the simple interest on Rs 400 for 5 years at 6 per cent.

Soln. $SI = \dfrac{400 \times 5 \times 6}{100} =$ Rs 120

Interest for a number of days

When the time is given in days or in years and days, 365 days are reckoned to a year. But when the time is given in months and days, 12 months are reckoned to a year and 30 days to the month. The day on which the money is paid back should be included but not the day on which it is borrowed, ie, in counting, the first day is omitted.

Ex.2: Find the simple interest on Rs 306. 25 from March 3rd to July 27th at $3\frac{3}{4}\%$ per annum.

Sol. Interest = Rs $306\frac{1}{4} \times \dfrac{146}{365} \times \dfrac{15}{4} \times \dfrac{1}{100}$

$= \text{Rs}\frac{1225}{4} \times \frac{2}{5} \times \frac{15}{4} \times \frac{1}{100} = \text{Rs}\frac{147}{32}$

$= \text{Rs } 4.59$ nearly Ans.

Note: 73, 146, 219 and 292 days are respectively $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}$ and $\frac{4}{5}$ of a year.

The interest I on principal P for d days at r p.c. is given by

$$I = P \times r \times \frac{d}{365} \times \frac{1}{100} = P \times d \times \frac{2r}{73000}$$

Hence we deduce the following rule for calculating the total interest on **different principals for different number of days**, the rate of interest being the same in each case.

Rule. *Multiply each principal by its own number of days, and find the sum of these products. Multiply this sum by twice the rate and divide the product by 73000.*

The four quantities involved in questions connected with interest are P, t, r and I. If any three of these be given, the fourth can be found.

To find principal

Since $I = \frac{Ptr}{100}$ $\therefore$ $\boxed{P = \frac{100\,I}{tr}}$

Ex.3: What sum of money will produce Rs 143 interest in $3\frac{1}{4}$ years at $2\frac{1}{2}$ p.c. simple interest ?

Sol. Let the required sum be Rs P. Then

$$\text{Rs } P = \text{Rs}\frac{100 \times 143}{3\frac{1}{4} \times 2\frac{1}{2}} = \text{Rs}\frac{100 \times 143 \times 4 \times 2}{13 \times 5} = \text{Rs } 1760. \text{ Ans.}$$

To find rate %

Since $I = \frac{Prt}{100}$ $\therefore$ $\boxed{r = \frac{100\,I}{Pt}}$

Ex.4: A sum of Rs 468.75 was lent out at simple interest and at the end of 1 year 8 months the total amount was Rs 500. Find the rate of interest per cent per annum.

Sol. Here P = Rs 468.75, $t = 1\frac{2}{3}$ or $\frac{5}{3}$ years.

I = Rs (500 - 468.75) = Rs 31.25.

$\therefore$ rate p.c. $= \frac{100 \times 31.25}{468.75 \times \frac{5}{3}} = 100 \times \frac{3125}{46875} \times \frac{3}{5} = 4$. Ans.

Ex. 5: A lent Rs 600 to B for 2 years, and Rs 150 to C for 4 years and received altogether from both Rs 90 as interest. Find the rate of interest, simple interest being calculated.

Sol. Rs 600 for 2 years = Rs 1200 for 1 year

and Rs 150 for 4 years = Rs 600 for 1 year.

Int. = Rs 90.

$\therefore$ Rate $= \frac{90 \times 100}{1800 \times 1} = 5\%$. Ans.

To find Time

Since $I = \frac{Ptr}{100}$. $\therefore$ $\boxed{t = \frac{100\,I}{Pr}}$

Ex.6: In what time will Rs 8500 amount to Rs 15767.50 at $4\frac{1}{2}$ per cent per annum ?

Sol. Here interest = Rs 15767.50 - Rs 8500 = Rs 7267.50.

$\therefore\ t = \frac{7267.50 \times 100}{8500 \times 4.5} = 19$ years. Ans.

Miscellaneous Examples

Ex.7: The simple interest on a sum of money is $\frac{1}{9}$ of the principal, and the number of years is equal to the rate per cent per annum. Find the rate per cent.

Soln: Let Principal = P, time = t years, rate = t

Then $\frac{Ptt}{100} = \frac{P}{9}$

$\therefore\ t^2 = \frac{100}{9}$, $\therefore\ t = \frac{10}{3} = 3\frac{1}{3}$ $\therefore$ rate $= 3\frac{1}{3}\%$

Direct formula :

Rate = time $= \sqrt{100 \times \frac{1}{9}} = \frac{10}{3} = 3\frac{1}{3}\%$.

Ex.8: What annual payment will discharge a debt of Rs 770 due in 5 years, the rate of interest being 5% per annum?

Soln: Let the annual payment be P rupees.

The amount of Rs P in 4 years at 5% $= \frac{100 + 4 \times 5P}{100} = \frac{120\ P}{100}$

" " " 3 yrs " " $= \frac{115\ P}{100}$

" " " 2 yrs " " $= \frac{110\ P}{100}$

" " " 1 yrs " " $= \frac{105\ P}{100}$

These four amounts together with the last annual payment of Rs P will discharge the debt of Rs 770.

$\therefore \frac{120\ P}{100} + \frac{115\ P}{100} + \frac{110\ P}{100} + \frac{105\ P}{100} + P = 770$

$\therefore \frac{550\ P}{100} = 770 \quad \therefore P = \frac{770 \times 100}{550} = 140$

Hence annual payment = Rs 140

Theorem : *The annual payment that will discharge a debt of Rs A due in t years at the rate of interest r% per annum is* $\frac{100A}{100t + \frac{rt(t-1)}{2}}$.

Proof : Let the annual payment be x rupees.

The amount of Rs x in (t - 1) yrs at r% $= \frac{100 + (t-1)r}{100} x$

The amount of Rs x in (t - 2) yrs at r% $= \frac{100 + (t-2)r}{100} x$

- -

- -

The amount of Rs x in 2 yrs at r% $= \frac{100 + 2r}{100} x$

The amount of Rs x in 1 year at r% $= \frac{100 + 1r}{100} x$

These (t - 1) amounts together with the last annual payment of Rs x will discharge the debt of Rs A.

$\therefore \frac{100 + (t-1)r}{100} x + \frac{100 + (t-2)r}{100} x \ldots + \frac{100 + r}{100} x + x = A$

or, $x [\{100 + (t-1) r\} + \{100 + (t-2)r\} + \ldots \{100 + r\} + \{100\}] = 100A$

or, $x \left[100t + \frac{r(t-1)(t)}{2}\right] = 100A \quad \therefore x = \frac{100A}{100t + \frac{r(t-1)(t)}{2}}$

Note : $1 + 2 + 3 + \ldots + m = \frac{m(m+1)}{2}$

Using the above theorem :

Annual payment $= \frac{100 \times 770}{5 \times 100 + \frac{5(4)(5)}{2}} = \frac{770 \times 100}{550} = $ Rs 140

Ex.9: What annual payment will discharge a debt of Rs 848 in 4 yrs at 4% per annum?

Soln : By the theorem :

Annual payment $= \frac{848 \times 100}{4 \times 100 + \frac{4(3)(4)}{2}} = $ Rs 200

Ex.10 : The annual payment of Rs 80 in 5 yrs at 5% per annum simple interest will discharge a debt of ... .

Soln : Putting the values in the above formula :

$80 = \frac{A \times 100}{5 \times 100 + \frac{5(4)(5)}{2}}$

or, $A = \frac{80 \times 550}{100} = $ Rs 440

Ex.11 : The rate of interest for the first 2 yrs is 3% per annum, for the next 3 years is 8% per annum and for the period beyond 5 years 10% per annum. If a man gets Rs.1520 as a simple interest for 6 years, how much money did he deposit ?

Soln: Let his deposit = Rs 100

Interest for first 2 yrs = Rs 6

Interest for next 3 yrs = Rs 24

Interest for the last year = Rs 10

Total interest = Rs 40

When interest is Rs 40, deposited amount is Rs 100

$\therefore$ when interest is Rs 1520, deposited amount

$= \frac{100}{40} \times 1520 = $ Rs 3800

Direct formula: Principal $= \frac{\text{Interest} \times 100}{t_1 r_1 + t_2 r_2}$

$$= \frac{1520 \times 100}{2 \times 3 + 3 \times 8 + 1 \times 10}$$

$$= \frac{1520 \times 100}{40} = \text{Rs } 3800$$

Ex.12: A sum of money doubles itself in 10 years at simple interest. What is the rate of interest?

Soln: Let the sum be Rs 100.

After 10 years it becomes Rs 200,

$\therefore$ Interest = 200 - 100 = 100

Then, rate $= \frac{100\ I}{P\ t} = \frac{100 \times 100}{100 \times 10} = 10\%$

Direct formula :

Time $\times$ Rate = 100 (Multiple number of principal -1)

or, Rate $= 100 \times \frac{\text{Multiple number of principal} - 1}{\text{time}}$

Using the above formula : rate $= \frac{100\ (2-1)}{10} = 10\%$

Ex.13: A sum of money trebles itself in 20 yrs at SI. Find the rate of interest.

Soln: Rate $= \frac{100\ (3-1)}{20} = 10\%$

Note: A generalised form can be shown as:

If a sum of money becomes 'x' times in 't' years at SI, the rate of interest is given by $\frac{100\ (x-1)}{t}\ \%$.

Ex.14: In what time does a sum of money become four times at the simple interest rate of 5% per annum ?

Soln: Using the above formula,

$$\text{Time} = \frac{100(\text{Multiple number of prinicipal} - 1)}{\text{Rate}}$$

$$= \frac{100\ (4-1)}{5} = 60 \text{ yrs}$$

Ex.15: Divide Rs 2379 into three parts so that their amounts after 2, 3 and 4 years respectively may be equal, the rate of interest being 5% per annum.

Soln: Amount of 1st part $= \frac{110}{100} \times$ 1st part

" " 2nd part $= \frac{115}{100} \times$ 2nd part

" " 3rd part $= \frac{120}{100} \times$ 3rd part

According to the question, these amounts are equal

$\therefore$ 110 $\times$ 1st part = 115 $\times$ 2nd part = 120 $\times$ 3rd part

$\therefore$ 1st part : 2nd part : 3rd part $= \frac{1}{110} : \frac{1}{115} : \frac{1}{120}$

= 276 : 264 : 253

Hence, dividing Rs 2379 into three parts in the ratio 276 : 264 : 253, we have 1st part = Rs 828, 2nd part = Rs 792, 3rd part = Rs 759.

Ex.16: A certain sum of money amounts to Rs 756 in 2 yrs and to Rs 873 in 3.5 yrs. Find the sum and the rate of interest.

Soln: P + SI for 3.5 yrs = Rs 873

P + SI for 2 yrs = Rs 756

On subtracting, SI for 1.5 yrs = Rs 117

Therefore, SI for 2 yrs = Rs $\frac{117}{1.5} \times 2$ = Rs 156

$\therefore$ P = 756 - 156 = Rs 600

and rate $= \frac{100 \times 156}{600 \times 2} = 13\%$ per annum.

Ex.17: A sum was put at SI at a certain rate for 2 yrs. Had it been put at 3 % higher rate, it would have fetched Rs 300 more. Find the sum.

Soln: Let the sum be Rs x and the original rate be y% per annum. Then, new rate = (y + 3)% per annum.

$$\therefore \frac{x\ (y+3) \times 2}{100} - \frac{x\ (y) \times 2}{100} = 300$$

xy + 3x - xy = 15,000 or, x = 5000

Thus, the sum = Rs 5000

Quicker Method: Direct Formula

$$\text{Sum} = \frac{\text{More Interest} \times 100}{\text{Time} \times \text{More rate}} = \frac{300 \times 100}{2 \times 3} = 5000$$

Ex. 18 : A sum of money doubles itself in 7 yrs. In how many years will it become fourfold?

Soln : Rate $= \frac{100(2-1)}{7} = \frac{100}{7}$ $\therefore$ Time $= \frac{100\ (4-1)}{\frac{100}{7}} = 21$ year

Other Method : This question can be solved without writing anythi
Think like.
Doubles in 7 years
Trebles in 14 years
4 times in 21 years
5 times in 28 years
and so on.

Ex. 19 : Rs 4000 is divided into two parts such that if one part be inves at 3% and the other at 5%, the annual interest from both investments is Rs 144. Find each part.

Soln : Let the amount lent at 3% rate be Rs x, then
3% of x + 5% of (4000 - x) = 144
or, $3x + 5 \times 4000 - 5x = 14400$
or, $2x = 5600$ $\therefore x = 2800$
Thus, the two amounts are Rs 2800 and Rs (4000 - 2800) or R 1200.

By the Method of Alligation : See example 25 in ALLIGATION chapter.

Ex.20 : At a certain rate of simple interest Rs 800 amounted to Rs 92 in 3 years. If the rate of interest be increased by 3%, what will the amount after 3 years ?

Soln : First rate of interest $= \frac{120 \times 100}{800 \times 3} = 5\%$
New rate = 5 + 3 = 8%
$\therefore$ New interest $= \frac{800 \times 3 \times 8}{100} =$ Rs 192
$\therefore$ New amount = 800 + 192 = Rs 992.

Ex. 21 : The simple interest on a sum of money will be Rs 300 after 5 years. In the next 5 yrs principal is trebled, what will be the tot interest at the end of the 10th year?

Soln : Simple interest for 5 years = Rs 300
Now, when principal is trebled, the simple interest for 5 year will also be treble the simple interest on original principal for th same period. Thus SI for last 5 years when principal is trebled
$= 3 \times 300 =$ Rs 900
$\therefore$ Total SI for 10 yrs = 300 + 900 = Rs 1200

Theorem : *A sum of Rs X is lent out in n parts in such a way that the interest on first part at r_1 % for t_1 yrs, the interest on second part at r_2 % for t_2 years the interest on third part at r_3 % for t_3 years, and so on, are equal, the ratio in which the sum was divided in n parts is given by* $\frac{1}{r_1 t_1} : \frac{1}{r_2 t_2} : \frac{1}{r_3 t_3} : \dots \frac{1}{r_n t_n}$.

Proof : Let the sum be divided into $S_1, S_2, \dots S_n$. Then,

$$S_1 = \frac{\text{Int} \times 100}{r_1 t_1}$$

$$S_2 = \frac{\text{Int} \times 100}{r_2 t_2}$$

$$S_3 = \frac{\text{Int} \times 100}{r_3 t_3}$$

$$S_n = \frac{\text{Int} \times 100}{r_n t_n}$$

[Since interests of all parts are equal]

$$\therefore S_1 : S_2 : S_3 : \dots : S_n = \frac{\text{Int} \times 100}{r_1 t_1} : \frac{\text{Int} \times 100}{r_2 t_2} : \frac{\text{Int} \times 100}{r_3 t_3} \dots : \frac{\text{Int} \times 100}{r_n t_n}$$

$$= \frac{1}{r_1 t_1} : \frac{1}{r_2 t_2} : \frac{1}{r_3 t_3} : \dots : \frac{1}{r_n t_n}$$

Ex. 22 : A sum of Rs 2600 is lent out in two parts in such a way that the interest on one part at 10% for 5 years is equal to that on another part at 9% for 6 years. Find the two sums.

Soln : Each Interest $= \frac{\text{1st Part} \times 5 \times 10}{100} = \frac{\text{2nd Part} \times 6 \times 9}{100}$

or, $\frac{\text{1st Part}}{\text{2nd Part}} = \frac{6 \times 9}{5 \times 10} = \frac{27}{25} = 27 : 25$

$\therefore$ 1st part $= \frac{2600}{27 + 25} \times 27 =$ Rs 1350

and 2nd part = 2600 - 1350 = Rs 1250

Note : If we use the above theorem, $S_1 : S_2 = \frac{1}{50} : \frac{1}{54} = 54 : 50 = 27:25$

Ex 23 : A certain sum of money amounted to Rs 575 at 5% in a time in which Rs 750 amounted to Rs 840 at 4%. If the rate of interest is simple, find the sum.

Soln : Interest = Rs 840 - Rs 750 = Rs 90

$\therefore$ Time $= \frac{90 \times 100}{750 \times 4} = 3$ yrs

Now, by the formula :

Sum $= \frac{100 \times \text{Amount}}{100 + rt} = \frac{100 \times 575}{100 + 3 \times 5} =$ Rs 500

Note : There is a direct relationship between the principal and the amount and is given by $\boxed{\text{Sum} = \frac{100 \times \text{Amount}}{100 + rt}}$

Ex. 24 : A certain sum of money amounts to Rs 2613 in 6 yrs at 5% per annum. In how many years will it amount to Rs 3015 at the same rate?

Soln : Use the formula ;

Principal $= \frac{100 \times \text{Amount}}{100 + rt} = \frac{100 \times 2613}{100 + 30} = \frac{100 \times 2613}{130} =$ Rs 2010

Again by using the same formula ;

$2010 = \frac{100 \times 3015}{100 + 5t}$

or, $100 + 5t = \frac{100 \times 3015}{2010}$

$\therefore t = \frac{1}{5}\left[\frac{100 \times 3015 - 100 \times 2010}{2010}\right]$

$= \frac{100 \times (3015 - 2010)}{2010 \times 5} = \frac{100 \times 1005}{2010 \times 5} = 10$ years.

Ex. 25 : A person lent a certain sum of money at 4% simple interest; and in 8 years the interest amounted to Rs 340 less than the sum lent. Find the sum lent.

Soln : Let the sum be Rs x.

$\therefore$ Interest $= \frac{x \times 8 \times 4}{100} = \frac{32x}{100}$

$x - \frac{32x}{100} = \frac{68x}{100}$

When interest is $\frac{68x}{100}$ less, the sum is Rs x.

$\therefore$ when interest is 340 less, the sum is $\frac{x}{68x} \times 100 \times 340 =$ Rs 500

Direct Formula : Sum $= \frac{100}{100 - 8 \times 4} \times 340 = \frac{100 \times 340}{68} =$ Rs 500

Ex. 26 : The simple interest on Rs 1650 will be less than the interest on Rs 1800 at 4% simple interest by Rs 30. Find the time.

Soln : We may consider that Rs (1800 - 1650) gives interest of Rs 30 at 4% per annum.

$\therefore$ Time $= \frac{30 \times 100}{150 \times 4} = 5$ yrs.

Ex. 27 : Arun and Ramu are friends. Arun borrowed a sum of Rs 400 at 5% per annum simple interest from Ramu. He returns the amount with interest after 2 yrs. Ramu returns to Arun 2% of the total amount returned. How much did Arun receive?

Soln : After 2 yrs amount returned to Ramu

$= 400 + \frac{400 \times 5 \times 2}{100} =$ Rs 440

Amount returned to Arun = 2% of Rs 440 = Rs 8.80

Ex. 28 : A man invests an amount of Rs 15,860 in the names of his three sons A, B, and C in such a way that they get the same amount after 2, 3, and 4 years respectively. If the rate of simple interest is 5% then find the ratio in which the amount was invested for A, B and C?

Theorem: *When different amounts mature to the same amount at simple rate of interest, the ratio of the amounts invested are in inverse ratio of (100 + time × rate). That is, the ratio in which the amounts are invested is* $\frac{1}{100 + r_1 t_1} : \frac{1}{100 + r_2 t_2} : \frac{1}{100 + r_3 t_3} : \dots : \frac{1}{100 + r_n t_n}$

Proof : We know that Sum $= \frac{100 \times \text{Amount}}{100 + rt}$

Let the sums invested be $S_1, S_2, S_3, \dots S_n$, at the rate of $r_1, r_2, r_3, \dots, r_n$ for the time $t_1, t_2, t_3, \dots t_n$ yrs respectively. Then $S_1 : S_2 : S_3 : \dots : S_n$

$= \frac{100 \times A}{100 + r_1 t_1} : \frac{100 \times A}{100 + r_2 t_2} : \frac{100 \times A}{100 + r_3 t_3} : \dots : \frac{100 \times A}{100 + r_n t_n}$

[Since the amount (A) is the same for all]

$= \frac{1}{100 + r_1 t_1} : \frac{1}{100 + r_2 t_2} : \frac{1}{100 + r_3 t_3} : \dots : \frac{1}{100 + r_n t_n}$

Soln : Therefore, the required ratio is this case is

$$\frac{1}{100+2\times5}:\frac{1}{100+3\times5}:\frac{1}{100+4\times5}=\frac{1}{110}:\frac{1}{115}:\frac{1}{120}$$

Ex. 29 : A sum of money doubles itself in 4 yrs at a simple interest. In how many yrs will it amount to 8 times itself?

Soln : Doubles in 4 yrs

3 times in $4 \times 2 = 8$ yrs.

4 times in $4 \times 3 = 12$ yrs.

8 times in $4 \times 7 = 28$ yrs.

Thus direct formula : x times in = No. of yrs to double (x - 1)

$\therefore$ 8 times in = 4 (8 - 1) = $4 \times 7 = 28$ yrs.

Ex. 30 : Two equal amounts of money are deposited in two banks each at 15% per annum for 3.5 yrs and 5 yrs respectively. If the difference between their interests is Rs 144, find each sum.

Soln : Let the sum be Rs x, then

$$\frac{x\times15\times5}{100}-\frac{x\times15\times7}{200}=144$$

or, $150x - 105x = 144 \times 200$ $\qquad \therefore x=\frac{144\times200}{45}=$ Rs 640

Direct formula : Two equal amounts of money are deposited at r_1 % and r_2 % for t_1 and t_2 yrs respectively. If the difference between their interest is I_d then $\text{Sum}=\frac{I_d\times100}{r_1t_1-r_2t_2}$

Thus, in this case: $\text{Sum}=\frac{144\times100}{15\times5-15\times3.5}=\frac{144\times100}{22.5}=$ Rs 640

Ex. 31 : The difference between the interest received from two different banks on Rs 500 for 2 yrs is Rs 2.5. Find the difference between their rates.

Soln : $I_1=\frac{500\times2\times r_1}{100}=10\,r_1$

$$I_2=\frac{500\times2\times r_2}{100}=10\,r_2$$

$$I_1-I_2=10r_1-10r_2=2.5$$

$$\text{or, } r_1-r_2=\frac{2.5}{10}=0.25\%$$

By Direct formula (Used in previous example) : When $t_1 = t_2$,

$$(r_1-r_2)=\frac{I_d\times100}{\text{Sum}\times t}=\frac{2.5\times100}{500\times2}=0.25\%$$

Ex. 32 : The simple interest on a certain sum of money at 4% per annum for 4 yrs is Rs 80 more than the interest on the same sum for 3 yrs at 5% per annum. Find the sum.

Soln : Let the sum be Rs x, then at 4% rate for 4 yrs the simple interest $=\frac{x\times4\times4}{100}=\text{Rs}\,\frac{4x}{25}$

At 5% rate for 3 yrs the simple interest $=\frac{x\times5\times3}{100}=\text{Rs}\,\frac{3x}{20}$

Now, we have, $\frac{4x}{25}-\frac{3x}{20}=80$

or, $\frac{16x-15x}{100}=80$

$\therefore$ x = Rs 8000

Quicker Method : For this type of question

$$\text{Sum}=\frac{\text{Difference}\times100}{|r_1t_1-r_2t_2|}=\frac{80\times100}{4\times4-3\times5}=\text{Rs } 8000$$

Ex. 33: A sum of money at simple interest amounts to Rs 600 in 4 years and Rs 650 in 6 years. Find the rate of interest per annum.

Soln : Suppose the rate of interest = r% and the sum = Rs A

Now, $A+\frac{A\times r\times4}{100}=600$; or, $A+\frac{Ar}{25}=600$

or, $A\left[1+\frac{r}{25}\right]=600$ ----- (1)

And, $A+\frac{A\times r\times6}{100}=650$; or, $A\left[1+\frac{3r}{50}\right]=650$ ----- (2)

Dividing (1) by (2), we have

$$\frac{1+\frac{r}{25}}{1+\frac{3r}{50}}=\frac{600}{650};\ \text{or, } \frac{(25+r)\times2}{50+3r}=\frac{12}{13}$$

or, $(50+2r)\times13=(50+3r)\times12$

or, $650+26r=600+36r$; or, $10r=50$ $\quad\therefore r=5\%$

Direct Formula : If a sum amounts to Rs A_1 in t_1 years and Rs A_2 in t_2 years at simple rate of interest,

then rate per annum $= \dfrac{100[A_2 - A_1]}{(A_1 t_2 - A_2 t_1)}$

In the above case,

$$r = \frac{100[650 - 600]}{6 \times 600 - 4 \times 650} = \frac{100 \times 50}{1000} = 5\%$$

Ex. 34: Ramesh borrows Rs 7000 from a bank at SI. After 3 yrs he pai Rs 3000 to the bank and at the end of 5 yrs from the date o borrowing he paid Rs 5450 to the bank to settle the account. Fin the rate of interest.

Soln : Any sum that is paid back to the bank before the last instalmen is deducted from the *principal* and not from the *interest.* Thus,
Total interest = Interest on Rs 7000 for 3 yrs + Interest on (Rs 7000 Rs 3000 =) Rs 4000 for 2 yrs.

Or, $(5450 + 3000 - 7000) = \dfrac{7000 \times 3 \times r}{100} + \dfrac{4000 \times 2 \times r}{100}$

or, $1450 = 210r + 80r \qquad \therefore r = \dfrac{1450}{290} = 5\%.$

Ex. 35. Some amount out of Rs 7000 was lent at 6% per annum and the remaining at 4% per annum. If the total simple interest from both the fractions in 5 yrs was Rs 1600, find the sum lent at 6% per annum.

Soln : Suppose Rs x was lent at 6% per annum.

Thus, $\dfrac{x \times 6 \times 5}{100} + \dfrac{(7000 - x) \times 4 \times 5}{100} = 1600$

or, $\dfrac{3x}{10} + \dfrac{7000 - x}{5} = 1600$

or, $\dfrac{3x + 14,000 - 2x}{10} = 1600 \quad \therefore x = 16000 - 14000 = \text{Rs } 2000$

By Method of Alligation: Overall rate of interest $= \dfrac{1600 \times 100}{5 \times 7000} = \dfrac{32}{7}\%$

6%   4%

$\frac{32}{7}\%$

$\frac{4}{7}\%$ $\frac{10}{7}\%$

$\therefore$ ratio of two amounts = 2 : 5

$\therefore$ amount lent at 6% $= \dfrac{7000}{7} \times 2 = \text{Rs } 2000$

EXERCISES

1. The simple interest on a certain sum for 3 years at 14% per annum is Rs. 235.20.The sum is ____.
2. If Rs. 64 amounts to Rs. 83.20 in 2 years, what will Rs. 86 amount to in 4 years at the same rate per cent per annum ?
3. A sum of money amounts to Rs. 850 in 3 years and Rs. 925 in 4 years. What is the sum?
4. A sum amounts to Rs.702 in 2 years and Rs. 783 in 3 years. The rate per cent is ______.
5. A money-lender finds that due to a fall in the rate of interest from 13% to $12\frac{1}{2}\%$, his yearly income diminishes by Rs. 104. What is his capital?
6. If the amount of Rs. 360 in 3 years is Rs. 511.20, what will be the amount of Rs. 700 in 5 years?
7. A sum of Rs. 2540 is lent out in two parts, one at 12% and the other at $12\frac{1}{2}\%$. If the total annual income is Rs. 312.42 the money lent at 12% is ______.
8. A sum of Rs. 2600 is lent out in two parts in such a way that the interest on one part at 10% for 5 years is equal to that on the other part at 9% for 6 years. The sum lent out at 10% is ________.
9. The simple interest on a sum of money is $\frac{1}{16}$ of the principal and the number of years is equal to the rate per cent per annum.The rate per cent per annum is ________.
10. The simple interest on a certain sum at a certain rate is $\frac{9}{16}$ of the sum. If the number representing rate per cent and time in years be equal, then the time is ________.
11. A sum of money will double itself in 16 years at simple interest at an yearly rate of ________.
12. A sum of money put at simple interest trebles itself in 15 years.The rate per cent per annum is ________.
13. At a certain rate of simple interest, a certain sum doubles itself in 10 years. It will treble itself in ________.
14. Rs. 800 amounts to Rs. 920 in 3 years at simple interest. If the interest rate is increased by 3%, to how much would it amount?

15. A lent Rs. 600 to B for 2 years and Rs. 150 to C for 4 years and received altogether from both Rs. 90 as simple interest. The rate of interest is ________.

16. A man invested $\frac{1}{3}$ of his capital at 7% , $\frac{1}{4}$ at 8% and the remainder at 10% . If his annual income is Rs. 561, the capital is ________.

17. The simple interest at x% for x years will be Rs. x on a sum of ______.

18. If the interest on Rs. 1200 be more than the interest on Rs. 1000 by Rs. 50 in 3 years, the rate per cent is _______.

19. A sum was put at simple interest at a certain rate for 2 years. Had it been put at 1% higher rate, it would have fetched Rs.24 more. The sum is __________.

20. A sum of money becomes 8/5 of itself in 5 years at a certain rate of interest. The rate per cent per annum is _________.

21. A man lends Rs. 10000 in four parts. If he gets 8% on Rs. 2000, $7\frac{1}{2}$% on Rs. 4000 and $8\frac{1}{2}$% on Rs. 1400, what per cent must he get for the remainder, if the average interest is 8.13%?

22. The simple interest on a sum of money at 8% per annum for 6 years is half the sum.The sum is ____.

23. The difference between the interest received from two different banks on Rs. 500 for 2 years is Rs. 2.50. The difference between their rates is ________.

24. Two equal amounts of money are deposited in two banks, each at 15% per annum, for $3\frac{1}{2}$ years and 5 years respectively. If the difference between their interests is Rs. 144,each sum is _________.
(a) Rs. 460 (b) Rs. 500 (c) Rs. 640 (d) Rs. 720

25. The rate of interest on a sum of money is 4% per annum for the first 2 years, 6% per annum for the next 4 years and 8% per annum for the period beyond 6 years. If the simple interest accrued by the sum for a total period of 9 years is Rs.1120.what is the sum?

Solutions

1. Principal = $\frac{235.20 \times 100}{3 \times 14}$ = Rs 560

2. Rate of interest = $\frac{19.20 \times 100}{64 \times 2}$ = 15%

Interest on Rs 86 = $\frac{86 \times 15 \times 4}{100}$ = Rs 51.6

$\therefore$ Amount = 86 + 51.6 = Rs 137.6

Quicker Maths:

Interest on Rs 64 for 2yrs is Rs 19.2. Hence by **Rule of fraction,** interest on Rs 86 for 4 yrs is 19.2 $\left(\frac{86}{64}\right)\left(\frac{4}{2}\right)$ = Rs 51.6

$\therefore$ Amount = 86 + 51.6 = 137.6

3. P + SI for 4 yrs = Rs 925
P + SI for 3 yrs = Rs 850
On subtracting, SI for 1 yr = Rs 75
$\therefore$ SI for 3 yrs = 3 × 75 = Rs 225
$\therefore$ P = 850 - 225 = Rs 625

4. Follow the same as in Q. 3. You get the sum, interest and time. Find the rate of interest.

5. Re $\frac{1}{2}$ decreases on Rs 100

$\therefore$ Rs 104 decreases on Rs $\frac{100}{\frac{1}{2}}$ × 104 = 100 × 2 × 104 = Rs 20800

6. Same as Q. 2.

7. Solve it by the method of alligation.

Overall rate of interest = $\frac{312.42}{2540}$ × 100 = 12.3%

Now,

I part		II part
12%		12.5%
	12.3%	
0.2		0.3

Therefore, the sum will be divided into the ratio of 0.2 : 0.3 or 2 : 3

Then sum lent at 12% = $\frac{2540}{5}$ × 2 = Rs 1016

and sum lent at $12\frac{1}{2}$% = $\frac{2540}{5}$ × 3 = Rs 1524

8. **Quicker Method:**

Ratio of two parts = $r_2 t_2 : r_1 t_1$ = 54 : 50 = 27 : 25

$\therefore$ Sum lent out at 10% = $\frac{2600}{52}$ × 27 = Rs 1350

9. Let the rate of interest = r%

$\therefore$ times = r yrs.

Now, $\frac{S}{16} = \frac{S \times r \times r}{100}$

or, $r^2 = \frac{100}{16}$ $\therefore$ $r = \frac{25}{4} = 6\frac{1}{4}\%$

10. Same as Q. 9

11. **Quicker Maths:**

Rate of interest $= \frac{100\,(2-1)}{16} = 12\frac{1}{2}\%$

12. **Quicker Maths:**

Rate of interest $= \frac{100\,(3-1)}{15} = \frac{200}{15} = 13\frac{1}{3}\%$

13. It doubles in 10 yrs.
Then trebles in 20 yrs.

14. Rate of interest $= \frac{120 \times 100}{800 \times 3} = 5\%$

Now, the new rate becomes 8%. Then interest

$= \frac{800 \times 8 \times 3}{100} = \text{Rs } 192$

$\therefore$ Amount = 800 + 192 = Rs 992.

Quicker Method:

Increase in interest $= \frac{800 \times 3 \times 3}{100} = \text{Rs } 72$

$\therefore$ Increased amount = 920 + 72 = Rs 992

15. Let the rate of interest be r% per annum.

Then $\frac{600 \times 2 \times r}{100} + \frac{150 \times 4 \times r}{100} = 90$

or, 12r + 6r = 90

$\therefore$ r = 5%

Note: Solve it by method of 'Alligation'

16. Let the capital be Rs 120

Then total interest $= 7\% \text{ of } \frac{120}{3} + 8\% \text{ of } \frac{120}{4} + 10\% \text{ of remainder}$

$= 7\% \text{ of } 40 + 8\% \text{ of } 30 + 10\% \text{ of } 50$

$= 2.8 + 2.4 + 5 = \text{Rs } 10.2$

$\therefore$ actual capital $= \frac{120}{10.2} \times 561 = 6600$

Quicker Method:

Capital $= \frac{561}{7\% \text{ of } \frac{1}{3} + 8\% \text{ of } \frac{1}{4} + 10\% \text{ of } \left(\frac{5}{12}\right)}$

$= \frac{561 \times 100}{\frac{7}{3} + \frac{8}{4} + \frac{25}{6}} = \frac{561 \times 100 \times 12}{28 + 24 + 50} = \text{Rs } 6600$

17. Sum $= \frac{x \times 100}{x \times x} = \text{Rs } \frac{100}{x}$

18. Let the rate of interest be x%

Then, $\frac{1200 \times 3x}{100} = \frac{1000 \times 3x}{100} + 50$

$\therefore$ 6x = 50 $\therefore$ $x = 8\frac{1}{3}\%$

Quicker Maths:

Rate $= \frac{\text{Difference in Interest} \times 100}{\text{Time (Difference in Principal)}}$

$= \frac{50 \times 100}{3\,(1200 - 1000)} = \frac{25}{3} = 8\frac{1}{3}\%$

Note : The above-used formula is simply based on

Rate $= \frac{\text{Interest} \times 100}{\text{Time} \times \text{Principals}}$

19. Let the sum be Rs P and rate be r%. Then

$\frac{P \times 2 \times r}{100} = \frac{P\,(r+1) \times 2}{100} - 24$

or, 2Pr = 2Pr + 2P - 2400

or, 2P = 2400

$\therefore$ P = 1200

Thus the sum is Rs 1200

Quicker Maths:

Sum $= \frac{\text{Difference in Interests} \times 100}{\text{Times} \times \text{Difference in rate}} = \frac{24 \times 100}{2 \times 1} = \text{Rs } 1200$

Note: The above formula is simply based on Sum $= \frac{\text{Int.} \times 100}{\text{Time} \times \text{Rate}}$

20. **Quicker Method:**

Rate $= 100\,\frac{\left(\frac{8}{5} - 1\right)}{\text{Time}} = \frac{100 \times \frac{3}{5}}{5} = \frac{300}{25} = 12\%$

21. Total Interest = 8.13% of 10000 = Rs 813

Remainder money = 10000 - (2000 + 4000 + 1400) = 2600

Then, 8% of 2000 + 7.5% of 4000 + 8.5% of 1400 + x% of 2600 = 813

or, 160 + 300 + 119 + 26x = 813

$$\therefore x = \frac{234}{26} = 9\%$$

22. $$\frac{S}{2} = \frac{S \times 8 \times 6}{100}$$

We can't find the value of S. We also see that the above relationship is not correct. Thus we conclude that the question is wrong.

23. **Quicker Maths:** Use the formula given in Q 19.

$$\text{Sum} = \frac{\text{Difference in Interests} \times 100}{\text{Times} \times \text{Difference in rates}}$$

$$\text{or, } 500 = \frac{2.5 \times 100}{2 \times x} \quad \therefore \; x = \frac{2.5 \times 100}{2 \times 500} = 0.25\%$$

24. **Quicker Maths:**

$$\text{Sum} = \frac{\text{Difference in Interests} \times 100}{\text{Rate} \times \text{Difference in times}} = \frac{144 \times 100}{15 \times 1.5} = \text{Rs } 640$$

25. **Quicker Maths:**

$$\text{Sum} = \frac{\text{Interest} \times 100}{r_1t_1 + r_2t_2 + r_3t_3 + \ldots\ldots}$$

$$= \frac{1120 \times 100}{4 \times 2 + 6 \times 4 + 8 \times 3} = \frac{1120 \times 100}{56} = \text{Rs } 2000$$

Compound Interest

Money is said to be lent at Compound Interest (CI) when at the end of a year or other fixed period the interest that has become due is not paid to the lender, but is added to the sum lent, and the amount thus obtained becomes the principal for the next period. The process is repeated until the amount for the last period has been found. The difference between the original principal and the final amount is called **Compound Interest (CI).**

Important Formulae

Let Principal = Rs P, Time = t yrs and Rate = r% per annum

Case I: When interest is compounded annually :

$$\text{Amount} = P\left[1 + \frac{r}{100}\right]^{t}$$

Case II: When interest is compounded half-yearly.

$$\text{Amount} = P\left[1 + \frac{\frac{r}{2}}{100}\right]^{2t} = P\left[1 + \frac{r}{200}\right]^{2t}$$

Case III: When interest is compounded quarterly :

$$\text{Amount} = P\left[1 + \frac{\frac{r}{4}}{100}\right]^{4t} = P\left[1 + \frac{r}{400}\right]^{4t}$$

Case IV: When rate of interest is r_1%, r_2% and r_3% for 1st year, 2nd year and 3rd year respectively,

$$\text{then Amount} = P\left[1 + \frac{r_1}{100}\right] \times \left[1 + \frac{r_2}{100}\right] \times \left[1 + \frac{r_3}{100}\right]$$

The above-mentioned formulae are not new for you. We think that all of you know their uses. When dealing with the above formulae, some mathematical calculations become lengthy and take more time. To simplify the calculations and save the valuable time we are giving some extra informations. Study the following sections carefully and apply them during your calculations.

The problems are generally asked upto the period of 3 ye
and the rates of interest are 10%, 5% and 4%.
We have the basic formula :

$$\text{Amount} = \text{Principal}\left(1+\frac{\text{rate}}{100}\right)^{\text{time}}$$

If the principal is Re 1, the amount for first, second and third yea
will be

$\left(1+\frac{r}{100}\right)$, $\left(1+\frac{r}{100}\right)^2$ and $\left(1+\frac{r}{100}\right)^3$ respectively.

And if the rate of interest is 10%, 5% and 4%, these values will be

$$\left(\frac{11}{10}\right), \left(\frac{11}{10}\right)^2, \left(\frac{11}{10}\right)^3$$

$$\left(\frac{21}{20}\right), \left(\frac{21}{20}\right)^2, \left(\frac{21}{20}\right)^3$$

$$\left(\frac{26}{25}\right), \left(\frac{26}{25}\right)^2, \left(\frac{26}{25}\right)^3$$

The above informations can be put in the tabular form as given below
Principal = Re 1, then CI:

Time	1Year	2 Years	3 Years
r	$\left(1+\frac{r}{100}\right)$	$\left(1+\frac{r}{100}\right)^2$	$\left(1+\frac{r}{100}\right)^3$
10	$\frac{11}{10}$	$\frac{121}{100}$	$\frac{1331}{1000}$
5	$\frac{21}{20}$	$\frac{441}{400}$	$\frac{9261}{8000}$
4	$\frac{26}{25}$	$\frac{676}{625}$	$\frac{17576}{15625}$

The above table should be remembered. The use of the above
table can be seen in the following examples.

Ex.: Rs. 7500 is borrowed at CI at the rate of 4% per annum. What will be the amount to be paid after 2 yrs ?

Soln : As the rate of interest is 4% per annum and the time is 2 yrs, our concerned fraction would be $\frac{676}{625}$. From the above table, you know that Re 1 becomes Rs $\frac{676}{625}$ at 4% per annum after 2 yrs. So after 2 yrs Rs 7500 will produce $7500 \times \frac{676}{625} = \text{Rs } 8112$.

Miscellaneous Examples

To find time

Ex. 1. In what time will Rs 390625 amount to Rs 456976 at 4% compound interest ?

Soln : $\therefore\ P\left(1+\frac{r}{100}\right)^t = A$

$$\therefore\ 390625\left(1+\frac{4}{100}\right)^t = 456976$$

$$\therefore\ \left(1+\frac{1}{25}\right)^t = \frac{456976}{390625} \qquad \therefore\ \left(\frac{26}{25}\right)^t = \left(\frac{26}{25}\right)^4$$

$\therefore\ t = 4$ $\quad\therefore$ the required time is 4 years.

Ex. 2. A sum of money placed at compound interest doubles itself in 4 yrs. In how many years will it amount to eight times itself ?

Soln : We have $P\left(1+\frac{r}{100}\right)^4 = 2P \qquad \therefore\ \left(1+\frac{r}{100}\right)^4 = 2$

Cubing both sides, we get

$$\left(1+\frac{r}{100}\right)^{12} = 2^3 = 8 \qquad \text{or,} \quad P\left(1+\frac{r}{100}\right)^{12} = 8P$$

Hence the required time is 12 yrs.

Quicker Approach :

x becomes 2x in 4 yrs.
2x becomes 4x in next 4 yrs.

4x becomes 8x in yet another 4 yrs.

Thus, x becomes 8x in 4 + 4 + 4 = 12 yrs.

Ex. 3: Find the least number of complete years in which a sum of mone at 20% CI will be more than doubled.

Soln. We have, $P\left(1+\frac{20}{100}\right)^t > 2P \quad \therefore \left(\frac{6}{5}\right)^t > 2$

By trial, $\frac{6}{5}\times\frac{6}{5}\times\frac{6}{5}\times\frac{6}{5} > 2$

$\therefore$ the required time is 4 yrs.

Ex. 4: A sum of money at compound interest amounts to thrice itself in three years. In how many years will it be 9 times itself?

Soln: Detail Method:

Suppose the sum = Rs x

Then, we have

$$3x = x\left(1+\frac{r}{100}\right)^3 \quad \text{or } 3 = \left(1+\frac{r}{100}\right)^3$$

Squaring both sides

$$(3)^2 = \left\{\left(1+\frac{r}{100}\right)^3\right\}^2 \quad \text{or } 9 = \left(1+\frac{r}{100}\right)^6$$

Now multiply both sides by x; then $9x = x\left(1+\frac{r}{100}\right)^6$

$\therefore$ the sum x will be 9 times in 6 years.

Quicker Method: Remember the following conclusion:

If a sum becomes x *times in* y *years at CI then it will be* $(x)^n$ *times in* ny *years.*

Thus, if a sum becomes 3 times in 3 years it will be $(3)^2$ times in $2 \times 3 = 6$ years

Sample Questions:

1. If a sum deposited at compound interest becomes double in 4 years when will it be 4 times at the same rate of interest?

Soln: Using the above conclusion, we say that the sum will be $(2)^2$ times in $2 \times 4 = 8$ years.

2. In the above question, when will the sum be 16 times?

Soln: $(2)^4 = 16$ times in $4 \times 4 = 16$ years.

To find rate

Ex. 5: At what rate per cent compound interest does a sum of money become nine-fold in 2 years?

Soln: Detail method: Let the sum be Rs x and the rate of compound interest be r% per annum; then

$$9x = x\left(1+\frac{r}{100}\right)^2 \quad \text{or, } 9 = \left(1+\frac{r}{100}\right)^2$$

$$\text{or, } 3 = 1+\frac{r}{100}; \quad \text{or, } \frac{r}{100} = 2 \quad \therefore \; r = 200\%$$

Direct Formula: The general formula of comound interest can be changed to the following form:

If a certain sum becomes 'm' times in 't' years, the rate of compound interest r is equal to $100\left[(m)^{1/t} - 1\right]$

In this case, $r = 100\left[(9)^{1/2} - 1\right] = 100\,(3 - 1) = 200\%$

Ex. 6: At what rate percentage (compound interest) will a sum of money become eight times in three years?

Soln : By Direct Formula :

$$\text{Rate \%} = \left[(8)^{1/t} - 1\right] \times 100$$

$$= \left[(8)^{1/3} - 1\right] \times 100 = (2 - 1) \times 100 = 100\%$$

Ex. 7: At what rate per cent compounded yearly will Rs 80,000 amount to Rs 88,200 in 2 yrs ?

Soln : We have $80{,}000\left(1+\frac{r}{100}\right)^2 = 88{,}200$

$$\text{or, } \left(1+\frac{r}{100}\right)^2 = \frac{88{,}200}{80{,}000} = \frac{441}{400} = \left(\frac{21}{20}\right)^2$$

$$\text{or, } \quad 1+\frac{r}{100} = \frac{21}{20} \quad \therefore \; r = 5\%$$

Given CI, to find SI and vice versa

Ex. 8: If the CI on a certain sum for 2 yrs at 3% be Rs 101.50, what would be the SI ?

Soln : CI on 1 rupee $= \left(1+\frac{3}{100}\right)^2 - 1 = \left(\frac{103}{100}\right)^2 - 1 = \text{Re } \frac{609}{10000}$

SI on 1 rupee $= \text{Re } \frac{2 \times 3}{100} = \text{Re } \frac{6}{100}$

$\therefore \frac{SI}{CI} = \frac{6}{100} \times \frac{10000}{609} = \frac{200}{203}$

$\therefore SI = \frac{200}{203}$ of $CI = \frac{200}{203} \times 101.5 = $ Rs 100

Note: If you don't want to go through the details of the above method, remember the following direct formula and get the answer quickly.

$$\text{Simple Interest} = \frac{rt}{100\left[\left(1+\frac{r}{100}\right)^t - 1\right]} \times \text{Compound Interest}$$

Given CI and SI, to find sum and rate

Ex. 9: The compound interest on a certain sum for 2 yrs is Rs 40.80 and simple interest is Rs 40.00. Find the rate of interest per annum and the sum.

Soln : A little reflection will show that the difference between the simple and compound interests for 2 yrs is the interest on the first year's interest.

First year's SI = Rs $\frac{40}{2}$ = Rs 20

CI - SI = Rs 40.8 - Rs 40 = Re 0.80

Interest on Rs 20 for 1 year = Re 0.80

$\therefore$ Interest on Rs 100 for 1 yr = Rs $\frac{80 \times 100}{100 \times 20}$ = Rs 4

$\therefore$ rate = 4%

Now, principal P is given by

$P = \frac{100 \times I}{tr} = \frac{100 \times 40}{2 \times 4} = $ Rs 500

Quicker Method (Direct Formula) [For 2 yrs only]:

$$\text{Rate} = \frac{2 \times \text{Difference in CI and SI}}{\text{SI}} \times 100$$

Thus, in this case, rate $= \frac{2 \times 0.8}{40} \times 100 = 4\%$

And Sum $= \frac{40 \times 100}{4 \times 2} = $ Rs 500

Division of sum

Ex. 10: Divide Rs 3903 between A and B, so that A's share at the end of 7 yrs may equal B's share at the end of 9 yrs, compound interest being at 4%.

Soln : We have, (A's share at present)$\left(1+\frac{4}{100}\right)^7$

= (B's share at present)$\left(1+\frac{4}{100}\right)^9$

$\therefore \frac{\text{A's share at present}}{\text{B's share at present}} = \left(1+\frac{4}{100}\right)^2 = \left(\frac{26}{25}\right)^2 = \frac{676}{625}$

Dividing Rs 3903 in the ratio 676 : 625

A's present share $= \frac{676}{676+625} \times 3903 = $ Rs 2028

B's present share = Rs 3903 - Rs 2028 = Rs 1875

When Difference Between SI and CI is given

Theorem : *When difference between the compound interest and simple interest on a certain sum of money for 2 years at r% rate is Rs x, then the sum is given by :*

$$\text{Sum} = \frac{\text{Difference} \times 100 \times 100}{\text{Rate} \times \text{Rate}} = \frac{x(100)^2}{r^2} = x\left(\frac{100}{r}\right)^2$$

And when sum is given and difference between SI and CI is asked, then

$$\text{Difference} = \text{Sum}\left(\frac{r}{100}\right)^2$$

Proof : Let the sum be Rs A.

Then $SI = \frac{A \times 2 \times r}{100} = \frac{2Ar}{100}$

$CI = A\left[1+\frac{r}{100}\right]^2 - A$

$= A\left[1 + \frac{r^2}{100^2} + \frac{2r}{100}\right] - A$

$= A + \frac{Ar^2}{100^2} + \frac{2Ar}{100} - A = \frac{Ar^2}{100^2} + \frac{2Ar}{100}$

$$\text{Now, CI - SI} = \frac{Ar^2}{100^2} + \frac{2Ar}{100} - \frac{2Ar}{100} = A\left(\frac{r}{100}\right)^2$$

$$\therefore A = \text{Difference}\left(\frac{100}{r}\right)^2$$

Ex. 11: The difference between the compound interest and the simple interest on a certain sum of money at 5% per annum for 2 years is Rs 1.50. Find the sum.

Soln : Using the above theorem :

$$\text{Sum} = 1.5\left(\frac{100}{5}\right)^2 = 1.5 \times 400 = \text{Rs } 600$$

Ex. 12: Find the difference between the compound interest and the simple interest for the sum Rs 1500 at 10% per annum for 2 years.

Soln : $\text{Difference} = \text{Sum}\left(\frac{r}{100}\right)^2 = 1500\left(\frac{10}{100}\right)^2 = \text{Rs } 15$

Ex 13 : The difference between the simple and the compound interests on a certain sum of money for 2yrs at 4% per annum is Re 1. Find the sum.

Soln : $\text{Sum} = \text{difference}\left(\frac{100}{r}\right)^2 = 1\left(\frac{100}{4}\right)^2 = \text{Rs } 625$

Theorem : *If the diffrerence between CI and SI on a certain sum for 3 years at r% is Rs x, the sum will be* $\frac{\text{Difference} \times (100)^3}{r^2(300+r)}$ *and if the sum is given and the difference is asked, then*

$$\textit{Difference} = \frac{Sr^2(300+r)}{(100)^3}$$

Proof : Let the sum be Rs S.

$$\text{Then SI} = \frac{S \times 3 \times r}{100} = \frac{3Sr}{100}$$

$$\text{CI} = S\left[1 + \frac{r}{100}\right]^3 - S$$

$$= S\left[\left(1 + \frac{r}{100}\right)^3 - 1\right]$$

$$= S\left[1 + \frac{r^3}{(100)^3} + \frac{3r}{100} + \frac{3r^2}{100^2} - 1\right]$$

$$= S\left[\frac{r^3}{100^3} + \frac{3r^2}{100^2} + \frac{3r}{100}\right]$$

$$\text{Now, CI - SI} = S\left[\frac{r^3}{100^3} + \frac{3r^2}{100^2} + \frac{3r}{100}\right] - \frac{3Sr}{100}$$

$$= S\left[\frac{r^3}{100^3} + \frac{3r^2}{100^2}\right]$$

$$\text{or, Difference} = \frac{Sr^2}{100^2}\left[\frac{r}{100} + 3\right]$$

$$= \frac{Sr^2(300+r)}{(100)^3}$$

$$\therefore S = \frac{\text{Difference } (100)^3}{r^2(300+r)}$$

Ex 14 : If the difference between CI and SI on a certain sum of money for 3yrs at 5% p.a. is Rs 122, find the sum.

Soln : By the above theorem :

$$\text{Sum} = \frac{122 \times 100 \times 100 \times 100}{5^2(300+5)} = \text{Rs } 16{,}000$$

Ex 15: Find the difference between CI and SI on Rs 8000 for 3 yrs at 2.5% p.a.

Soln : $\text{Difference} = \frac{\text{Sum} \times r^2(300+r)}{(100)^3}$

$$= \frac{8000 \times 2.5 \times 2.5\,(300+2.5)}{100 \times 100 \times 100}$$

$$= \frac{8 \times 25 \times 25 \times 3025}{100 \times 100 \times 100} = \frac{121}{8} = \text{Rs } 15.125$$

Ex 16 : Find the difference between CI and SI on Rs 2000 for 3 yrs at 5% p.a.

Soln : Difference $= \frac{2000 \times 5 \times 5(305)}{100 \times 100 \times 100} = $ Rs 15.25

Ex 17: The simple interest on a sum at 4% per annum for 2 yrs is Rs 80. Find the compound interest on the same sum for the same period.

Soln : Recall the formula used in Ex 6.

$$\text{Rate} = \frac{2 \times \text{Diff in CI \& SI}}{\text{SI}} \times 100$$

or, $\text{Diff in CI and SI} = \frac{\text{Rate} \times \text{SI}}{2 \times 100}$

$\therefore$ Difference in CI and SI $= \frac{4 \times 80}{2 \times 100} = 1.6$

$\therefore$ CI = 80 + 1.6 = Rs 81.6

Other approach : If you don't remember the formula, then understand the following conclusion : "Compound interest differs from simple interest for 2 yrs only because under compound interest, simple interest over first year's simple interest is also included."

Simple interest for 2 yrs = Rs 80

$\therefore$ Simple interest for 1 yr = Rs 80 ÷ 2 = Rs 40

$\therefore$ CI for 2yrs = SI for 2 yrs + SI over SI of first year = $80 + \frac{40 \times 4 \times 1}{100} = 80 + 1.6 = $ Rs 81.6

If you have understood the above conclusion, you may write the direct solution as

$$\text{CI} = 80 + \frac{80}{2} \times \frac{4}{100} = 80 + 1.6 = \text{Rs } 81.6$$

Ex 18: The compound interest on a certain sum of money for 2 years at 10% per annum is Rs 420. Find the simple interest at the same rate and for the same time.

Soln : By the formula given in Ex 8. we have

$$\text{SI} = \frac{rt}{100\left[\left(1+\frac{r}{100}\right)^t - 1\right]} \times \text{CI}$$

When t = 2,

$$\text{SI} = \frac{2r}{100\left[1+\frac{r^2}{100^2}+\frac{2r}{100}-1\right]} \times \text{CI} = \frac{2r \times \text{CI} \times 100}{r^2 + 200r}$$

$$\text{SI} = \frac{200r}{r(r+200)} \times \text{CI}$$

Now, it is easy to use this form of the above formula. Therefore,

$$\text{SI} = \frac{200 \times 10 \times 420}{10 \times 210} = \text{Rs } 400$$

Ex 19: If the compound interest on a certain sum of money for 2 years at 5% is Rs 246, find the simple interest at the same rate for the same time.

Soln : Using the above formula :

$$\text{SI} = \frac{200 \times 5 \times 246}{5(205)} = \text{Rs } 240$$

Note : (1) When CI is given and SI is asked then we apply the above used formulae in Ex 18 and Ex 19

(2) But when SI is given and CI is asked we use the method as used in Ex 17.

Ex 20: If the simple interest on a certain sum of money for 3 yrs at 5% is Rs 150, find the corresponding CI.

Soln : Whenever the relationship between CI and SI is asked for 3 yrs of time, we use the formula :

$$\text{SI} = \frac{rt}{100\left[\left(1+\frac{r}{100}\right)^t - 1\right]} \times \text{CI}$$

$$150 = \frac{5 \times 3}{100\left[\left(1+\frac{r}{100}\right)^3 - 1\right]} \times \text{CI}$$

$$\therefore \text{CI} = \frac{150 \times 100\left[\frac{9261 - 8000}{8000}\right]}{5 \times 3}$$

$$= \frac{150 \times 100 \times 1261}{5 \times 3 \times 8000} = \frac{1261}{8} = \text{Rs } 157.62$$

Ex 21: The CI on a certain sum is Rs 104 for 2 yrs and SI is Rs 100. What is the rate per cent ?

Soln : Difference in CI and SI = 104 - 100 = Rs 4.

Therefore by using the formula (used in Ex 5 & Ex 14)

$$\text{Rate} = \frac{2 \times \text{Diff} \times 100}{\text{SI}} = \frac{2 \times 4 \times 100}{100} = 8\%$$

Ex 22: An amount of money grows upto Rs 4840 in 2 yrs and upto Rs 5324 in 3 yrs on compound interest. Find the rate per cent

Soln : We have,

P + CI of 3 yrs = Rs 5324 ---- (1)

P + CI of 2 yrs = Rs 4840 ---- (2)

Subtracting (2) from (1), we get

CI of 3rd year = 5324 - 4840 = Rs 484.

Thus, the CI calculated in the third year which is Rs 484 basically the amount of interest on the amount generated aft years which is Rs 4840.

$$\therefore r = \frac{484 \times 100}{4840 \times 1} = 10\%$$

Quicker Method (Direct Formula) :

$$\text{Rate} = \frac{\text{Difference of amount after n yrs and (n + 1) yrs} \times 100}{\text{Amount after n yrs}}$$

In this case, n = 2.

$$\therefore \text{rate} = \frac{\text{Difference of amount after 2 yrs and 3 yrs} \times 100}{\text{Amount after 2 yrs}}$$

$$= \frac{(5324 - 4840)}{4840} \times 100 = \frac{484 \times 100}{4840} = 10\%$$

Note : The above generalised formula can be used for any positive val of n. See in the following example.

Ex 23: A certain amount of money at compound interest grows upto R 51168 in 15 yrs and upto Rs 51701 in 16 yrs. Find the rate p cent per annum.

Soln : Using the above formula :

$$\text{Rate} = \frac{(51701 - 51168) \times 100}{51168} = \frac{533 \times 100}{51168}$$

$$= \frac{100}{96} = \frac{25}{24} = 1\frac{1}{24}\%$$

Ex 24 : Find the compound interest on Rs 18,750 in 2 yrs, the rate of interest being 4% for the first year and 8% for the second year.

Soln : After first year the amount $= 18750\left(1 + \frac{4}{100}\right) = 18750\left(\frac{104}{100}\right)$

After 2nd year the amount $= 18750\left(\frac{104}{100}\right)\left(\frac{108}{100}\right)$

$$= 18750\left(\frac{26}{25}\right)\left(\frac{27}{25}\right) = \text{Rs } 21060$$

$\therefore$ CI = 21060 - 18,750 = Rs 2310.

Ex. 25: Rs 4800 becomes Rs 6000 in 4 years at a certain rate of compound interest. What will be the sum after 12 years?

Soln : Detail method : We have:

$$4800\left(1 + \frac{r}{100}\right)^4 = 6000$$

$$\text{or, } \left(1 + \frac{r}{100}\right)^4 = \frac{6000}{4800} = \frac{5}{4}$$

$$\text{Now, } \left(1 + \frac{r}{100}\right)^{4 \times 3} = \left(\frac{5}{4}\right)^3 = \frac{125}{64}$$

$$\text{or, } \left(1 + \frac{r}{100}\right)^{12} = \frac{125 \times 75}{64 \times 75} = \frac{9375}{4800}$$

$$\text{or, } 4800\left(1 + \frac{r}{100}\right)^{12} = 9375$$

The above equation shows that Rs 4800 becomes Rs 9375 after 12 years.

Direct formula :

$$\text{Required amount} = \frac{(6000)^{12/4}}{(4800)^{12/4 - 1}} = \frac{(6000)^3}{(4800)^2} = \text{Rs } 9375$$

Note : Thus, we can say that:

"If a sum 'A' becomes 'B' in t_1 years at compound rate of interest, then after t_2 years the sum becomes

$$\frac{(B)^{t_2/t_1}}{(A)^{t_2/t_1 - 1}} \text{ rupees."}$$

Ex. 26: Find the compound interest on Rs 10000 for 3 years if the rate of interest is 4% for the first year, 5% for the second year and 6% for the third year.

...I. The Compound Interest on Rs x in 't' years if the rate of inter... is r_1% for the first year, r_2% for the second year ... and r_t% ... the tth year is given by

$$x\left(1+\frac{r_1}{100}\right)\left(1+\frac{r_2}{100}\right)\cdots\left(1+\frac{r_t}{100}\right)-x$$

In this case

Compound interest

$$=10000\left(1+\frac{4}{100}\right)\left(1+\frac{5}{100}\right)\left(1+\frac{6}{100}\right)-10000$$

$$=10000\left(\frac{26}{25}\right)\left(\frac{21}{20}\right)\left(\frac{53}{50}\right)-10000$$

$= 11,575.20 - 10,000 =$ Rs 1,575.2

Note : The more general formula for this type of question can be give... as:

If the compound rate of interest for the first t_1 years is r_1%, for the next t_2 years is r_2%, for the next t_3 years is r_3%, ... and th... last t_n years is r_n%, then compound interest on Rs x for $(t_1 + t_2 + t_3 + \ldots t_n)$ years is

$$\left[x\left(1+\frac{r_1}{100}\right)^{t_1}\left(1+\frac{r_2}{100}\right)^{t_2}\cdots\left(1+\frac{r_n}{100}\right)^{t_n}\right]-x$$

In the above case, $t_1 = t_2 = t_3 = 1$ year.

Ex. 27: What sum of money at compound interest will amount to R... 2249.52 in 3 years, if the rate of interest is 3% for the first yea... 4% for the second year, and 5% for the third year?

Soln : The general formula for such question is:

$$A=P\left(1+\frac{r_1}{100}\right)\left(1+\frac{r_2}{100}\right)\left(1+\frac{r_3}{100}\right)\ldots$$

where A = Amount, P = Principal and r_1, r_2, r_3 are the rates o... interest for different years.

In the above case:

$$2249.52=P\left(1+\frac{3}{100}\right)\left(1+\frac{4}{100}\right)\left(1+\frac{5}{100}\right)$$

or, $2249.52 = P\,(1.03)\,(1.04)\,(1.05)$

$$\therefore\ P=\frac{2249.52}{1.03\times1.04\times1.05}=\text{Rs } 2000$$

Direct formula : By the rule of fraction:

$$\text{Principal}=2249.52\left(\frac{100}{103}\right)\left(\frac{100}{104}\right)\left(\frac{100}{105}\right)=\text{Rs } 2000.$$

Ex. 28: A man borrows Rs 3000 at 10% compound rate of interest. At the end of each year he pays back Rs 1000. How much amount should he pay at the end of the third year to clear all his dues?

Soln : The general formula for the above question may be written as: If a man borrows Rs P at r% compound interest and pays back Rs A at the end of each year, then at the end of the nth year he should pay Rs

$$P\left[1+\frac{r}{100}\right]^n-A\left[\left(1+\frac{r}{100}\right)^{n-1}+\left(1+\frac{r}{100}\right)^{n-2}+\ldots\ldots+\left(1+\frac{r}{100}\right)^1\right]$$

In the above case:

$$3000\left[1+\frac{10}{100}\right]^3-1000\left[\left(1+\frac{10}{100}\right)^2+\left(1+\frac{10}{100}\right)^1\right]$$

$$=3000\left(\frac{11}{10}\times\frac{11}{10}\times\frac{11}{10}\right)-1000\left[\left(\frac{11}{10}\right)^2+\frac{11}{10}\right]$$

$$=3993-\left\{1000\times\frac{121}{100}+1000\times\frac{11}{10}\right\}$$

$= 3993 - 1210 - 1100 =$ Rs 1683

Note: The above question may be solved like:

Principal at the beginning of the 2nd year

$$=3000\left(1+\frac{10}{100}\right)-1000=3300-1000=\text{Rs } 2300$$

Principal at the beginning of the third year

$$=2300\left(1+\frac{10}{100}\right)-1000=\text{Rs } 1530$$

$\therefore$ at the end of the third year he should pay

$$\text{Rs } 1530\left(1+\frac{10}{100}\right)=\text{Rs } 1683$$

Ex. 29: Geeta deposits Rs 20,000 in a private company at the rate of 16% compounded yearly; whereas Meera deposits an equal sum in PNB

Housing Finance Ltd at the rate of 15% compounded half-yearl
If both deposit their money for 1½ years only, calculate whic
deposit earns better interest.

Soln : If you are not asked to find the absolute values, the questio becomes easier.

Amount after $1\frac{1}{2}$ years in both the cases will be

$$20000\left(1+\frac{16}{100}\right)^{3/2} \text{ and } 20000\left(1+\frac{7.5}{100}\right)^{3}$$

Now move for inequality:

$$20000\left(1+\frac{16}{100}\right)^{3/2} \Leftrightarrow 20000\left(1+\frac{7.5}{100}\right)^{3}$$

$$\text{or, } \left(1+\frac{16}{100}\right)^{3/2} \Leftrightarrow \left(1+\frac{7.5}{100}\right)^{3}$$

On raising both sides to the power ⅔,

$$\text{or, } \left(1+\frac{16}{100}\right) \Leftrightarrow \left(1+\frac{7.5}{100}\right)^{2}$$

$$\text{or, } 1+\frac{16}{100} \Leftrightarrow 1+\left(\frac{7.5}{100}\right)^{2}+2\left(\frac{7.5}{100}\right)$$

$$\text{or, } \frac{16}{100}-\frac{15}{100} \Leftrightarrow \frac{7.5}{100}\times\frac{7.5}{100}$$

$$\text{or, } 1 \Leftrightarrow \frac{7.5}{100}\times 7.5 \qquad \text{or, } 1 \Leftrightarrow \frac{56.25}{100}$$

Clearly LHS is greater than RHS. Thus Geeta gets better interes

Note : If you are asked to find the value by which Geeta earns more tha Meera, you will have to calculate

$$20000\left(1+\frac{16}{100}\right)^{3/2} \text{ and } 20000\left(1+\frac{7.5}{100}\right)^{3}.$$

Ex. 30: A sum of money is lent out at compound interest rate of 20% per annum for 2 years. It would fetch Rs 482 more if interest i compounded half-yearly. Find the sum.

Soln : Suppose the sum is Rs P.

$$\text{CI when interest is compounded yearly} = P\left[1+\frac{20}{100}\right]^{2} - P$$

CI when interest is compounded half-yearly

$$= P\left[1+\frac{10}{100}\right]^{4} - P$$

$$\text{Now, we have, } P\left[1+\frac{10}{100}\right]^{4} - P\left[1+\frac{20}{100}\right]^{2} = 482$$

$$\Rightarrow P\left[(1.1)^{4} - (1.2)^{2}\right] = 482$$

$$\Rightarrow P\left[\{(1.1)^2-(1.2)\}\{(1.1)^2+(1.2)\}\right] = 482$$

$$\Rightarrow P[\{1.21-1.2\}\{1.21+1.2\}] = 482$$

$$\Rightarrow P[(0.01)(2.41)] = 482 \qquad \therefore P = \frac{482}{2.41\times 0.01} = \text{Rs } 20{,}000.$$

EXERCISES

1. Find the amount of Rs 6400 in 1 year 6 months at 5 p.c. compound interest, interest being calculated half-yearly.
2. Find the compound interest on Rs 10000 in 9 months at 4 p.c., interest payable quarterly.
3. Find the difference between the simple and the compound interests on Rs 1250 for 2 years at 4 p.c. per annum.
4. I give a certain person Rs 8000 at simple interest for 3 years at $7\frac{1}{2}$ p.c. How much more should I have gained had I given it at compound interest ?
5. A merchant commences with a certain capital and gains annually at the rate of 25 p.c. At the end of 3 years he has Rs 10,000. What was his original capital ?
6. In what time will Rs 1200 amount to Rs 1323 at 5 p.c. compound interest ?
7. In what time will Rs 2000 amount to Rs 2431.0125 at 5 p.c. per annum compound interest?
8. In what time will Rs 6250 amount to Rs 6632.55 at 4 p.c. compound interest payable half-yearly?
9. At what rate per cent compound interest will Rs 400 amount to Rs 441 in 2 years?
10. At what rate per cent. compound interest will Rs 625 amount to Rs 676 in 2 years?

11. At what rate per cent compound interest does a sum of money become $\frac{9}{4}$ times itself in 2 years?
12. At what rate per cent compound interest does a sum of money become fourfold in 2 years?
13. If the difference between the simple interest and the compound interest on a certain sum of money for 3 years at 5 per cent per annum is Rs 122, find the sum.
14. The simple interest on a certain sum of money for 4 years at 4 per cent per annum exceeds the compound interest on the same sum for 3 years at 5 per cent. per annum by Rs 57. Find the sum.
15. A sum of money at compound interest amounts in two years to Rs 2809, and in three years to Rs 2977.54. Find the rate of interest and the original sum.
16. A sum is invested at compound interest payable annually. The interest in two successive years was Rs 225 and Rs 236.25. Find the rate of interest and the principal.

ANSWERS

1. Amount $= 6400\left(1+\frac{2.5}{100}\right)^3$

$$= 6400\left(\frac{41}{40}\right)^3 = \frac{6400\times41\times41\times41}{40\times40\times40} = \text{Rs } 6892.1$$

2. $CI = 10000\left\{\left(1+\frac{1}{100}\right)^3 - 1\right\}$

$$= 10000\left\{\frac{30301}{100\times100\times100}\right\} = \text{Rs } 303.01$$

3. **Quicker Maths:** Use the formula (Diff for 2 yrs)

$$\text{Difference} = \text{Sum}\left(\frac{r}{100}\right)^2 = 1250\left(\frac{4}{100}\right)^2 = \frac{1250}{625} = \text{Rs } 2$$

4. **Quicker Maths:** Use the formula (Diff for 3 yrs)

$$\text{Difference} = \frac{\text{Sum}\times r^2(300+r)}{(100)^3}$$

$$= \frac{8000\times(7.5)^2(300+7.5)}{(100)^3} = 138.375 = \text{Rs } 138.38$$

Therefore, I will get Rs 138.38 more.

5. $10000 = x\left(1+\frac{25}{100}\right)^3 \quad \therefore\ x = \frac{10000\times4\times4\times4}{5\times5\times5} = \text{Rs } 5120$

6. $1323 = 1200\left(1+\frac{5}{100}\right)^t$

$$\frac{1323}{1200} = \left(\frac{21}{20}\right)^t$$

$$\text{or, } \frac{441}{400} = \left(\frac{21}{20}\right)^t \quad \text{or, } \left(\frac{21}{20}\right)^2 = \left(\frac{21}{20}\right)^t \quad \therefore\ t = 2 \text{ years}$$

7. Same as Q 6.

8. $6632.55 = 6250\left(1+\frac{2}{100}\right)^t$

$$\text{or, } \frac{6632.55}{625000} = \left(\frac{51}{50}\right)^t$$

$$\text{or, } \frac{663255}{625000} = \left(\frac{51}{50}\right)^t \quad \text{or, } \frac{132651}{125000} = \left(\frac{51}{50}\right)^3 = \left(\frac{51}{50}\right)^t \quad \therefore\ t = 3$$

Hence the time is $\frac{t}{2} = \frac{3}{2}$

Note: As the interest was compounded half-yearly; we changed r to $\frac{r}{2}$ and t to 2t.

9. From the table we find the answer directly as 5%.
10. From the table we find the answer directly as 4%.

11. $\frac{9}{4}S = S\left(1+\frac{r}{100}\right)^2 \quad \text{or, } \left(\frac{3}{2}\right)^2 = \left(1+\frac{r}{100}\right)^2$

$$\text{or, } 1+\frac{r}{100} = \frac{3}{2} \quad \text{or, } \frac{r}{100} = \frac{1}{2} \quad \therefore\ r = 50\%$$

12. Same as Q. 11

13. Sum $= \dfrac{\text{Difference} \times (100)^3}{r^2(300+r)} = \dfrac{122 \times 100 \times 100 \times 100}{25 \times 305} =$ Rs 16000

14. Let the sum be Rs x

Then, $\dfrac{x \times 4 \times 4}{100} - 57 = x\left\{\left(1+\dfrac{5}{100}\right)^3 - 1\right\}$

or, $\dfrac{4x}{25} - 57 = x\left\{\dfrac{1261}{8000}\right\}$

or, $x\left[\dfrac{4}{25} - \dfrac{1261}{8000}\right] = 57$

or, $x\left[\dfrac{1280-1261}{8000}\right] = 57 \quad \therefore \quad x = \dfrac{57 \times 8000}{19} =$ Rs 24000

Note: As the time is different for simple and compound interests, we didn't find the quicker method (direct formula).

15. Difference in amounts = 2977.54 - 2809 = Rs 168.54

Now, we see that Rs 168.54 is the interest on Rs 2809 in one year (it is either simple or compound interest because both are the same for a year).

Hence, rate of interest $= \dfrac{168.54 \times 100}{2809} = 6\%$

Now, for the original sum,

$2809 = x\left(1+\dfrac{6}{100}\right)^2$

or, $2809 = x\left(\dfrac{53}{50}\right)^2 \quad \therefore \quad x = \dfrac{2809 \times 50 \times 50}{53 \times 53} =$ Rs 2500

16. Method is the same as in Q. 15.

Difference in interest = 236.25 - 225 = Rs 11.25

This difference is the simple interest over Rs 225 for one year. Hence rate of interest

$= \dfrac{11.25 \times 100}{225 \times 1} = 5\%$

Now, since any particular number of years is not mentioned, we can't find the sum.

Alligation

Alligation is the rule that enables us

(i) to find the mean or average value of mixture when the prices of two or more ingredients which may be mixed together and the proportion in which they are mixed are given (this is **Alligation Medial**); and

(ii) to find the proportion in which the ingredients at given prices must be mixed to produce a mixture at a given price. This is **Alligation Alternate.**

Note: (1) The word *Alligation* literally means *linking*. The rule takes its name from the lines or links used in working out questions on mixture.

(2) Alligation method is applied for **percentage value, ratio, rate, prices, speed** etc and not for absolute values. That is, whenever per cent, per hour, per kg, per km etc are being compared, we can use Alligation.

Rule of Alligation : If the gradients are mixed in a ratio, then

$$\frac{\text{Quantity of cheaper}}{\text{Quantity of dearer}} = \frac{\text{CP of dearer} - \text{Mean price}}{\text{Mean price} - \text{CP of cheaper}}$$

We represent it as under :

CP of unit quantity of cheaper (c)   CP of unit quantity of dearer (d)

Mean price

(m)

(d - m)   (m - c)

Then, (cheaper quantity) : (dearer quantity) = (d - m) : (m - c)

Solved Problems

Ex.1: In what proportion must rice at Rs 3.10 per kg be mixed with rice at Rs 3.60 per kg, so that the mixture be worth Rs 3.25 a kg ?

Soln: C.P. of 1 kg. cheaper rice   C.P. of 1 kg. dearer rice

(310 paise)   (360 paise)

Mean price

(325 paise)

35   15

By the alligation rule : $\dfrac{\text{(Quanitity of cheaper rice)}}{\text{(Quantity of dearer rice)}} = \dfrac{35}{15} = \dfrac{7}{3}.$

$\therefore$ They must be mixed in the ratio 7 : 3.

Ex.2: How many kg. of salt at 42 P per kg. must a man mix with 25 kg of salt at 24 P per kg. so that he may, on selling the mixture at 40 P per kg, gain 25% on the outlay ?

Sol. Cost price of mixture $= 40 \times \frac{100}{125}$ P = 32 P per kg.

----- (By the rule of fraction)

42   24

32

8   10

Ratio = 4 : 5

Thus for every 5 kg. of salt at 24 P, 4 kg. of salt at 42 P is used.

$\therefore$ the required no. of kg $= 25 \times \frac{4}{5} = 20$. Ans.

Milk and Water

Ex.3 : A mixture of a certain quantity of milk with 16 litres of water is worth 90 P per litre. If pure milk be worth Rs 1.08 per litre how much milk is there in the mixture ?

Sol. The mean value is 90 P and the price of water is 0 P.

milk   water

108   0

90

90 - 0   108 - 90

By the Alligation Rule, milk and water are in the ratio of 5 : 1.

$\therefore$ quantity of milk in the mixture $= 5 \times 16 = 80$ litres. Ans.

Ex.4 : In what proportion must water be mixed with spirit to gain $16\frac{2}{3}\%$ by selling it at cost price ?

Sol. Let C.P. of spirit be Re 1 per litre.

Then S.P. of 1 litre of mixture = Re 1. Gain $= 16\frac{2}{3}\%$.

C.P of 1 litre of mixture $= \text{Rs}\left(\frac{100 \times 3 \times 1}{350}\right) = \text{Rs}\left(\frac{6}{7}\right)$

C.P. of 1 litre water (Re.0)   C. P. of 1 litre pure spirit (Re 1)

Mean Price $\left(\text{Re } \frac{6}{7}\right)$

$\frac{1}{7}$ $\frac{6}{7}$

$$\frac{(\text{Quantity of water})}{(\text{Quantity of spirit})} = \frac{\frac{1}{7}}{\frac{6}{7}} = \frac{1}{6}.$$

or Ratio of water and spirit = 1 : 6.

Ex.5: A butler stole wine from a butt of sherry which contained 40% of spirit. He replaced what he had stolen by wine containing only 16% spirit. The butt was then of 24% strength only. How much of the butt did he steal ?

Soln : Wine containing 40 % Spirit   Wine Containing 16 % Sprit

Wine Containing 24 % Spirit

8   16

$\therefore$ By alligation rule : $\frac{\text{wine with 40\% spirit}}{\text{wine with 16\% spirit}} = \frac{8}{16} = \frac{1}{2}$

i.e., they must be mixed in the ratio (1 : 2). Thus $\frac{1}{3}$ of the butt of sherry was left and hence the butler drew out $\frac{2}{3}$ of the butt.

Three ingredients — Number of proportions unlimited

Ex.6: In what proportion may three kinds of wheat at Rs 1.27, Rs 1.29 and Rs 1.32 per kg be mixed to produce mixture worth Rs 1.30 per kg?

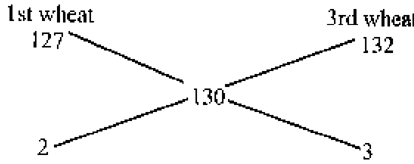
Soln:

1st wheat	2nd wheat	3rd wheat	Mean Price
127 P	129 P	132 P	130 P

Here the first two prices are less and the third price is greater than the mean price.

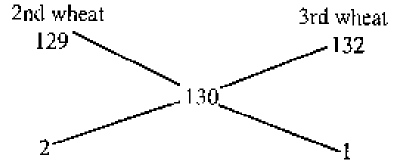
We first find the proportion in which wheat at 127 P and 132 P must be mixed to produce a mixture at 130 P.

(i)



The proportion is 2 : 3

We next find the proportion in which wheat at 129 P and 132 P must be mixed to produce a mixture at 130 P.



The proportion is 2 : 1

Now in whatever proportion these two mixtures are mixed, the price of the resulting mixture will always be 130 P per kg because both mixtures cost 130 P/kg. Now 5 kg of the first mixture is composed of 2 kg of wheat at 127 P and 3 kg of wheat at 132 P, and 3 kg of second mixture is composed of 2 kg of wheat at 129 P and 1 kg of wheat at 132 P; hence 5+3 or 8 kg of the resulting mixture is composed of 2 kg at 127 P, 2 kg at 129 P and (3+1) or 4 kg at 132 P. Hence the required proportion is 2:2:4 or 1:1:2

Take another case :

If we use (say) 4 kg of the first wheat we must use 6 kg of the third wheat. Again if we use (say) 10 kg of the second wheat, we must use 5 kg of third wheat. There is thus another proportion.

1st		2nd		3rd
4 kg	:	10 kg	:	6 + 5 = 11 kg.
or 4	:	10	:	11

The student can verify this result also.

In fact, we can use any number of kg of the 1st or 2nd wheat as long as we use the necessary corresponding number of kg of the 3rd and hence the number of proportions is unlimited.

Note : The above calculations can be simplified further. For this follow the following rule:

Rule: Reduce the several prices to one denomination (like, Rs 1.24, Rs 1.31, Rs 1.20 can be written as 124, 131 and 120) and place them under one another in order of magnitude, the least being uppermost. Set down the mean price to the left of the prices. Link the prices in pairs so that the prices greater and lesser than the average price go together. Then find the difference between each price and the mean price and place it opposite to the price with which it is linked. These differences will give the required answer. For example, the above example can be solved as :

	127	2	(= 132 - 130)
130	129	2	(= 132 - 130)
	132	3+ 1= 4	[= (130 - 127) + (130 - 129)]

∴ the required proportion is 2 : 2 : 4 or 1 : 1 : 2

Ex.7: In what ratio must a person mix three kinds of wheat costing him Rs 1.20, Rs 1.44 and Rs 1.74 per kg, so that the mixture may be worth Rs 1.41 per kg ?

Soln:

1st wheat	2nd wheat	3rd wheat
120	144	174

following the above rule, we have,

	120	3 + 33	[(144 - 141) + (174 - 141)]
141	144	21	[= 141 - 120]
	174	21	[= 141 - 120]

Therefore, the required ratio = 36 : 21 : 21 = 12 : 7 : 7

Note : Try to get the other ratios which satisfy the conditions.

Four ingredients

Ex.8: How must a grocer mix 4 types of rice worth 54 P, 72 P, Rs 1.20 and Rs 1.44 per kg so as to obtain a mixture at 96 P per kg?

Soln: **First solution :**

	54	48	[= 144 - 96]
96	72	24	[= 120 - 96]
	120	24	[= 96 - 24]
	144	42	[= 96 - 54]

∴ required proportion is 48 : 24 : 24 : 42 = 8 : 4 : 4 : 7

Second solution :

$$
\begin{array}{llll}
 & 54 & 24 & [= 120 - 96] \\
96 & 72 & 48 & [= 144 - 96] \\
 & 120 & 42 & [= 96 - 54] \\
 & 144 & 24 & [= 96 - 72]
\end{array}
$$

$\therefore$ required proportion is 24 : 48 : 42 : 24 = 4 : 8 : 7 : 4

Note: Different ways of linking will give different solutions.

Mixture from two vessels

Ex.9: Milk and water are mixed in a vessel A in the proportion 5 : 2, and in vessel B in the proportion 8 : 5. In what proportion should quantities be taken from the two vessels so as to form a mixture in which milk and water will be in the proportion of 9 : 4?

Soln: In vessel A, milk = $\frac{5}{7}$ of the weight of mixture

In vessel B, milk = $\frac{8}{13}$ of the weight of mixture. Now we want to form a mixture in which milk will be $\frac{9}{13}$ of the weight of this mixture.

By alligation rule :

$$
\begin{array}{ccc}
\frac{5}{7} & & \frac{8}{13} \\
 & \frac{9}{13} & \\
\frac{1}{13} & & \frac{2}{91}
\end{array}
$$

$\therefore$ required proportion is $\frac{1}{13} : \frac{2}{91} = 7 : 2$

A butler stealing wine

Ex.10: A butler stores wine from a butt of sherry which contained 30% of spirit and he replaced what he had stolen by wine containing only 12% of spirit. The butt was then 18% strong only. How much of the butt did he steal ?

Soln : By the alligation rule we find that wine containg 30% of spirit and wine containing 12% of spirit should be mixed in the ratio 1 : 2 to produce a mixture containing 18% of spirit.

$$
\begin{array}{ccc}
30\% & & 12\% \\
 & 18\% & \\
6\% & & 12\%
\end{array}
$$

Ratio = 6 : 12 = 1 : 2

This means that $\frac{1}{3}$ of the butt of sherry was left, i.e. to say, the butler drew out $\frac{2}{3}$ of the butt.

$\therefore$ $\frac{2}{3}$ of the butt was stolen.

Ex.11: A goldsmith has two qualities of gold — one of 12 carats and another of 16 carats purity. In what proportion should he mix both to make an ornament of 15 carats purity ?

Soln :

$$
\begin{array}{ccc}
\text{I} & & \text{II} \\
12 & & 16 \\
 & 15 & \\
1 & & 3
\end{array}
$$

$\therefore$ he should mix both the qualities in the ratio 1 : 3.

Ex. 12 : 300 gm of sugar solution has 40% sugar in it. How much sugar should be added to make it 50% in the solution ?

Soln: The existing solution has 40% sugar. And sugar is to be mixed; so the other solution has 100% sugar. So by alligation method :

$$
\begin{array}{ccc}
40\% & & 100\% \\
 & 50\% & \\
50\% & & 10\%
\end{array}
$$

$\therefore$ The two mixtures should be added in the ratio 5 : 1.

Therefore, required sugar = $\frac{300}{5} \times 1 = 60$ gm.

Direct formula :

Quantity of sugar added

$$= \frac{\text{Solution (required\% value} - \text{present \% value)}}{(100 - \text{required\% value})}$$

In this case,

$$\text{Ans} = \frac{300\ (50-40)}{100-50} = 60 \text{ gms.}$$

Ex.13: There are 65 students in a class. 39 rupees are distributed among them so that each boy gets 80 P and each girl gets 30 P. Find the number of boys and girls in that class.

Soln: Here alligation is applicable for "money per boy or girl."

Mean value of money per student $= \frac{3900}{65} = 60$ P

Boys 80, Girls 30 (mean 60); 30, 20

$$\begin{array}{ccccc} \text{Boys} & & & & \text{Girls} \\ 80 & & & & 30 \\ & & 60 & & \\ 30 & & & & 20 \end{array}$$

$\therefore$ Boys : Girls = 3 : 2

$\therefore$ Number of boys $= \frac{65}{3+2} \times 3 = 39$

and number of girls = 65 - 39 = 26.

Ex.14: A person has a chemical of Rs 25 per litre. In what ratio should water be mixed in that chemical so that after selling the mixture at Rs 20/litre he may get a profit of 25%?

Soln: In this question the alligation method is applicable on prices, so we should get the average price of mixture.

SP of mixture = Rs 20/ litre; profit = 25%

$\therefore$ average price $= 20 \times \frac{100}{125} =$ Rs 16 /litre.

Applying the alligation rule :

$$\begin{array}{ccccc} \text{Chemical} & & & & \text{Water} \\ 25 & & & & 0 \\ & & 16 & & \\ 16 & & & & 9 \end{array}$$

$\therefore$ C : W = 16 : 9 Ans.

Ex.15: A person travels 285 km in 6 hrs in two stages. In the first part of the journey, he travels by bus at the speed of 40 km per hr. In the second part of the journey, he travels by train at the speed of 55 km per hr. How much distance did he travel by train ?

Soln: In this question, the alligation method is applicable for the speed.

$$\begin{array}{ccccc} \text{Speed of Bus} & & & & \text{Speed of train} \\ 40 & & & & 55 \\ & & \text{Average speed} & & \\ & & \frac{285}{6} & & \\ \frac{45}{6} & & & & \frac{45}{6} \end{array}$$

$\therefore$ time spent in bus : time spent in train $= \frac{45}{6} : \frac{45}{6} = 1 : 1$

$\therefore$ distance travelled by train $= \frac{285}{2} = 142.5$ km.

Ex.16: In what ratio should milk and water be mixed so that after selling the mixture at the cost price a profit of $16\frac{2}{3}\%$ is made?

Soln : See soln 4.

Short-cut Method : In such questions the ratio is

Water : milk $= 16\frac{2}{3} : 100 = 1 : 6$.

Ex.17: In what ratio should water and wine be mixed so that after selling the mixture at the cost price a profit of 20% is made ?

Soln: Water : Wine = 20 : 100 = 1 : 5

Ex.18: A trader has 50 kg of pulses, part of which he sells at 8% profit and the rest at 18% profit. He gains 14% on the whole. What is the quantity sold at 18% profit ?

Soln: Detail Method

Let the quantity sold at 18% profit be x kg. Then the quantity sold at 8% profit will be (50-x) kg.

For a matter of convenience suppose that the price of pulse is 1 rupee per kg.

Then price of x kg pulse = Rs x and price of (50-x) kg pulse = Rs (50-x)

Now we get an equation.

18% of x + 8 % of (50 - x) = 14% of 50

$\Rightarrow$ 18x + 8 (50 - x) = 14 × 50

$\Rightarrow$ 10x = 300 $\quad \therefore$ x = 30

By Alligation Method :

$$\begin{array}{ccccc} \text{I part} & & & & \text{II part} \\ 8\ \%\ \text{profit} & & & & 18\ \%\ \text{profit} \\ & & 14\ \% & & \\ & & \text{(mean profit)} & & \\ 4\ \% & & & & 6\ \% \end{array}$$

= 4 : 6 = 2 : 3

Therefore the quantity sold at 18% profit $= \frac{50}{2+3} \times 3 = 30$ kg.

Note: For the above example, both the detailed and alligation methods

are given so that you can compare them and understand t importance of alligation method in Quicker Maths.

Ex.19: A trader has 50 kg of rice, a part of which he sells at 10% pro and the rest at 5% loss. He gains 7 % on the whole. What is t quantity sold at 10% gain and 5% loss ?

Soln :

I part II part

10 (-) 5

7

12 3

$\therefore$ Ratio of quantities sold at 10% profit and 5% loss = 12 : 3 = 4 : 1.

Therefore, the quantity sold at 10% profit $= \frac{50}{4+1} \times 4 = 40$ kg and t quantity sold at 5% loss = 50 - 40 = 10 kg.

Note: Whenever there is loss, take the negative value. Here, differen between 7 and (-5) = 7-(-5) = 7+5 = 12. Never take the differen that counts negative value.

Ex.20: A trader has 50 kg of rice, a part of which he sells at 14% prof and the rest at 6% loss. On the whole his loss is 4%. What is th quantity sold at 14% profit and that at 6% loss?

Soln.

I part II part

14 (-) 6

(-) 4 (as there is loss on whole

2 18

$\therefore$ ratio of quantities sold at 14% profit and 6% loss = 2 : 18 = 1 : 9.

$\therefore$ quantity sold at 14% profit $= \frac{50}{1+9} \times 1 = 5$ kg and sold at 6% loss = 50-5 = 45 kg.

Note: Numbers in the third line should always be + ve. That is why (-) 6-(-)4 = -2 is not taken under consideration.

Ex.21: Mira's expenditure and savings are in the ratio 3 : 2. Her income increases by 10%. Her expenditure also increases by 12%. By how many % does her saving increase?

Soln:

Expenditure Saving

12 x

(% increase in exp) (% increase in saving)

10

(% increase in income)

3 2 (given)

We get two values of x, 7 and 13. But to get a viable answer, we must keep in mind that the central value (10) must lie between x and 12. Thus the value of x should be 7 and not 13.

$\therefore$ required % increase = 7%

Ex.22: A vessel of 80 litre is filled with milk and water. 70% of milk and 30% of water is taken out of the vessel. It is found that the vessel is vacated by 55%. Find the initial quantity of milk and water.

Soln: Here the % values of milk and water that is taken from the vessel should be taken into consideration.

milk water

70% 30%

55%

25% 15%

$\Rightarrow$ 5 : 3

Ratio of milk to water = 5 : 3

$\therefore$ quantity of milk $= \frac{80}{5+3} \times 5 = 50$ litres

and quantity of water $= \frac{80}{5+3} \times 3 = 30$ litres

Ex.23: A container contained 80 kg of milk. From this container 8 kg of milk was taken out and replaced by water. This process was further repeated two times. How much milk is now contained by the container ?

Soln. Amount of liquid left after n operations, when the container originally contains x units of liquid from which y units is taken out each time is

$$x\left(\frac{x-y}{x}\right)^n \text{ units.}$$

Thus, in the above case, amount of milk left

$$= 80\left[\frac{80-8}{80}\right]^3 \text{ kg } = 58.32 \text{ kg.}$$

Ex.24: Nine litres are drawn from a cask full of water and it is then filled with milk. Nine litres of mixture are drawn and the cask is again filled with milk. The quantity of water now left in the cask is to that of the milk in it as 16 : 9. How much does the cask hold ?

Soln: Let there be x litres in the cask. From the above formula we have, after n operations :

$$\frac{\text{Water left in vessel after n operations}}{\text{Whole quantity of milk in vessel}} = \left(\frac{x-y}{x}\right)^n$$

Thus in this case, $\left(\frac{x-9}{x}\right)^2 = \left(\frac{16}{16+9}\right) = \frac{16}{25}$

$\therefore$ x = 45 litres.

Ex.25: Rs. 1500 is invested in two such parts that if one part be invested at 6 %, and the other at 5% , the total interest in one year from both investments is Rs 85. How much is invested at 5%?

Soln: If the whole money is invested at 6% , the annual income is 6% of Rs. 1,500 = Rs. 90. If the whole money is invested at 5% , the annual income is 5% of Rs 1,500 = Rs 75

But real income = Rs 85.

$\therefore$ Applying the alligation rule, we have

6% Rs 90 — 5% Rs 75

Rs 85

Rs 10 — Rs 5

2 : 1

$\therefore$ Money invested at 5% = $\frac{1}{3}$ × Rs 1,500 = Rs 500

Note: Solve with the help of average rate of interest.

Ex 26 : A mixture of 40 litres of milk and water contains 10% water. How much water must be added to make 20% water in the new mixture ?

Soln : This question is the same as Ex 12.

The existing mixture has 10% water. Water is to be added, so the other solution has 100% water. So by alligation method

10% — 100%

20%

80% — 10%

$\therefore$ The two mixture should be added in the ratio 8 : 1.

i.e. for every 8 litres of first mixture, 1 litre of water should be added.

Therefore for 40 litres of first mixture $\frac{40}{8}\times 1 = 5$ litres of water should be added.

By Direct formula : By the formula given in Ex 12.

Required quantity of water = $\frac{40\,(20-10)}{100-20}$ = 5 litres.

Note : (1) Both of the above methods are fast-working. But usually, it seems to be difficult to recall the formula in the examination hall. So we suggest you to solve this type of questions by the Rule of Alligation.

(2) In Ex 12, sugar was added in a solution of sugar, but in the above example water is added in a mixture of milk and water. In both the cases this method works. The only thing you should keep in mind is that the % value should be given for the same component in both the mixtures. For example see the following case:

"A mixture of 40 litres of milk and water contains 90% milk. How much water must be added to make 20% water in the new mixture?"

In the above example, percentage value of milk (90%) is given in the first mixture and percentage value of water (20%) is given in the resulting mixture.

So, the above example can be solved by both the ways.

(1) By finding the % value of water in first mixture,

or, (2) by finding the % value of milk in second mixture.

For case (1) : Water % in the mixture = 100 - 90 = 10%

Now, the rest is the same as given in Soln (26).

For case (2) : Percentage value of milk in the resulting mixture = 100 - 20 = 80%. Now we can apply the alligation rule.

The second mixture is water which has 0% milk; so

$$\begin{array}{ccccc} \text{1st mixture} & & & & \text{2nd mixture (water)} \\ 90\% & & & & 0\% \\ & & 80\% & & \\ 80\% & & & & 10\% \end{array}$$

$\therefore$ Ratio in which the two mixtures should be added is 8 : 1. Thus we get the same result by this method also.

Ex 27 : In a zoo, there are rabbits and pigeons. If heads are counted, there are 200 and if legs are counted, there are 580. How many pigeons are there ?

Soln : This question can be solved in many ways. If you suppose the quantities to be x and y, then you get two equations and by solving them you get the required answer.

We give you a **direct formula** for the questions when 2-legged and 4-legged creatures are counted together.

$$\text{No. of 4-legged creatures} = \frac{\text{Total legs} - 2 \times \text{Total heads}}{2}$$

$$\text{No. of 2-legged creatures} = \frac{4 \times \text{Total heads} - \text{Total legs}}{2}$$

$$\therefore \text{ number of pigeons (2-legged)} = \frac{4 \times 200 - 580}{2} = 110$$

By Alligation Rule : Rule of Alligation is applicable on number of legs per head.

$$\text{Average number of legs per head} = \frac{580}{200} = \frac{29}{10}$$

$$\begin{array}{ccccc} \text{Rabbit} & & & & \text{Pigeons} \\ 4 & & & & 2 \\ & & \frac{29}{10} & & \\ \frac{9}{10} & & & & \frac{11}{10} \end{array}$$

$\therefore$ Rabbit : Pigeons = 9 : 11

$$\therefore \text{ number of pigeons} = \frac{200}{9+11} \times 11 = 110$$

Ex 28 : A jar contains a mixture of two liquids A and B in the ratio 4 : 1. When 10 litres of the mixture is taken out and 10 litres of liquid B is poured into the jar, the ratio becomes 2 : 3. How many litres of liquid A was contained in the jar?

Soln : This question should have been discussed under the chapter "**Ratio and Proportion**". But as it is easy to solve it by the method of alligation, it is being discussed here. First we see the method of alligation.

Method I :

$$\text{In original mixture, \% of liquid B} = \frac{1}{4+1} \times 100 = 20\%$$

$$\text{In the resultant mixture, \% of liquid B} = \frac{3}{2+3} \times 100 = 60\%$$

Replacement is made by the liquid B, so the % of B in second mixture = 100%

Then by the method of Alligation :

$$\begin{array}{ccccc} 20\% & & & & 100\% \\ & & 60\% & & \\ 40\% & & & & 40\% \end{array}$$

$\therefore$ Ratio in which first and second mixtures should be added is 1:1. What does it imply? It simply implies that the reduced quantity of the first mixture and the quantity of mixture B which is to be added are the same.

$\therefore$ Total mixture = 10 + 10 = 20 litres.

$$\text{and liquid A} = \frac{20}{5} \times 4 = 16 \text{ litres.}$$

Method II :

The above method is explained through percentage. Now, method II will be explained through fraction.

$$\text{Fraction of B in original mixture} = \frac{1}{5}$$

Fraction of B in second mixture (liquid B) = 1

$$\text{Fraction of B in resulting mixture} = \frac{3}{5}$$

So,

$$\begin{array}{ccc} \frac{1}{5} & & 1 \\ & \frac{3}{5} & \\ \frac{2}{5} & & \frac{2}{5} \end{array}$$

Thus, we see that the original mixture and liquid B are mixed in the same ratio. That is, if 10 litres of liquid B is added then after taking out 10 litres of mixture from the jar, there should have been 10 litres of mixture left.

So, the quantity of mixture in the jar = 10 + 10 = 20 litres

and quantity of A in the jar $= \frac{20}{5} \times 4 = 16$ litres.

Method III :

This method is different from the Method of Alligation. Let the quantity of mixture in the jar be 5x litre. Then

$$4x - 10\left(\frac{4}{4+1}\right) : x - 10\left(\frac{1}{4+1}\right) + 10 = 2:3 \text{ ---(*)}$$

or, $4x - 8 : x - 2 + 10 = 2 : 3$

or, $\frac{4x-8}{x+8} = \frac{2}{3}$

$\therefore x = 4$

Then quantity of A in the mixture $= 4x = 4 \times 4 = 16$ litre

Note (*) : Liquid A in original mixture = 4x

Liquid A taken out with 10 litres of mixture $= 10 \times \frac{4}{4+1}$ litres

$\therefore$ Remaining quantity of A in the mixture $= 4x - 10\left(\frac{4}{5}\right)$

Liquid B in original mixture = x

Liquid B taken out with 10 litres of mixture $= 10\left(\frac{1}{5}\right)$ litres

Liquid B added = 10 litres

$\therefore$ Total quantity of liquid B $= x - 10\left(\frac{1}{5}\right) + 10$

And the ratio of the two should be 2 : 3.

Ex 29 : 729 litres of a mixture contains milk and water in the ratio 7 : 2. How much water is to be added to get a new mixture containing milk and water in the ratio 7 : 3 ?

Soln : Similar questions were discussed in examples 12 and 26. Previously, the percentage of components of mixture were given, but in this example components are given in ratio. Some methods to solve this question are being discussed below.

Method I :

Change the ratio in percentage and use the formula given in Ex. 12.

% of water in the original mixture $= \frac{2}{7+2} \times 100 = \frac{200}{9}\%$

% of water in the resulting mixture $= \frac{3}{10} \times 100 = 30\%$

$\therefore$ Quantity of water to be added $= \dfrac{729\left(30 - \frac{200}{9}\right)}{100 - 30}$

$= \frac{729 \times 70}{9 \times 70} = 81$ litres.

Method II :

It is a little easier than the above method. You don't need to find the percentage value of water. You can use the fractional value of water in the mixture. Use the formula given below :

Required quantity of water to be added

$$= \frac{\text{Solution (Required fractional value} - \text{Present fractional value)}}{1 - \text{(Required fractional value)}}$$

$$= \frac{729\left(\frac{3}{3+7} - \frac{2}{2+7}\right)}{1 - \frac{3}{3+7}}$$

$$= \frac{729\left(\frac{3}{10} - \frac{2}{9}\right)}{1 - \frac{3}{10}}$$

$$= \frac{729\left(\frac{7}{90}\right)}{\frac{7}{10}} = \frac{729}{9} = 81 \text{ litres.}$$

Method III :

To solve this question by the method of alligation, we can use either of the two, percentage or fractional value.

$\frac{200}{9}\%$ 100%

30%

70% $\frac{70}{9}\%$

Therefore, the ratio in which the mixture and water are to be added is $1 : \frac{1}{9}$ or 9 : 1

Then quantity of water to be added $= \frac{729}{9} \times 1 = 81$ litres.

Note : Solve this question by this method. You can use the fractional value also. Try it.

Theorem : *If x glasses of equal size are filled with a mixture of spirit and water. The ratio of spirit and water in each glass are as follows: $a_1 : b_1, a_2 : b_2, \ldots, a_x : b_x$. If the contents of all the x glasses are emptied into a single vessel, then proportion of spirit and water in it is given by*

$$\left(\frac{a_1}{a_1+b_1}+\frac{a_2}{a_2+b_2}+\ldots+\frac{a_x}{a_x+b_x}\right):\left(\frac{b_1}{a_1+b_1}+\frac{b_2}{a_2+b_2}+\ldots+\frac{b_x}{a_x+b_x}\right)$$

Ex 30 : In three vessels each of 10 litres capacity, mixture of milk and water is filled. The ratios of milk and water are 2 : 1, 3 : 1 and 3 : 2 in the three respective vessels. If all the three vessels are emptied into a single large vessel, find the proportion of milk and water in the mixture.

Soln : By the above theorem the required ratio is

$$\left(\frac{2}{2+1}+\frac{3}{3+1}+\frac{3}{3+2}\right):\left(\frac{1}{2+1}+\frac{1}{3+1}+\frac{2}{3+2}\right)$$

$$=\left(\frac{2}{3}+\frac{3}{4}+\frac{3}{5}\right):\left(\frac{1}{3}+\frac{1}{4}+\frac{2}{5}\right)=\frac{40+45+36}{3\times4\times5}:\frac{20+15+24}{3\times4\times5}$$

$$= 121 : 59$$

Note : This question can also be solved without using this theorem. For convenience in calculation, you will have to suppose the capacity of the vessels to be the LCM of (2 + 1), (3 + 1) and (3 + 2), i.e. 60 litres.

Because it hardly matters whether the capacity of each vessel is 10 litres or 60 litres or 1000 litres. The only thing is that they should have equal quantity of mixture.

Ex 31 : If 2 kg of metal, of which $\frac{1}{3}$ is zinc and the rest is copper, be mixed with 3 kg of metal, of which $\frac{1}{4}$ is zinc and the rest is copper, what is the ratio of zinc to copper in the mixture?

Soln : Quantity of zinc in the mixture

$$=2\left(\frac{1}{3}\right)+3\left(\frac{1}{4}\right)=\frac{2}{3}+\frac{3}{4}=\frac{8+9}{12}=\frac{17}{12}$$

Quantity of copper in the metal

$$=3+2-\frac{17}{12}=5-\frac{17}{12}=\frac{43}{12}$$

$$\therefore \text{ ratio} = \frac{17}{12}:\frac{43}{12}=17:43$$

Ex 32 : A man mixes 5 kilolitres of milk at Rs 600 per kilolitre with 6 kilolitres at Rs 540 per kilolitre. How many kilolitres of water should be added to make the average value of the mixture Rs 480 per kilolitre?

Soln : This question should be solved by the method of alligation.

Cost of milk when two qualities are mixed $= \frac{5\times600+6\times540}{5+6}$

$= \text{Rs } \frac{6240}{11}$ per kilolitre.

Cost of water = Rs 0/ kilolitre.

So, First mixture (milk)   second mixture (water)

$\frac{6240}{11}$ 0

480

480      $\frac{960}{11}$

$\therefore$ Ratio of milk and water $= 480 : \frac{960}{11} = 1 : \frac{2}{11} = 11 : 2$

Which implies that 11 kilolitres of milk should be mixed with 2 kilolitres of water. Thus 2 kilolitres of water should be added.

Ex 33 : If goods be purchased for Rs 450 and one-third be sold at a loss of 10%, what per cent of profit should be taken on the remainder so as to gain 20% on the whole transaction ?

Soln :

1st Part   2nd Part

-10%   x%

20%

$\frac{1}{3}$ $\frac{2}{3}$

or 1 : 2

We see that 20 - (-10) = 20 + 10 = 30. As 2 is written in place of 30, there should be 15 in place of 1. Therefore, x = 20 + 15 = 35%

Ex 34 : If goods be purchased for Rs 840 and $\frac{1}{4}$ of the goods be sold at a loss of 20%, at what gain per cent should the remainder be sold so as to gain 20% on the whole?

Soln :

1st Part   2nd Part

-20%   x%

20%

$\frac{1}{4}$ $\frac{3}{4}$

or 1 : 3

We see that, 20 - (-20) = 40 is replaced by 3, so there should be $\frac{40}{3}$ in place of 1. Then $x = 20 + \frac{40}{3} = \frac{100}{3} = 33\frac{1}{3}$ %.

Note : To find the value of x, you may use

$$\frac{20-(-20)}{x-20} = \frac{3}{1}$$

or, $x - 20 = \frac{40}{3}$ $\therefore x = \frac{40}{3} + 20 = \frac{100}{3} = 33\frac{1}{3}$ %

And in Ex 33 : $\frac{20-(-10)}{x-20} = \frac{2}{1}$

or, 2x - 40 = 30; $\therefore x = \frac{70}{2} = 35\%$

Ex 35 : A man buys two horses for Rs 1350 and sells one so as to lose 6% and the other so as to gain 7.5% and on the whole he neither gains nor loses. What does each horse cost ?

Soln :

1st horse   2nd horse

-6%   7.5%

0

7.5   6

or, 5 : 4

Thus, we see that the ratio of the costs of the two horses is 5 : 4.

$\therefore$ Cost of 1st horse $= \frac{1350}{5+4} \times 5 =$ Rs 750

and cost of 2nd horse $= \frac{1350}{5+4} \times 4 =$ Rs 600

Ex 36 : A merchant borrowed Rs 2500 from two money lenders. For one loan he paid 12% p.a. and for the other 14% p.a. The total interest paid for one year was Rs 326. How much did he borrow at each rate ?

Soln : This example is similar to example 25. But we will solve it differently. Previously, the amount was used, but in this we will use the rate of interest.

The merchant paid Rs 326 as interest for his total borrowed amount.

Then average per cent of interest paid $= \frac{326}{2500} \times 100 = \frac{326}{25}\%$

12%   14%

$\frac{326}{25}\%$

$\frac{24}{25}$ $\frac{26}{25}$

$\therefore$ ratio in which the amount should be divided is

$$\frac{24}{25} : \frac{26}{25} = 12 : 13$$

Thus the amount lent at 12% $= \frac{2500}{12+13} \times 12 =$ Rs 1200

and amount lent at 14% $= \frac{2500}{12+13} \times 13 =$ Rs 1300

Ex 37 : How many kg of tea at Rs 42 per kg must a man mix with 25 kg of tea at Rs 24 per kg so that he may, on selling the mixture at Rs 40 per kg, gain 25% on the outlay?

Soln : Solve yourself (see Ex 2).

Ex 38 : Rs 1050 was divided among 1400 men and women so that each man gets Re 1 and each woman 50 paise. Find the number of women. (Ans. 700)

Soln : Solve yourself (see Ex 13).

Ex 39 : A man travelled a distance of 80 km in 7 hours partly on foot at the rate of 8 km per hour and partly on bicycle at 16 km per hour. Find the distance travelled on foot.

Soln : Average speed = $\frac{80}{7}$ km/hr.

By alligation method :

Foot Bicycle

8 16

$\frac{80}{7}$

$\frac{32}{7}$ $\frac{24}{7}$

Ratio of time travelled on foot and by bicycle = $\frac{32}{7} : \frac{24}{7} = 4 : 3$

$\therefore$ Time travelled on foot = $\frac{7}{4+3} \times 4 = 4$ hrs.

$\therefore$ Distance travelled on foot = $8 \times 4 = 32$ km.

Ex 40 : Some amount out of Rs 7000 was lent at 6% per annum and the remaining was lent at 4% per annum. The total simple interest from both the parts in 5 yrs was Rs 1600. Find the sum lent at 6% p.a. (Ans : Rs 2000)

Soln : Solve it yourself by both the methods discussed in Ex 25 and Ex 36.

Ex 41 : Milk and water are mixed in a vessel A as 4 : 1 and in vessel B as 3 : 2. For vessel C, if one takes equal quantities from A and B, find the ratio of milk to water in C. (Ans : 7 : 3)

Soln : Try yourself (see Ex 30 and the theorem used in it).

Ex. 42: An army of 12,000 consists of Europeans and Indians. The average height of Europeans is 5ft 10 inches and that of an Indian is 5ft 9 inches. The average height of the whole army is 5ft $9\frac{3}{4}$ inches. Find the number of Indians in the army.

Soln. Detail method: Let the number of Indians be x; then

$$\frac{x\,(5\text{ ft }9\text{ in}) + (12000 - x)\,(5\text{ ft }10\text{ in})}{12000} = 5\text{ ft }9\tfrac{3}{4}\text{ in}$$

or, x (69 in) + (12000 - x) (70 in) = 69.75 in × 12000

or, x = 12000 (70 - 69.75) = 12000 × 0.25 = 3000

By Method of Alligation (Quicker Method):

Europeans Indians

70 69

69.75

0.75 0.25

$\therefore$ ratio = 0.75 : 0.25 = 3:1

$\therefore$ no. of Indians = $\frac{12000}{3+1} \times 1 = 3000$

Ex. 43: In an alloy, zinc and copper are in the ratio 1:2. In the second alloy the same elements are in the ratio 2:3. In what ratio should these two alloys be mixed to form a new alloy in which the two elements are in ratio 5:8?

Soln: Detail Method: Let them be mixed in the ratio x : y

Then, in 1st alloy, Zinc = $\frac{x}{3}$ and Copper = $\frac{2x}{3}$

2nd alloy : Zinc = $\frac{2y}{5}$ and Copper = $\frac{3y}{5}$

Now, we have $\frac{x}{3} + \frac{2y}{5} : \frac{2x}{3} + \frac{3y}{5} = 5 : 8$

or, $\frac{5x + 6y}{10x + 9y} = \frac{5}{8}$

or, $40x + 48y = 50x + 45y$ or, $10x = 3y$ $\therefore \frac{x}{y} = \frac{3}{10}$

Thus, the required ratio = 3 : 10

By Method of Alligation (Quicker Method):

You must know that we can apply this rule over the fractional value of either zinc or copper. Let us consider the fractional value of zinc.

1st alloy 2nd alloy

$\frac{1}{3}$ $\frac{2}{5}$

$\frac{5}{13}$ (mixture)

$\frac{1}{65}$ $\frac{2}{39}$

Therefore, they should be mixed in the ratio

$\frac{1}{65} : \frac{2}{39}$ *or,* $\frac{1}{65} \times \frac{39}{2} = \frac{3}{10}$ or 3 : 10

Note: Try to solve it by taking fractional value of Copper.

Ex. 44: Jayashree purchased 150 kg of wheat at the rate of Rs 7 per kg. She sold 50 kg at a profit of 10%. At what rate per kg should she sell the remaining to get a profit of 20% on the total deal?

Soln : Selling price of 150 kg wheat at 20% profit

$$= 150 \times 7 \left(\frac{120}{100}\right) = \text{Rs } 1260$$

Selling price of 50 kg wheat at 10% profit

$$= 50 \times 7 \left(\frac{110}{100}\right) = \text{Rs } 385$$

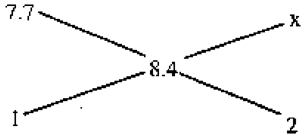
$\therefore$ Selling price per kg of remaining 100 kg wheat

$$= \frac{1260 - 385}{100} = \text{Rs } 8.75$$

By Method of Alligation : Selling price per kg at 10% profit = Rs 7.70

Selling price per kg at 20% profit = Rs 8.40

Now, the two lots are in ratio = 1 : 2

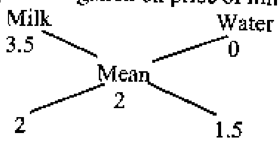


$\Rightarrow \frac{8.4 - 7.7}{x - 8.4} = \frac{2}{1}$ $\quad \therefore x - 8.4 = \frac{0.7}{2} = 0.35; \quad \therefore x = 8.75$

$\therefore$ Selling price per kg of remaining 100 kg = Rs 8.75

Ex. 45: How much water must be added to a cask which contains 40 litres of milk at cost price Rs 3.5/litre so that the cost of milk reduces to Rs 2/litre?

Soln: This question can be solved in so many different ways. But the method of alligation method is the simplest of all the methods. We will apply the alligation on price of milk, water and mixture



$\therefore$ ratio of milk and water should be 2 : 1.5 = 4 : 3

$\therefore$ added water $= \frac{40}{4} \times 3 = 30$ litres.

EXERCISES

1. Gold is 19 times as heavy as water and copper 9 times. In what ratio should these metals be mixed so that the mixture may be 15 times as heavy as water ?
2. How much chicory at Rs. 4 a kg. should be added to 15 kg. of tea at Rs. 10 a kg. so that the mixture be worth Rs. 6.50 a kg. ?
3. A mixture of 40 litres of milk and water contains 10% water. How much water must be added to make water 20% in the new mixture?
4. A sum of Rs. 6.40 is made up of 80 coins which are either 10-paise or five-paise coins. How many are of 5 P.?
5. In a zoo, there are rabbits and pigeons. If heads are counted, there are 200 and if legs are counted, there are 580. How many pigeons are there ?
6. A man has 30,000 rupees to lend on loan. He lent some of his capital to Mohan at an interest rate of 20% per annum and the rest to Suresh at an interest rate of 12% per annum. At the end of one year he got 17% of his capital as interest. How much did he lend to Mohan ?
7. A vessel of 80 litre is filled with milk and water. 70% of milk and 30% of water is taken out of the vessel. It is found that the vessel is vacated by 55% . Find the initial quantity of milk and water.
8. Mohan's expenditures and savings are in the ratio of 4 : 1. His income increases by 20% . If his savings increase by 12% , by how much % should his expenditure increase ?
9. Mukesh earned Rs 4000 per month. From the last month his income increased by 8% . Due to rise in prices, his expenditures also increased by 12% and his savings decreased by 4% . Find his increased expenditure and initial saving.
10. A man has 60 pens. He sells some of these at a profit of 12% and the rest at 8% loss. On the whole, he gets a profit of 11% . How many pens were sold at 12% profit and how many at 8% loss ?
11. A man has 40 kg of tea, a part of which he sells at 5% loss and the rest at the cost price. In this business he gets a loss of 3% . Find the quantity which he sells at the cost price.
12. The ratio of milk and water in 66 kg. of adulterated milk is 5 : 1. Water is added to it to make the ratio 5 : 3. The quantity of water added is ____.

13. Some amount out of Rs. 7000 was lent at 6% p.a. and the remaining at 4% p.a. If the total simple interest from both the fractions in 5 years was Rs. 1600, the sum lent at 6% p.a. was ____.

14. 729 ml. of a mixture contains milk and water in the ratio 7:2. How much more water is to be added to get a new mixture containing milk and water in the ratio 7:3 ?

15. A dishonest milkman professes to sell his milk at cost price but he mixes it with water and thereby gains 25%. The percentage of water in the mixture is ________.

16. A sum of Rs 41 was divided among 50 boys and girls. Each boy gets 90 paise and each girl 65 paise. The number of boys is ______.

17. A can contains a mixture of two liquids A and B in proportion 7:5. When 9 litres of mixture are drawn off and the can is filled with B, the proportion of A and B becomes 7:9. How many litres of liquid A was contained by the can initially ?

18. In a mixture of 60 litres, the ratio of milk and water is 2:1. If the ratio of milk and water is to be 1:2, then the amount of water to be further added is ________.

19. A vessel contains 56 litres of a mixture of milk and water in the ratio 5:2. How much water should be mixed with it so that the ratio of milk to water be 4:5 ?

20. A sum of Rs. 39 was divided among 45 boys and girls. Each girl gets 50 P. whereas each boy gets one rupee . How many girls are there?

21. I mixed some water in pure milk and sold the mixture at the cost price of the milk. If I gained $16\frac{2}{3}\%$, in what ratio did I mix water in the milk?

22. Milk and water are mixed in vessel A in the ratio of 5:2 and in vessel B in the ratio of 8:5. In what ratio should quantities be taken from the two vessels so as to form a mixture in which milk and water will be in the ratio of 9:4 ?

SOLUTION

1.

Gold   Copper
19   9
15
6   4

$\therefore$ Gold : Copper = 6:4 = 3:2

2.

Chicory   Tea
4   10
6.5
3.5   2.5

$\therefore$ Chicory : Tea = 3.5 : 2.5 = 7:5

Therefore, when tea is 15 kg, chicory should be 21 kg.

3.

Impure Milk   Water
10%   100%
20%
80%   10%

Impure Milk : Water = 8:1

Therefore, if there is 40 litres of impure milk, quantity of water is $\frac{40}{8} \times 1 = 5$ litres.

4. Average value per coin $= \frac{640}{80} = 8P$

10   5
8
3   2

10-paise coin : 5-paise coin = 3:2

$\therefore$ 5-paise coins $= \frac{80}{5} \times 2 = 32$

5. Average legs per head $= \frac{580}{200} = 2.9$

Rabbits   Pigeons
4   2
2.9
0.9   1.1

$\therefore$ Rabbits : Pigeons = 0.9 : 1.1 = 9:11

$\therefore$ No. of pigeons $= \frac{200}{20} \times 11 = 110$

6.

Mohan   Suresh
20%   12%
17%
5%   3%

Ratio = 5:3

$\therefore$ amount given to Mohan $= \frac{30000}{8} \times 5 =$ Rs 18750

7.

Milk Water

70% 30%

55%

25% 15%

Therefore, ratio of milk and water in the vessel = 5:3

Thus, milk = $\frac{80}{8} \times 5 = 50$ litres

Water = $\frac{80}{8} \times 3 = 30$ litres.

8.

Expenditure Savings

x 12

(% increase in exp) (% increase in savings)

20

(% increase in income)

4 1 (given)

or, 8 2

We see that x = 22%

9.

% Inc. in Exp. **% Inc in Sav.**

12 -4

8

% Inc in income

12 4

Therefore, expenditure : savings = 12 : 4 = 3 : 1

$\therefore$ Expenditure = $\frac{4000}{4} \times 3$ = Rs 3000

and saving = 4000 - 3000 = Rs 1000

Now, increased expenditure = $3000\left(\frac{112}{100}\right)$ = Rs 3360

10.

Profit Loss

12% -8%

11%

19% 1%

Therefore, ratio of pens sold at profit & loss = 19 : 1

$\therefore$ number of pens sold at 12% profit = $\frac{60}{20} \times 19 = 57$

11.

At loss At cost

5% 0%

3%

3 2

Ratio of quantity of tea sold at loss and cost price = 3:2

$\therefore$ quantity sold at cost price = $\frac{40}{5} \times 2 = 16$ kg

Note: In Q. 10 we took loss as -ve because there was overall profit and thus each was presented in term of profit. [Profit = - (loss)] But in Q. 11 there is overall loss, and each is presented in terms of loss, therefore loss is taken as positive.

12. F_1 = Fraction of milk in the adulterated milk = $\frac{5}{6}$

F_2 = Fraction of milk in water = 0

F_3 = Fraction of milk in the new mixture = $\frac{5}{8}$

F_1 F_2

$\frac{5}{6}$ 0

$\frac{5}{8}$

$\frac{5}{8}$ $\frac{5}{24}$

$\therefore \quad F_1 : F_2 = \frac{5}{8} : \frac{5}{24} = 3 : 1$

If we have 66 kg of adulterated milk, water = $\frac{66}{3} \times 1 = 22$ litres.

13. Overall rate of interest = $\frac{1600 \times 100}{5 \times 7000} = \frac{32}{7}\%$

Rate for I amount Rate for II amount

6% 4%

$\frac{32}{7}\%$

(average rate)

$\frac{4}{7}\%$ $\frac{10}{7}\%$

$\therefore$ ratio of two amounts = 2:5

$\therefore$ amount lent at 6% = $\frac{7000}{7} \times 2$ = Rs 2000

14. Same as Q. 12

15. We will apply alligation on % profit. If he sells the milk at CP he gains 0% but if he sells water at CP he gains 100%.

$$\begin{array}{ccc} \text{Milk} & & \text{Water} \\ 0\% & & 100\% \\ & 25\% & \\ 75\% & & 25\% \end{array}$$

Ratio of milk to water in the mixture should be 3:1

$\therefore$ % of water in mixture $= \frac{1}{3+1} \times 100 = 25\%$

16. Apply the alligation method on paise per head.

Paise per student $= \frac{4100}{50} = 82$

$$\begin{array}{ccc} \text{Boys} & & \text{Girls} \\ 90 & & 65 \\ & 82 & \\ 17 & & 8 \end{array}$$

Boys : Girls = 17 : 8

$\therefore$ no. of boys $= \frac{50}{25} \times 17 = 34$

17. Apply alligation on fraction of A in each mixture.

$$\begin{array}{ccc} \text{Original mixture} & & \text{B} \\ \frac{7}{12} & & 0 \\ & \text{New mixture} & \\ & \frac{7}{16} & \\ \frac{7}{16} & & \frac{7}{48} \end{array}$$

Ratio of original mixture and B $= \frac{7}{16} : \frac{7}{48} = 3 : 1$

When 9 litres of B is mixed, original mixture should be

$\frac{9}{1} \times 3 = 27$ litres.

Therefore initial quantity in can = 27 + 9 = 36 litres.

18. Apply the alligation on fraction of milk in each mixture.

$$\begin{array}{ccc} \text{Mixture} & & \text{Water} \\ \frac{2}{3} & & 0 \\ & \frac{1}{3} & \\ \frac{1}{3} & & \frac{1}{3} \end{array}$$

Ratio of mixture and water = 1 : 1

Therefore, if there is 60 litres of solution, 60 litres of water should be added.

19. Same as Q. 18.

20. Apply the alligation on paise per head.
Same as Q. 16.

21. Same as Q. 15.

22. Apply the alligation on fraction of milk in each vessel.

$$\begin{array}{ccc} \textbf{Vessel A} & & \textbf{Vessel B} \\ \frac{5}{7} & & \frac{8}{13} \\ & \frac{9}{13} & \\ \frac{1}{13} & & \frac{2}{91} \end{array}$$

Ratio of quantity taken from vessel A and vessel B

$= \frac{1}{13} : \frac{2}{91} = 7 : 2$

Time and Work

If 'M_1' persons can do 'W_1' works in 'D_1' days and 'M_2' persons can do 'W_2' works in 'D_2' days then we have a very general formula in the relationship of $M_1 D_1 W_2 = M_2 D_2 W_1$ The above relationship can be taken as a very basic and all-in-one formula. We also derive

1) *More men less days* and conversely *more days less men.*
2) *More men more work* and conversely *more work more men.*
3) *More days more work* and conversely *more work more days.*

If we include the working hours (say T_1 and T_2) for the two groups then the relationship is

$$M_1D_1T_1W_2 = M_2D_2T_2W_1$$

Again, if the efficiency (say E1 and E2) of the persons in two groups is different then the relationship is

$$M_1D_1T_1E_1W_2 = M_2D_2T_2E_2W_1$$

Now, we should go ahead starting with simpler to difficult and more difficult questions.

Ex.1: 'A' can do a piece of work in 5 days. How many days will he take to complete 3 works of the same type ?

Soln: We recall the statement: "More work more days"

It simply means that we will get the answer by multiplication.

Thus, our answer = $5 \times 3 = 15$ days.

This way of solving the question is very simple, but you should know how the "basic formula" could be used in this question.

Recall the basic formula: $M_1D_1W_2 = M_2 D_2 W_1$

As 'A' is the only person to do the work in both the cases, so

$M_1 = M_2 = 1$ (Useless to carry it)

$D_1 = 5$ days, $W_1 = 1$, $D_2 = ?$ and $W_2 = 3$

Putting the values in the formula we have,

$5 \times 3 = D_2 \times 1$

or, $D_2 = 15$ days.

Ex.2: 16 men can do a piece of work in 10 days. How many men are needed to complete the work in 40 days ?

Soln: To do a work in 10 days, 16 men are needed.

Or, to do the work in 1 day, 16×10 men are needed.

So to do the work in 40 days, $\frac{16 \times 10}{40} = 4$ men are needed.

This was the method used for non-objective exams.

We should see how the "basic formula" works here.
$M_1 = 16, D_1 = 10, W_1 = 1$ and $M_2 = 7, D_2 = 40, W_2 = 1$
Thus, from $\mathbf{M_1D_1W_2 = M_2D_2W_1}$
$16 \times 10 = M_2 \times 40$
or, $M_2 = \frac{16 \times 10}{40} = 4$ men.

By rule of fractions : To do the work in 40 days we need less number of men than 10. So we should multiply 10 with a fraction which is less than 1. And that fraction is $\frac{10}{40}$. Therefore, required number of men

$$= 16 \times \frac{10}{40} = 4$$

Ex.3: 40 men can cut 60 trees in 8 hrs. If 8 men leave the job how many trees will be cut in 12 hours?

Soln: 40 men - working 8 hrs - cut 60 trees
or, 1 man - working 1 hr - cuts $\frac{60}{40 \times 8}$ trees
Thus, 32 men - working 12 hrs - cut $\frac{60 \times 32 \times 12}{40 \times 8} = 72$ trees.

By our "basic - formula"
$M_1 = 40, D_1 = 8$ (As days and hrs both denote time)
$W_1 = 60$ (cutting of trees is taken as work)
$M_2 = 40 - 8 = 32, D_2 = 12, W_2 = ?$
Putting the values in the formula
$M_1 D_1 W_2 = M_2 D_2 W_1$
We have, $40 \times 8 \times W_2 = 32 \times 12 \times 60$
or, $W_2 = \frac{32 \times 12 \times 60}{40 \times 8} = 72$ trees.

By rule of fractions : First, there were 40 men, but when 8 men leave the job we are left with 32 men. As the number of men is reduced, less number of trees will be cut by them. So, 60 should be multiplied with less-than-one fraction, $\frac{32}{40}$. Furthermore, as the number of hours increases, more number of trees will be cut. So the previous product will be multiplied by more-than-one fraction, $\frac{12}{8}$. Therefore, the required number of trees $= 60\left(\frac{32}{40}\right)\left(\frac{12}{8}\right) = 72$ trees.

Note: Try to solve this question without writing the initial steps.

Ex.4: 5 men can prepare 10 toys in 6 days working 6 hrs a day. Then in how many days can 12 men prepare 16 toys working 8 hrs a day?

Soln: This example has an extra variable 'time' (hrs a day), so the 'basic-formula' can't work in this case. An extended formula is being given:
$\mathbf{M_1 D_1 T_1 W_2 = M_2 D_2 T_2 W_1}$
Here, $5 \times 6 \times 6 \times 16 = 12 \times D_2 \times 8 \times 10$
$\therefore D_2 = \frac{5 \times 6 \times 6 \times 16}{12 \times 8 \times 10} = 3$ days.

Note: Number of toys is considered as work in the above example.

By rule of fractions : See the steps :

1. We have to find number of days, so write the given number of days first.

2. Number of men increases $\Rightarrow$ work will be done in less days $\Rightarrow$ multiplying fraction should be less than 1, which is $\frac{5}{12}$.

3. Number of toys increases $\Rightarrow$ it will take more days $\Rightarrow$ multiplying fraction should be more than 1, which is $\frac{16}{10}$.

4. Number of working hours increases $\Rightarrow$ it will take less days $\Rightarrow$ multiplying fraction should be less than 1, which is $\frac{6}{8}$.

Thus, required number of days $= 6\left(\frac{5}{12}\right)\left(\frac{16}{10}\right)\left(\frac{6}{8}\right) = 3$ days

Note : If you understand the method of fraction, your writing work reduces and you need to write only $6\left(\frac{5}{12}\right)\left(\frac{16}{10}\right)\left(\frac{6}{8}\right) = 3$ days.

Theorem : *If A can do a piece of work in x days and B can do it in y days then A and B working together will do the same work in $\frac{xy}{x+y}$ days.*

Proof: A's work in 1 day $= \frac{1}{x}$

B's work in 1 day $= \frac{1}{y}$

(A+B)'s work in 1 day $= \frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy}$

$\therefore$ (A+B) do the whole work in $\frac{xy}{x+y}$ days.

Ex. 5: A can do a piece of work in 5 days, and B can do it in 6 days. How long will they take if both work together ?

Soln: 'A' can do $\frac{1}{5}$ work in 1 day.

'B' can do $\frac{1}{6}$ work in 1 day.

Thus 'A' and 'B' can do $\left(\frac{1}{5}+\frac{1}{6}\right)$ work in 1 day.

$\therefore$ 'A' and 'B' can do the work in $\frac{1}{\frac{1}{5}+\frac{1}{6}}$ days $=\frac{30}{11}=2\frac{8}{11}$ days

By the theorem : A+B can do the work in $\frac{5\times 6}{5+6}$ days $=\frac{30}{11}=2\frac{8}{11}$ days

Theorem : *If A, B and C can do a work in x, y and z days respectively then all of them working together can finish the work in $\frac{xyz}{xy+yz+xz}$ days.*

Proof: Try yourself.

Ex.6: In the above question, if C, who can do the work in 12 days, joins them, how long will they take to complete the work?

Soln: By the theorem :

'A', 'B' and 'C' can do the work in

$$\frac{5\times 6\times 12}{5\times 6+6\times 12+5\times 12}=\frac{360}{162}=2\frac{2}{9}\text{ days.}$$

Note: Do you find it easier to remember the direct formula in examples 5 and 6? Try to solve some more examples by this method.

Ex.7: Mohan can do a piece of work in 10 days and Ramesh can do the same work in 15 days. How long will they take if both work together?

Sol: Ans $=\frac{10\times 15}{10+15}=\frac{150}{25}=6$ days.

Ex.8: In the above question if Suresh, who can finish the same work in 30 days, joins them, how long will they take to complete the work?

Sol: Ans $=\frac{10\times 15\times 30}{10\times 15+10\times 30+15\times 30}=\frac{10\times 15\times 30}{900}=5$ days.

Theorem : *If A and B together can do a piece of work in x days and A alone can do it in y days, then B alone can do the work in $\frac{xy}{y-x}$ days.*

Proof: Try yourself.

Ex.9: A and B together can do a piece of work in 6 days and A alone can do it in 9 days. In how many days can B alone do it?

Soln: A and B can do $\frac{1}{6}$ of the work in 1 day.

A alone can do $\frac{1}{9}$ of the work in 1 day.

$\therefore$ B alone can do $\left(\frac{1}{6}-\frac{1}{9}\right)=\frac{1}{18}$ of the work in 1 day.

$\therefore$ B alone can do the whole work in 18 days.

By the theorem :

B alone can do the whole work in

$\frac{6\times 9}{9-6}=\frac{54}{3}=18$ days.

More uses of the above formula

Ex.10: A and B can do a piece of work in 12 days, B and C in 15 days, C and A in 20 days. How long would each take separately to do the same work?

Soln : A + B can do in 12 days.

B + C can do in 15 days.

A + C can do in 20 days.

By the theorem: We see that 2 (A + B + C) can do the work in

$$\frac{12\times 15\times 20}{12\times 15+12\times 20+15\times 20}=5\text{ days}$$

$\therefore$ A + B + C can do the work in 5 $\times$ 2 = 10 days

(Less men more days)

Now, A can do the work in $\frac{10\times 15}{15-10}=30$ days (As in Ex.7)

[As A = (A + B + C) - (B + C)]

B can do the work in $\frac{10\times 20}{20-10}=20$ days

[As, B = (A + B + C) - (A + C)]

C can do the work in $\frac{10 \times 12}{12 - 10} = 60$ days

[As, C = (A + B + C) - (A + B)]

Working alternately

Ex.11: Two women, Ganga and Saraswati, working separately can mow a field in 8 and 12 hrs respectively. If they work in stretches of one hour alternately, Ganga beginning at 9 a.m., when will the mowing be finished?

Soln: In the first hour Ganga mows $\frac{1}{8}$ of the field.

In the second hour Saraswati mows $\frac{1}{12}$ of the field.

$\therefore$ in the first 2 hrs $\left(\frac{1}{8} + \frac{1}{12} = \frac{5}{24}\right)$ of the field is mown.

$\therefore$ in 8 hrs $\frac{5}{24} \times 4 = \frac{5}{6}$ of the field is mown. --------(*)

Now, $\left(1 - \frac{5}{6}\right) = \frac{1}{6}$ of the field remains to be mown. In the 9th hour Ganga mows $\frac{1}{8}$ of the field.

$\therefore$ Saraswati will finish the mowing of $\left(\frac{1}{6} - \frac{1}{8}\right) = \frac{1}{24}$ of the field in $\left(\frac{1}{24} \div \frac{1}{12}\right)$ or $\frac{1}{2}$ of an hour.

$\therefore$ the total time required is $\left(8 + 1 + \frac{1}{2}\right)$ or $9\frac{1}{2}$ hrs.

Thus, the work will be finished at $9 + 9\frac{1}{2} = 18\frac{1}{2}$ or $6\frac{1}{2}$ p.m.

Note (*): We calculated the work for 4 pairs of hours only because if we calculate for 5 pairs of hours, the work done is more than 1. And it leads to absurd result.

Ex.12: A and B together can do a piece of work in 12 days which B and C together can do in 16 days. After A has been working at it for 5 days, and B for 7 days, C takes up and finishes it alone in 13 days. In how many days could each do the work by himself?

Soln: A and B in 1 day do $\frac{1}{12}$ work.

B and C in 1 day do $\frac{1}{16}$ work.

Now from the question,

A's 5 days' + B's 7 days' + C's 13 days' work = 1

or, A's 5 days' + B's 5 days' + B's 2 days' + C's 2 days' + C's 11 days' work = 1

(A +B)'s 5 days' + (B+C)'s 2 days' + C's 11 days' work = 1

$\therefore \frac{5}{12} + \frac{2}{16}$ + C's 11 days' work = 1

$\therefore$ C's 11 day's work = $1 - \left(\frac{5}{12} + \frac{2}{16}\right) = \frac{11}{24}$

$\therefore$ C's 1 day's work = $\frac{11}{24 \times 11} = \frac{1}{24}$

$\therefore$ B's 1 day's work = $\frac{1}{16} - \frac{1}{24} = \frac{1}{48}$

$\therefore$ A's 1 day's work = $\frac{1}{12} - \frac{1}{48} = \frac{1}{16}$

$\therefore$ A, B and C can do the work in 16, 48 and 24 days respectively.

Ex.13: To do a certain work B would take three times as long as A and C together and C twice as long as A and B together. The three men together complete the work in 10 days. How long would each take separately?

Soln: By the question

3 times B's daily work = (A + C)'s daily work.

Add B's daily work to both sides.

$\therefore$ 4 times B's daily work = (A + B + C)'s daily work = $\frac{1}{10}$

$\therefore$ B's daily work = $\frac{1}{40}$

Also, 2 times C's daily work = (A + B)'s daily work. Add C's daily work to both sides.

$\therefore$ 3 times C's daily work = (A + B + C)'s daily work = $\frac{1}{10}$

$\therefore$ C's daily work = $\frac{1}{30}$

Now A's daily work = $\frac{1}{10} - \left(\frac{1}{40} + \frac{1}{30}\right) = \frac{1}{24}$

$\therefore$ A, B and C can do the work in 24, 40 and 30 days respectively.

Quicker Method :

Number of days taken by B
= (Number of days taken by A+B+C) × (3+1)
= 10 (3+1) = 40 days
Similarly,
Number of days taken by C = 10 (2+1) = 30 days

Number of days taken by A = $\dfrac{1}{\dfrac{1}{10}-\left(\dfrac{1}{40}+\dfrac{1}{30}\right)}$ = 24 days

Ex.14: If 3 men or 4 women can reap a field in 43 days, how long will 7 men and 5 women take to reap it?

Soln: First Method

3 men reap $\frac{1}{43}$ of the field in 1 day.

$\therefore$ 1 man reaps $\frac{1}{43 \times 3}$ of the field in 1 day.

4 women reap $\frac{1}{43}$ of the field in 1 day.

$\therefore$ 1 woman reaps $\frac{1}{43 \times 4}$ of the field in 1 day.

$\therefore$ 7 men and 5 women reap $\left(\frac{7}{43 \times 3}+\frac{5}{43 \times 4}\right)=\frac{1}{12}$ of the field in 1 day.

$\therefore$ 7 men and 5 women will reap the whole field in 12 days.

Second Method

3 men = 4 women

$\therefore$ 1 man = $\frac{4}{3}$ women

$\therefore$ 7 men = $\frac{28}{3}$ women

$\therefore$ 7 men + 5 women = $\frac{28}{3}+5=\frac{43}{3}$ women

Now, the question becomes:

If 4 women can reap a field in 43 days, how long will $\frac{43}{3}$ women take to reap it?

The "basic-formula" gives

$$4 \times 43 = \frac{43}{3} \times D_2$$

or, $D_2 = \frac{4 \times 43 \times 3}{43}$ = 12 days.

Quicker Method :

Required number of days = $\dfrac{1}{\left[\dfrac{7}{43 \times 3}+\dfrac{5}{43 \times 4}\right]}$

$$= \frac{43 \times 3 \times 4}{7 \times 4 + 5 \times 3} = 12 \text{ days}$$

Note : The above formula is very easy to remember.
If we divide the question in two parts and call the first part as OR-part and the second part as AND-part then

$$\frac{7}{43 \times 3} = \frac{\text{Number of men in AND-part}}{\text{Number of days} \times \text{Number of men in OR-part}}$$

Similarly, you can look for the second part in denominator.

Ex.15: If 12 men and 16 boys can do a piece of work in 5 days and 13 men and 24 boys can do it in 4 days, how long will 7 men and 10 boys take to do it ?

Soln: 12 men and 16 boys can do the work in 5 days --(1)
13 men and 24 boys can do the work in 4 days --(2)
Now it is easy to see that if the no. of workers be multipled by any number, the time must be divided by the same number (derived from: *more workers less time*). Hence multiplying the no. of workers in (1) and (2) by 5 and 4 respectively, we get

5 (12 men + 16 boys) can do the work in $\frac{5}{5}$ = 1 day

4 (13 men + 24 boys) can do the work in $\frac{4}{4}$ = 1 day

or, 5(12 m + 16 b) = 4(13 m + 24 b)
or, 60 m + 80 b = 52 m + 96 b ----------(*)
or, 60 m - 52 m = 96 b - 80 b
or, 8 m = 16 b
$\therefore$ 1 man = 2 boys.
Thus, 12 men + 16 boys = 24 boys + 16 boys = 40 boys
and 7 men + 10 boys = 14 boys + 10 boys = 24 boys
The question now becomes:

"If 40 boys can do a piece of work in 5 days how long will 24 boys take to do it?"

Now, by "basic formula", we have

$40 \times 5 = 24 \times D_2$ -----------(*) (*)

or, $D_2 = \frac{40 \times 5}{24} = 8\frac{1}{3}$ days

Note: During practice session (*) should be your first step to be written down. Further calculations should be done mentally. Once you get that 1 man = 2 boys, your next step should be (*) (*). This way you can get the result within seconds.

Ex.16: A certain number of men can do a work in 60 days. If there were 8 men more it could be finished in 10 days less. How many men are there?

Soln: Let there be x men originally.

$(x + 8)$ men can finish the work in $(60 - 10) = 50$ days.

Now, 8 men can do in 50 days what x men can do in 10 days, then by "basic formula" we have

$8 \times 50 = x \times 10$

$\therefore x = \frac{8 \times 50}{10} = 40$ men.

Another Approach: We have:

x men do the work in 60 days and $(x + 8)$ men do the work in $(60 - 10=)$ 50 days. Then by "basic formula", $60x = 50(x + 8)$

$\therefore x = \frac{50 \times 8}{10} = 40$ men.

Quicker Method (Direct Formula): There exists a relationship:

Original number of workers

$= \frac{\text{No. of more workers} \times \text{Number of days taken by the second group}}{\text{No. of less days}}$

$= \frac{8 \times (60 - 10)}{10} = \frac{8 \times 50}{10} = 40$ man

Ex.17: A is thrice as fast as B, and is therefore able to finish a work in 60 days less than B. Find the time in which they can do it working together.

Soln: A is thrice as fast as B, means that if A does a work in 1 day then B does it in 3 days.

Hence if the difference be 2 days, then A does the work in 1 day and B in 3 days. But the difference is 60 days. Therefore A does the work in 30 days and B in 90 days.

$\therefore$ Now A and B together will do the work in

$\frac{30 \times 90}{30 + 90}$ days $= \frac{45}{2} = 22.5$ days

Ex.18: I can finish a work in 15 days at 8 hrs a day. You can finish it in $6\frac{2}{3}$ days at 9 hrs a day. Find in how many days we can finish it working together 10 hrs a day.

Soln: First suppose each of us works for only one hr a day.

Then I can finish the work in $15 \times 8 = 120$ days and you can finish the work in $\frac{20}{3} \times 9 = 60$ days

Now we together can finish the work in

$$\frac{120 \times 60}{120 + 60} = 40 \text{ days.}$$

But here we are given that we do the work 10 hrs a day. Then clearly we can finish the work in 4 days.

Ex. 19: A can do a work in 6 days. B takes 8 days to complete it. C takes as long as A and B would take working together. How long will it take B and C to complete the work together ?

Soln : (A+B) can do the work in $\frac{6 \times 8}{6 + 8} = \frac{24}{7}$ days

$\therefore$ C takes $\frac{24}{7}$ days to complete the work.

$\therefore$ (B+C) takes $\frac{\frac{24}{7} \times 8}{\frac{24}{7} + 8} = \frac{24 \times 8}{24 + 56} = 2\frac{2}{5}$ days.

Ex. 20: A is twice as good a workman as B. Together, they finish the work in 14 days. In how many days can it be done by each separately?

Soln: Let B finish the work in 2x days. Since A is twice as active as B therefore, A finishes the work in x days.

(A+B) finish the work in $\frac{2x^2}{3x} = 14$

or $x = 21$

$\therefore$ A finishes the work in 21 days and B finishes the work in $21 \times 2 = 42$ days.

Quicker Approach : Twice + One time = Thrice active person does the work in 14 days. Then one-time active person (B) will do it in $14 \times 3 =$ 42 days and twice active person (A) will do it in $\frac{42}{2} = 21$ days.

Note : Efficient person takes less time. In other words we may say that *"Efficiency (E) is indirectly proportional to number of days (D) taken to complete a work"*. Then mathematically,

$E \propto \frac{1}{D}$ or, $E = \frac{K}{D}$, where K is a constant.

or, ED = Constant

or, $E_1D_1 = E_2D_2 = E_3D_3 = E_4D_4 = ...E_nD_n$

And we see in the above case:

$E_1D_1 = E_2D_2 = E_3D_3$

or, $3 \times 14 = 2 \times 21 = 1 \times 42$

Thus, our answer verifies the above statement.

Ex. 21: 5 men and 2 boys working together can do four times as much work per hour as a man and a boy together. Compare the work of a man with that of a boy.

Soln : The first group is 4 times as much efficient as the second group. What does it mean? It simply means that the second group will take 4 times as many days as the first group (See the Note given under Ex. 20).

Therefore, (5m + 2b)'s 1 day's work = (1m + 1b)'s 4 days' work

or, (5m + 2b)'s 1 day's work = (4m + 4b)'s 1 day's work

or, 5m + 2b = 4m + 4b

or, m = 2b

$\therefore \frac{m}{b} = \frac{2}{1}$

That is, a man is twice as efficient as a boy.

Ex. 22: 12 men or 15 women can reap a field in 14 days. Find the number of days that 7 men and 5 women will take to reap it.

Soln : This example is the same as Ex. 14. Three methods have been discussed for Ex. 14. If you remember the direct formula, you get

the required number of days $= \dfrac{1}{\dfrac{7}{14 \times 12} + \dfrac{5}{14 \times 15}} = \dfrac{1}{\dfrac{1}{24} + \dfrac{1}{42}}$

$= \dfrac{24 \times 42}{24 + 42} = \dfrac{168}{11} = 15\frac{3}{11}$ days.

Ex. 23: 10 men can finish a piece of work in 10 days, whereas it takes 12 women to finish it in 10 days. If 15 men and 6 women undertake to complete the work, how many days will they take to complete it ?

Soln: 10 men do the work in 10 days

$\therefore$ 15 men do the work in $10\left(\frac{10}{15}\right) = \frac{20}{3}$ days (by rule of fraction)

Similarly, 6 women do the work in $10\left(\frac{12}{6}\right) = 20$ days (by rule of fraction)

$\therefore$ 15 men + 6 women do the work in $\dfrac{\frac{20}{3} \times 20}{\frac{20}{3} + 20} = \dfrac{20 \times 20}{80} = 5$ days.

Quicker Approach: We see that the above question is : "10 men or 12 women do a work in 10 days. In how many days can 15 men and 6 women complete the work?"

Thus, this question is the same as Ex 14 or Ex 22.

$\therefore$ required number of days $= \dfrac{1}{\dfrac{15}{10 \times 10} + \dfrac{6}{12 \times 10}}$

$= \dfrac{1}{\dfrac{3}{20} + \dfrac{1}{20}} = \dfrac{20}{4} = 5$ days

Ex 24: A and B can do a work in 45 and 40 days respectively. They began the work together, but A left after some time and B finished the remaining work in 23 days. After how many days did A leave?

Soln: B works alone for 23 days.

$\therefore$ Work done by B in 23 days $= \frac{23}{40}$ work

$\therefore$ A + B do together $1 - \frac{23}{40} = \frac{17}{40}$ work

Now, A + B do 1 work in $\dfrac{40 \times 45}{40 + 45} = \dfrac{40 \times 45}{85}$ days

$\therefore$ A + B do $\frac{17}{40}$ work in $\dfrac{40 \times 45}{85} \times \dfrac{17}{40} = 9$ days

Quicker Maths (Direct formula): If we ignore the intermediate steps, we can write a direct formula as: $\dfrac{40 \times 45}{40 + 45}\left(\dfrac{40 - 23}{40}\right) = 9$ days.

Ex. 25: A certain number of men complete a work in 160 days. If th were 18 men more, the work could be finished in 20 days le How many men were originally there?

Soln: This question is the same as in Ex 16. See the **Quicker Metho** (direct formula) and apply here.

Original number of men $= \frac{18 \times (160 - 20)}{20} = 126$

Ex. 26: 4 men and 6 women finish a job in 8 days, while 3 men and women finish in 10 days. In how many days will 10 women fin it?

Soln: Method I. Considering one day's work:

$4m + 6w = \frac{1}{8}$ -------(1)

$3m + 7w = \frac{1}{10}$ -------(2)

(1) × 3 - (2) × 4 gives

$18w - 28w = \frac{3}{8} - \frac{4}{10}$

or, $10w = \frac{1}{40}$

∴ 10 women can do the work in 40 days.

Method II: See the theory used in Ex 15. We find that

$8(4m + 6w) = 10(3m + 7w)$

or, $2m = 22w$

∴ $4m = 44w$

∴ 4men + 6women = 50 women do in 8 days

∴ 10 women do in $\frac{8 \times 50}{10} = 40$ days.

Ex. 27: 1 man or 2 women or 3 boys can do a work in 44 days. Then in how many days will 1 man, 1 woman and 1 boy do th work?

Soln: This is an extended form of Ex 14.

Thus, by our extended formula, number of required days

$= \frac{1}{\frac{1}{44 \times 1} + \frac{1}{44 \times 2} + \frac{1}{44 \times 3}} = \frac{44 \times 1 \times 2 \times 3}{6 + 3 + 2} = 24$ days

Note : $\frac{1}{44 \times 1} = \frac{\text{Number of men in AND–part}}{\text{No. of days} \times \text{No. of men in OR–part}}$

$\frac{1}{44 \times 2} = \frac{\text{Number of women in AND–part}}{\text{No. of days} \times \text{No. of women in OR–part}}$

Similarly, you can define $\frac{1}{44 \times 3}$.

Ex. 28: 3 men and 4 boys do a work in 8 days, while 4 men and 4 boys do the same work in 6 days. In how many days will 2 men and 4 boys finish the work?

Soln: This question is the same as Ex 15. Try yourself.

Ex. 29: A is thrice as good a workman as B. Together they can do a job in 15 days. In how many days will B finish the work?

Soln: This question is the same as Ex 20.

Thrice + One time = 4 times efficient person does in 15 days

∴ One-time efficient (B) will do in 15 × 4 = 60 days

Ex. 30: A group of men decided to do a work in 10 days, but five of them became absent. If the rest of the group did the work in 12 days, find the original number of men.

Soln: Suppose there were x men originally.

Then by "basic formula", $M_1D_1 = M_2D_2$, we have $10x = 12(x-5)$

∴ $x = \frac{(12 \times 5)}{2 - 10} = 30$ men.

Ex. 31: A builder decided to build a farmhouse in 40 days. He employed 100 men in the beginning and 100 more after 35 days and completed the construction in stipulated time. If he had not employed the additional men, how many days behind schedule would it have been finished?

Soln: Let 100 men only complete the work in x days.

Work done by 100 men in 35 days + Work done by 200 men in (40 - 35=) 5 days = 1.

or, $\frac{35}{x} + \frac{200 \times 5}{100x} = 1$

or, $\frac{45}{x} = 1$ ∴ $x = 45$ days.

Therefore, if additional men were not employed, the work would have lasted 45 - 40 = 5 days behind schedule time.

Quicker Approach:

200 men do the rest of the work in 40 - 35 = 5 days

$\therefore$ 100 men can do the rest of the work in $\frac{5 \times 200}{100} = 10$ days

$\therefore$ required number of days = 10 - 5 = 5 days.

Ex. 32: A team of 30 men is supposed to do a work in 38 days. After 25 days, 5 more men were employed and the work was finished one day earlier. How many days would it have been delayed if 5 more men were not employed ?

Soln: This question is similar to Ex 31. To solve it our quicker approach should be the same.

35 men do the rest of the job in 12 days. [12 = 38 - 25 - 1]

$\therefore$ 30 men can do the rest of the job in $\frac{12 \times 35}{30} = 14$ days

Thus the work would have been finished in 25 + 14 = 39 days, that is, (39 - 38 =) 1 day after the scheduled time.

Ex. 33: A can do a work in 25 days and B can do the same work in 20 days. They work together for 5 days and then A goes away. In how many days will B finish the work?

Soln: A + B can do the work in 5 days $= 5\left[\frac{1}{25} + \frac{1}{20}\right] = \frac{5 \times 45}{25 \times 20} = \frac{9}{20}$

Rest of the work $= 1 - \frac{9}{20} = \frac{11}{20}$

B will do the rest of the work in $20 \times \frac{11}{20} = 11$ days.

Ex. 34: A and B working separately can do a work in 9 and 12 days respectively. A starts the work and they work on alternate days. In how many days will the work be completed?

Soln: This question is the same as in Ex 11.

(A + B)'s 2 day's work $= \frac{1}{9} + \frac{1}{12} = \frac{7}{36}$

We see that $5 \times \frac{7}{36} = \frac{35}{36}$ (just less than 1), ie, (A + B) work for 5 pairs of days, ie, for 10 days.

Now rest of the work $\left(1 - \frac{35}{36} =\right) \frac{1}{36}$ is to be done by A.

A can do $\frac{1}{36}$ work in $9 \times \frac{1}{36} = \frac{1}{4}$ day.

$\therefore$ Total days $= 10 + \frac{1}{4} = 10\frac{1}{4}$ days.

Ex. 35: 8 children and 12 men complete a work in 9 days. Each child takes twice the time taken by a man. In how many days will 12 men finish the same work?

Soln: 2 children = 1 man

$\therefore$ 8 children + 12 men = 4 + 12 = 16 men.

$\therefore$ 12 men finish the work in $9\left(\frac{16}{12}\right) = 12$ days. -----------(*)

Note : (*) To find the result either use $M_1D_1 = M_2D_2$ (basic formula) or the "rule of fraction". We suggest you to use "rule of fraction". Since less men will do the work in more days. Therefore 9 should be multiplied by $\frac{16}{12}$ (a more-than-one fraction).

Ex. 36: 30 men working 7 hrs a day can do a work in 18 days. In how many days will 21 men working 8 hrs a day do the same work?

Soln: Using the formula :

$M_1D_1T_1W_2 = M_2D_2T_2W_1$

Since work is the same for the two cases,

$M_1D_1T_1 = M_2D_2T_2$ --------(*)

$\therefore D2 = \frac{M_1\ D_1\ T_1}{M_2\ T_2} = \frac{30 \times 18 \times 7}{21 \times 8} = 22\frac{1}{2}$ days.

Note(*): Man-day-hour is constant for a work.

Ex 37: A, B and C can do a work in 8, 16 and 24 days respectively. They all begin together. A continues to work till it is finished, C leaving off 2 days and B one day before its completion. In what time is the work finished?

Soln: Let the work be finished in x days.

Then A's x day's work + B's (x - 1) day's work + C's (x - 2) day's work = 1

or, $\frac{x}{8} + \frac{x-1}{16} + \frac{x-2}{24} = 1$

or, $\frac{6x + 3x - 3 + 2x - 4}{48} = 1$ or, $11x = 55$ $\therefore$ $x = 5$ days.

Ex. 38: There is sufficient food for 400 men for 31 days. After 28 days 280 men leave the place. For how many days will the rest of the food last for the rest of the men?

Soln: The rest of the food will last for (31-28=) 3 days if nobody leaves the place.

Thus the rest of the food will last for $3\left(\frac{400}{120}\right)$ days for the 12 men left.

$\therefore$ Ans $= 3\left(\frac{400}{120}\right) = 10$ days.

Note : For less persons the food will last longer, therefore 3 is multiplie by $\frac{140}{120}$, a more-than-one fraction.

Ex. 39: A takes as much time as B and C together take to finish a job. and B working together finish the job in 10 days. C alone can d the same job in 15 days. In how many days can B alone do th same work?

Soln: Quicker Method:

$(A + B) + (C)$ can do in $\frac{15 \times 10}{15 + 10} = 6$ days.

Since A's days = (B+C)'s days

B + C can do in $6 \times 2 = 12$ days.

$\therefore$ B [B = {B + C} - C] can do in $\frac{15 \times 12}{15 - 12} = 60$ days.

Ex. 40: A, B and C can do a work in 16 days, $12\frac{4}{5}$ days and 32 day respectively. They started the work together but after 4 days A left. B left the work 3 days before the completion of the work. In how many days was the work completed?

Soln : Suppose the work is completed in x days.

A's 4 days' work + B's (x - 3) days' work + C's x days' work = 1

or, $\frac{4}{16} + \frac{(x-3) \times 5}{64} + \frac{x}{32} = 1$

or, $\frac{16 + 5x - 15 + 2x}{64} = 1$ or, $7x + 1 = 64$ $\therefore x = 9$ days.

Ex. 41: Raju can do a piece of work in 16 days. Ramu can do the same work in $12\frac{4}{5}$ days while Gita can do in 32 days. All of them started to work together but Raju leaves after 4 days. Ramu leaves the job 3 days before the completion of the work. How long would the work last?

Soln : Suppose the work lasted for x days. Then, Raju's 4

days' work + Ramesh's (x - 3) days' work + Gita's x days' work = 1

or, $\frac{4}{16} + \frac{x-3}{12\frac{4}{5}} + \frac{x}{32} = 1$

or, $\frac{1}{4} + \frac{5(x-3)}{64} + \frac{x}{32} = 1$

or, $\frac{5(x-3) + 2x}{64} = \frac{3}{4}$

or, $7x - 15 = 48$

$\therefore x = \frac{48 + 15}{7} = \frac{63}{7} = 9$ days.

Ex. 42: A and B undertake to do a work for Rs 56. A can do it alone in 7 days and B in 8 days. If with the assistance of a boy they finish the work in 3 days then the boy gets Rs ______.

Soln : A's 3 days' work + B's 3 days' work + Boy's 3 days' work = 1

or, $\frac{3}{7} + \frac{3}{8}$ + Boy's 3 days' work = 1

or, Boy's 3 days' work $= 1 - \left(\frac{3}{7} + \frac{3}{8}\right) = \frac{11}{56}$

Ratio of shares $= \frac{3}{7} : \frac{3}{8} : \frac{11}{56} = \frac{3 \times 56}{7} : \frac{3 \times 56}{8} : \frac{11 \times 56}{56}$

$= 24 : 21 : 11$

$\therefore$ Boy's share $= \frac{56}{24 + 21 + 11} \times 11 = $ Rs 11

Ex. 43: A started a work and left after working for 2 days. Then B was called and he finished the work in 9 days. Had A left the work after working for 3 days, B would have finished the remaining work in 6 days. In how many days can each of them, working alone, finish the whole work?

Soln: Detailed Method: Suppose A and B do the work in x and y days respectively. Now, work done by A in 2 days + work done by B in 9 days = 1

or, $\frac{2}{x} + \frac{9}{y} = 1$ Similarly, $\frac{3}{x} + \frac{6}{y} = 1$

To solve the above equation put $\frac{1}{x} = a$ and $\frac{1}{y} = b$. Thus

$2a + 9b = 1$ --------(1) and $3a + 6b = 1$ --------(2)

Performing (2) × 3 - (1) × 3 we have

$5a = 1 \quad \therefore \; a = \frac{1}{5}$ or, $x = \frac{1}{a} = 5$ days.

and $y = \frac{1}{b} = 15$ days.

Quicker Method: Direct formula: In such case:

A will finish the work in $\frac{3 \times 9 - 2 \times 6}{9 - 6} = \frac{15}{3} = 5$ days.

For B, we should use the above result.

B does $1 - \frac{2}{5} = \frac{3}{5}$ work in 9 days.

$\therefore$ B does 1 work in $9 \times \frac{5}{3} = 15$ days.

Ex. 44: If 5 men and 3 boys can reap 23 acres in 4 days, and 3 men and 2 boys can reap 7 acres in 2 days, how many boys must assist 7 men in order that they may reap 45 acres in 6 days?

Soln: Firstly, we should find the relationship between man and boys. We may find two equations from the two given statements. The first statement implies that 5 men and 3 boys can reap $\frac{23}{4}$ acres in 1 day.

We write this as:

$5m + 3b = \frac{23}{4}$ ------(1)

Similarly, from the second statement

$3m + 2b = \frac{7}{2}$ -------(2)

Now, to find the relationship,

$\frac{5m + 3b}{3m + 2b} = \frac{23}{4} \div \frac{7}{2} = \frac{23}{14}$

or, $70m + 42b = 69m + 46b$

$\therefore \; m = 4b$ (or, one man is equal to 4 boys)

$\therefore \; 5m + 3b = 5 \times 4 + 3 = 23$ boys (for the first statement)

Now, use $M_1 D_1 W_2 = M_2 D_2 W_1$

$\therefore \; M_2 = \frac{M_1 D_1 W_2}{D_2 W_1} = \frac{23 \times 4 \times 45}{6 \times 23} = 30$ boys.

$\therefore \; 30 - 7 \times 4 = 2$ boys should assist them.

Ex. 45: A contractor undertakes to dig a canal 12 km long in 350 days and employs 45 men. After 200 days he finds that only 4.5 km of the canal has been completed. Find the number of extra men he must employ to finish the work in time.

Soln: To apply the rule of fraction or our direct formula, we rewrite the above question as:

45 men prepare 4.5 km canal in 200 days.

Then how many more persons are needed to prepare

12 - 4.5 = 7.5 km in 350 - 200 = 150 days?

By rule of fraction: $45\left(\frac{200}{150}\right)\left(\frac{7.5}{4.5}\right) = 100$ men

$\therefore$ required no. of persons to be added = 100 - 45 = 55 men.

By Direct Formula: $M_1 D_1 W_2 = M_2 D_2 W_1$

or, $45 \times 200 \times 7.5 = M_2 \times 150 \times 4.5$

$\therefore \; M_2 = \frac{45 \times 200 \times 7.5}{150 \times 4.5} = 100$

$\therefore$ required number of persons to be added

= 100 - 45 = 55 men.

Ex. 46: 8 men and 16 women can do a work in 8 days. 40 men and 48 women can do the same work in 2 days. How many days are required for 6 men and 12 women to do the same work?

Soln: Method I: The man-power (ie, no. of persons doing the job) is **indirectly** proportional to number of days (i.e., more man-power, less days or, less man-power, more days)

So, we **can't** write the equation like;

$8m + 16w = 8$ or $40m + 48w = 2$

Now, we have to find the two things which are **directly proportional** to each other. Clearly these two things in this respect are **man-power** and **work**. So, we change the relationship and find the work done by each group in one day. Then we have the equations

$8m + 16w = \frac{1}{8}$ .... (i)

and $40m + 48w = \frac{1}{2}$ .... (ii)

and we have to find: $6m + 12w = ?$

Now, (2) - 3 × (1), gives

$16\,m = \frac{1}{2} - \frac{3}{8} = \frac{1}{8}$ $\therefore$ $6\,m = \frac{6}{16 \times 8}$

Again, $5 \times (1) - (2)$, gives $32\,w = \frac{5}{8} - \frac{1}{2} = \frac{1}{8}$ $\therefore$ $12\,w = \frac{12}{32 \times 8}$

Now, $6m + 12w = \frac{6}{16 \times 8} + \frac{12}{32 \times 8} = \frac{3}{64} + \frac{3}{64} = \frac{6}{64} = \frac{3}{32}$

Therefore, 6 men and 12 women will do the job in $\frac{32}{3} = 10\frac{2}{3}$ days.

Method II: We will compare the capacity of a man and woman. To do so, we apply:

$8\,(8m + 16\,w) = 2\,(40\,m + 48\,w)$
or, $64m + 128\,w = 80\,m + 96\,w$
or, $16\,m = 32\,w$ $\therefore$ $1\,m = 2\,w$
Now, $8\,m + 16\,w = 16\,w + 16\,w = 32\,w$ (from first information)
$6\,m + 12\,w = 12\,w + 12\,w = 24\,w$
(from required information)
Now, apply the formula: $M_1 D_1 = M_2 D_2$

Then $32 \times 8 = 24 \times D_2$ $\therefore$ $D_2 = \frac{32 \times 8}{24} = \frac{32}{3} = 10\frac{2}{3}$ days.

Note: This method works very fast, so we suggest you to follow only this method. One more method for special cases (which is applicable in this case) is being discussed below:

Method III: (Very Quicker, but for special cases only):

First, you should know the type of question where this method can be applied. See the number of men and women in the question part. Find the ratio of these two numbers, like in this case:

men : women = 6 : 12 = 1 : 2.

Now, look at the question-parts for the same ratio. In this case the first question-part has the same ratio, i.e., 8 : 16 = 1 : 2. Now, we can use this method. If there is no such ratio in question part, we can't use this method.

$8\,m + 16\,w$ do the work in 8 days
or, $8\,(m + 2\,w)$ " " 8 days
or, $(m + 2\,w)$ " " $8 \times 8 = 64$ days
$\therefore$ $6\,(m + 2\,w)$ " " $\frac{64}{6} = \frac{32}{3}$ days
$\therefore$ $6m + 12w$ " " $10\frac{2}{3}$ days

Ex. 47: 38 men, working 6 hours a day can do a piece of work in 12 days. Find the number of days in which 57 men working 8 hrs a day can do twice the work. Assume that 2 men of the first group do as much work in 1 hour as 3 men of the second group do in $1\frac{1}{2}$ hrs.

Soln: Detailed Method:

2×1 men of first group = 3×1.5 men of second group
or, 2 men of first group = 4.5 men of second group

$\therefore$ 38 men of first group = $\frac{4.5}{2} \times 38 = 19 \times 4.5$

$\because$ (19×4.5) men do 1 work, working 6 hrs/day in 12 days.
$\therefore$ 1 man does 1 work working 1 hr/day in $(12 \times 19 \times 4.5 \times 6)$ days
$\therefore$ 57 men do 2 work working 8 hrs/day in
$\frac{12 \times 19 \times 4.5 \times 6}{57 \times 8} \times 2 = 27$ days

Quicker Method:

Ratio of efficiency of persons in first group to the second group $= E_1 : E_2 = (3 \times 1.5) : 2 \times 1 = 4.5 : 2$ ----- (*)
Now, use the formula: $\mathbf{M_1D_1T_1E_1W_2 = M_2D_2T_1E_2W_1}$ --- (*)(*)

$\therefore$ $D_2 = \frac{38 \times 12 \times 6 \times 4.5 \times 2}{57 \times 8 \times 2 \times 1} = 27$ days.

Note: (*) Less number of persons from the first group do the same work in less number of days, so they are more efficient.

(*)(*) M represents the number of men.
D represents the number of days.
T represents the number of working hours.
E represents the efficiency.
W represents the work.
and the suffix represents the respective groups.

EXERCISES

1. Mohan can do a job in 20 days and Sohan can do the same job in 30 days. How long would they take to do it working together?
2. Raju, Rinku and Ram can do a work in 6, 12 and 24 days respectively. In what time will they altogether do it?
3. A and B working together can do a piece of work in 6 days. B alone can do it in 8 days. In how many days A alone could finish?

4. A and B can finish a field work in 30 days, B and C in 40 days while C and A in 60 days. How long will they take to finish it together?
5. A can copy 75 pages in 25 hrs. A and B together can copy 135 pages in 27 hrs. In what time can B copy 42 pages?
6. A, B and C can finish a work in 10, 12 and 15 days respectively. If B stops after 2 days, how long, would it take A and C to finish the remaining work?
7. B can do a job in 6 hrs, B and C can do it in 4 hrs and A, B and C in $2\frac{2}{3}$ hrs. In how many hrs can A and B do it?
8. I can finish a work in 15 days at 8 hrs a day. You can finish it in $6\frac{2}{3}$ days at 9 hrs a day. Find in how many days we can finish it together, if we work 10 hrs a day?
9. A can do a work in 7 days. If A does twice as much work as B in a given time, find how long A and B would take to do the work.
10. A can do a work in 6 days. B takes 12 days. C takes as long as A and B would take working together. How long will it take B and C to complete the work together?
11. A is twice as good a workman as B; and together they finish a work in 16 days. In how many days can it be done by each separately?
12. If 3 men or 5 women can reap a field in 43 days, how long will 5 men and 6 women take to reap it?
13. If 5 men and 2 boys working together can do four times as much work per hour as a man and a boy together, compare the work of a man with that of a boy.
14. One man, 3 women and 4 boys can do a work in 96 hrs; 2 men and 8 boys can do it in 80 hrs; and 2 men and 3 women can do it in 120 hrs. In how many hours can it be done by 5 men and 12 boys?
15. A and B working separately can do a piece of work in 9 and 12 days respectively. If they work for a day alternately, A beginning, in how many days will the work be completed?
16. A sum of money is sufficient to pay A's wages for 21 days or B's wages for 28 days. The money is sufficient to pay the wages of both for _____days.
17. A does half as much work as B in three-fourth of the time. If together they take 18 days to complete a work, how much time shall B take to do it?
18. 10 men can finish a piece of work in 10 days, whereas it takes 12 women to finish it in 10 days. If 15 men and 6 women undertake to complete the work, how many days will they take to complete it?

19. A and B can do a piece of work in 30 and 40 days respectively. They began the work together, but A left after some days and B finished the remaining work in 12 days. After how many days did A leave?
20. A certain number of men complete a piece of work in 60 days. If there were 8 men more, the work could be finished in 10 days less. How many men were originally there ?
21. 8 children and 12 men complete a certain piece of work in 9 days. If each child takes twice the time taken by a man to finish the work, in how many days will 12 men finish the same work?
22. 2 men and 3 women can finish a piece of work in 10 days, while 4 men can do it in 10 days. In how many days will 3 men and 3 women finish it?
23. 3 men and 4 boys do a piece of work in 8 days, while 4 men and 4 boys finish it in 6 days. 2 men and 4 boys will finish it in ____ days.
24. If 1 man or 2 women or 3 boys can do a piece of work in 44 days, then the same piece of work will be done by 1 man, 1 woman and 1 boy in _______ days.

ANSWERS

1. $\frac{20 \times 30}{20 + 30} = 12$ days
2. $\frac{6 \times 12 \times 24}{6 \times 12 + 12 \times 24 + 6 \times 24} = \frac{6 \times 12 \times 24}{72 + 288 + 144} = \frac{6 \times 12 \times 24}{504} = 3\frac{3}{7}$ days
3. $\frac{6 \times 8}{8 - 6} = 24$ days
4. 2 (A + B + C) will do the work in $\frac{30 \times 40 \times 60}{30 \times 40 + 30 \times 60 + 40 \times 60} = \frac{40}{3}$

   $\therefore$ A + B + C will do in $\frac{80}{3} = 26\frac{2}{3}$ days
5. A can copy $\frac{75}{25} = 3$ pages in 1 hr.

   A + B can copy $\frac{135}{27} = 5$ pages in 1 hr.

   $\therefore$ B can copy 5 - 3 = 2 pages in 1 hr.

   $\therefore$ B can copy 42 pages in $\frac{42}{2} = 21$ hrs.

6. A + B + C in 2 days, do $2\left(\frac{1}{10}+\frac{1}{12}+\frac{1}{15}\right)$ work.

$= 2\left(\frac{1}{4}\right) = \frac{1}{2}$ work.

Now, B withdraws. A + B will do the whole work in

$\frac{10 \times 12}{12 + 10} = \frac{60}{11}$ days

$\therefore$ A + B will do $\frac{1}{2}$ work in $\frac{30}{11} = 2\frac{8}{11}$ days

7. A + B + C can do the work in $\frac{8}{3}$ hrs --- (1)

B + C can do the work in 4 hrs ---- (2)

B can do the work in 6 hrs ----- (3)

From (2) and (3), C can do it in $\frac{4 \times 6}{6 - 4} = 12$ hrs ----- (4)

From (1) & (4)

A + B can do it in $\dfrac{\frac{8}{3} \times 12}{12 - \frac{8}{3}} = \frac{32 \times 3}{28} = \frac{24}{7} = 3\frac{3}{7}$ hrs

8. Change the time into hours.

I finish in $15 \times 8 = 120$ hrs

You finish in $\frac{20}{3} \times 9 = 60$ hrs

$\therefore$ both of us working together finish the work in $\frac{120 \times 60}{120 + 60} = 40$ hrs.

$\therefore$ number of days $= \frac{40}{10} = 4$ days.

Neglecting the intermediate steps, the **direct formula** can be written as:

$$\frac{1}{10}\left[\frac{(15 \times 8) \times \left(\frac{20}{3} \times 9\right)}{(15 \times 8) + \left(\frac{20}{3} \times 9\right)}\right] = \frac{1}{10}\left[\frac{120 \times 60}{180}\right] = 4 \text{ days.}$$

Note: See Ex 18 (solved). The method is different there. You are suggested to adopt the earlier method.

9. A is twice as efficient as B, hence B will do the work in 14 days.

$\therefore$ A + B will do in $\frac{7 \times 14}{7 + 14} = \frac{14}{3} = 4\frac{2}{3}$ days.

10. A + B take $\frac{6 \times 12}{6 + 12} = 4$ days

$\therefore$ C takes 4 days

$\therefore$ B + C take $\frac{12 \times 4}{12 + 4} = 3$ days

11. Suppose B does in 2x days.

$\therefore$ A does in x days.

Now, working together, they can do in

$\frac{2x^2}{3x} = 16$ days

or, x = 24 days

$\therefore$ A does in 24 days and B does in 48 days.

Note: For quicker approach see solved Ex 20.

12. 3 men = 5 women

$\therefore$ 5 men $= \frac{5}{3} \times 5 = \frac{25}{3}$ women.

$\therefore$ 5 men + 6 women $= \frac{25}{3} + 6 = \frac{43}{3}$ women

Now, we are given that 5 women do in 43 days.

$\therefore \frac{43}{3}$ women do in $(43)\left(\frac{3}{43}\right) \times 5 = 15$ days.

Quicker Method: (See Ex 14)

Required no. of days $= \dfrac{1}{\left[\frac{5}{3 \times 43} + \frac{6}{5 \times 43}\right]} = 15$ days.

13. 5m + 2b = 4 (1m + 1b)

or, m = 2b

$\therefore \frac{m}{b} = \frac{2}{1}$

Therefore, a man does twice as much work as a boy does.

14. 1m + 3w + 4b in 96 hrs ---- (1)

2m + 8b in 80 hrs ---- (2)

or, 1m + 4b in 160 hrs ---- (3)

2m + 3w in 120 hrs ---- (4)

From (1) and (3), we have,

3w do the work in $\frac{160 \times 96}{160 - 96} = 240$ hrs --- (5)

From (4) and (5), we have;

2m do the work in $\frac{240 \times 120}{240 - 120} = 240$ hrs --- (6)

$\therefore$ 5m do the work in $240 \times \frac{2}{5} = 96$ hrs ---- (7)

From (2) and (6) we have,

8b do the work in $\frac{80 \times 240}{240 - 80} = 120$ hrs.

$\therefore$ 12b do the work in $\frac{120 \times 8}{12} = 80$ hrs ---- (8)

Now, from (7) and (8) we have,

5m + 12 b do the work in $\frac{96 \times 80}{96 + 80} = \frac{480}{11} = 43\frac{7}{11}$ hrs

15. (A + B)'s work in 2 days $= \frac{1}{9} + \frac{1}{12} = \frac{4+3}{36} = \frac{7}{36}$

In 5 pairs of days they will complete $\frac{7 \times 5}{36} = \frac{35}{36}$.

That is, after $5 \times 2 = 10$ days, $1 - \frac{35}{36} = \frac{1}{36}$ work is left which will be done by A alone.

A does 1 work in 9 days.

$\therefore$ A does $\frac{1}{36}$ work in $9 \times \frac{1}{36} = \frac{1}{4}$ days

$\therefore$ Total number of days $= 10 + \frac{1}{4} = 10\frac{1}{4}$ days.

16. Let the sum be equal to LCM of 21 and 28, ie. Rs 84.

Then A gets $\frac{84}{21}$ = Rs 4/day

and B gets $\frac{84}{28}$ = Rs 3/day

A + B get 4 + 3 = Rs 7/day

$\therefore$ Rs 84 is sufficient for $\frac{84}{7} = 12$ days to pay both of them.

Quicker Method (Direct formula) :

Number of days $= \frac{\text{Multiplication of no. of days}}{\text{Addition of no. of days}}$

$= \frac{21 \times 28}{21 + 28} = 12$ days.

17. Suppose B does the work in x days

Then, A does $\frac{1}{2}$ work in $\frac{3x}{4}$ days

$\therefore$ A does 1 work in $\frac{3x}{2}$ days

$\therefore$ A + B do the work in $\frac{x \times \frac{3x}{2}}{x + \frac{3x}{2}} = 18$ (given)

or, $\frac{\frac{3}{2}x^2}{\frac{5}{2}x} = 18$; $\therefore x = \frac{18 \times 5}{3} = 30$ days.

18. 15 men do the work in $\frac{10 \times 10}{15} = \frac{20}{3}$ days

6 women do the work in $\frac{12 \times 10}{6} = 20$ days

$\therefore$ 15 men + 6 women do in $\frac{\frac{20}{3} \times 20}{\frac{20}{3} + 20} = \frac{20 \times 20}{80} = 5$ days

19. See Ex 24.

By direct formula,

reqd no. of days $= \frac{30 \times 40}{30 + 40}\left[\frac{40 - 12}{40}\right] = 12$ days

20. Let there be x men originally, then 1 man will do the work in 60x days. In the second case, 1 man does the work in (x + 8) 50 days.

Now, 60x = 50 (x + 8) $\therefore x = \frac{400}{10} = 40$ men

Quicker Maths (Direct formula) :

Number of men $= \frac{\text{No. of more men} \times (60 - 10)}{10} = \frac{8 \times 50}{10} = 40$ men

21. If each child takes twice the time taken by a man, 8 children = 4 men.

$\therefore$ 8 children + 12 men = 16 men do the work in 9 days.

$\therefore$ 12 men finish the work in $\frac{9 \times 16}{12} = 12$ days

22. 4 men do in 10 days

$\therefore$ 2 men do in 20 days

$\therefore$ 3 women do in $\frac{10 \times 20}{20 - 10} = 20$ days

and 3 men do in $\frac{40}{3}$ days

$\therefore$ 3 men + 3 women do in $\frac{20 \times \frac{40}{3}}{20 + \frac{40}{3}} = \frac{20 \times 40}{100} = 8$ days.

23. 3 men + 4 boys do in 8 days --- (1)

4 men + 4 boys do in 6 days --- (2)

Subtracting (1) from (2) we have,

1 man does in $\frac{8 \times 6}{8 - 6} = 24$ days --- (3)

$\therefore$ 3 men do in $\frac{24}{3} = 8$ days --- (4)

From (1) and (4) we conclude that boys do no work.

$\therefore$ 2 men + 4 boys = 2 men will finish the work in $\frac{24}{2} = 12$ days.

24. 1 man = 2 women = 3 boys

$\therefore$ 1 man + 1 woman + 1 boy = 3 boys + $\frac{3}{2}$ boys + 1 boy = $\frac{11}{2}$ boys

Now, 3 boys do the work in 44 days

$\therefore$ $\frac{11}{2}$ boys do the work in $\frac{44 \times 3}{11} \times 2 = 24$ days

Work and Wages

Theorem: *Wages are distributed in proportion to the work done and in indirect (or inverse) proportion to the time taken by the individual.*

Ex.1: A can do a work in 6 days and B can do the same work in 5 days. The contract for the work is Rs 220. How much shall B get if both of them work together?

Soln: Method I:

A's 1 day's work = $\frac{1}{6}$; B's 1 day's work = $\frac{1}{5}$

$\therefore$ ratio of their wages = $\frac{1}{6} : \frac{1}{5} = 5 : 6$

$\therefore$ B's share = $\frac{220}{5 + 6} \times 6$ = Rs 120

Method II: As wages are distributed in inverse proportion of number of days, their share should be in the ratio 5 : 6.

$\therefore$ B's share = $\frac{220}{11} \times 6$ = Rs 120

Ex.2: A man can do a work in 10 days. With the help of a boy he can do the same work in 6 days. If they get Rs 50 for that work, what is the share of that boy?

Soln: The boy can do the work in $\frac{10 \times 6}{10 - 6} = 15$ days. [Recall the theorem]

Man's share : Boy's share = 15 : 10 = 3 : 2

Man's share = $\frac{50}{5} \times 3$ = Rs 30

Ex.3: A, B and C can do a work in 6, 8 and 12 days respectively. Doing that work together they get an amount of Rs 1350. What is the share of B in that amount?

Soln: A's one day's work = $\frac{1}{6}$; B's one day's work = $\frac{1}{8}$;

C's one day's work = $\frac{1}{12}$

A's share : B's share : C's share = $\frac{1}{6} : \frac{1}{8} : \frac{1}{12}$

Multiplying each ratio by the LCM of their denominators, the ratios become 4 : 3 : 2

$\therefore$ B's share $= \frac{1350}{9} \times 3 =$ Rs. 450.

Direct Method : A's share : B's share : C's share =
B's time × C's time : A's time × C's time : A's time × B's time
$= 96 : 72 : 48 = 4 : 3 : 2$ $\therefore$ B's share $= \frac{1350}{9} \times 3 =$ Rs. 450.

Ex.4: A, B and C contract a work for Rs 550. Together, A and B are supposed to do $\frac{7}{11}$ of the work. How much does C get ?

Soln: A + B did $\frac{7}{11}$ work and C did $\left(1 - \frac{7}{11} =\right) \frac{4}{11}$ work.

$\therefore$ (A + B)'s share : C's share $= \frac{7}{11} : \frac{4}{11} = 7 : 4$

$\therefore$ C's share $= \frac{550}{11} \times 4 =$ Rs 200.

Ex.5: Two men undertake to do a piece of work for Rs 200. One alone could do it in 6 days, the other in 8 days. With the assistance of a boy they finish it in 3 days. How should the money be divided?

Soln: 1st man's 3 days' work $= \frac{3}{6}$; 2nd man's 3 days' work $= \frac{3}{8}$

The boy's 3 days' work $= 1 - \left(\frac{3}{6} + \frac{3}{8}\right) = \frac{1}{8}$

Their share will be in the ratio $\frac{3}{6} : \frac{3}{8} : \frac{1}{8} = 4 : 3 : 1$

$\therefore$ 1st man's share $= \frac{200}{8} \times 4 =$ Rs 100

2nd man's share $= \frac{200}{8} \times 3 =$ Rs 75

The boy's share $= \frac{200}{8} \times 1 =$ Rs 25

Ex. 6: Wages for 45 women amount to Rs 15525 in 48 days. How many men must work 16 days to receive Rs 5750, the daily wages of a man being double those of a woman?

Soln: Wage of a woman for a day $= \frac{15525}{45 \times 48} =$ Rs $\frac{115}{16}$

Thus, wage of a man for a day $= 2 \times \frac{115}{16} =$ Rs $\frac{115}{8}$

Now, number of men

$= \frac{\text{Total wage}}{\text{No. of days} \times 1 \text{ man's 1 day's wage}} = \frac{5750 \times 8}{16 \times 115} = 25$ men

Note: We should remember the relationship:
Total wage = One person's one day's wage × No. of persons × No. of days.

Ex.7: 3 men and 4 boys can earn Rs 756 in 7 days. 11 men and 13 boys can earn Rs 3008 in 8 days. In what time will 7 men with 9 boys earn Rs 2480?

Soln: (3m + 4b) in 1 day earn Rs $\frac{756}{7} =$ Rs 108 ----(1)

(11m + 13b) in 1 day earn Rs $\frac{3008}{8} =$ Rs 376 ------(2)

From (1), we see that to earn Re 1 in 1 day there should be $\frac{3m + 4b}{108}$ persons. Similarly, from (2), to earn Re 1 in 1 day there should be $\frac{11m + 13b}{376}$ persons.

And also; $\frac{3m + 4b}{108} = \frac{11m + 13b}{376}$ ------- (*)

or, m (3 × 376 - 11 × 108) = b (108 × 13 - 4 × 376) ----- (*) (*)

$\therefore \frac{m}{b} = \frac{100}{60} = \frac{5}{3}$

Now, from (1)
(3m + 4b) in 1 day earn Rs 108

or, $3m + 4 \times \frac{3}{5}m$ in 1 day earn Rs 108

or, $\frac{27m}{5}$ in 1 day earn Rs 108

$\therefore$ 1m in 1 day earns Rs $\frac{108 \times 5}{27} =$ Rs 20

Thus, we get that a man earns Rs 20 daily and a boy earns Rs $20 \times \frac{3}{5} =$ Rs 12 daily.

$\because$ 7m + 9b earn Rs (7 × 20 + 9 × 12) = Rs 248 in 1 day.
$\therefore$ 7m + 9b earn Rs 2480 in 10 days.

Note : (*) Since both the LHS and the RHS denote the same quantity : "Number of persons earning Re 1 in 1 day".

(*) (*) We can arrive at this step directly be using cross-multipli cation-division rule. Arrange the given information as follows:

Men	Boys		Earning		Days
3	4	×	756	÷	7
11	13	×	3008	÷	8

Now,

$$\text{Men}\left(\frac{3\times3008}{8}-\frac{11\times756}{7}\right)=\text{Boys}\left(\frac{13\times756}{7}-\frac{4\times3008}{8}\right)$$

or, $m(3\times376-11\times108)=b(108\times13-4\times376)$ or, $\frac{m}{b}=\frac{5}{3}$

Ex 8: 12 men with 13 boys can earn Rs 326.25 in 3 days. 5 men with boys can earn Rs 237.5 in 5 days. In what time will 3 men wit 4 boys earn Rs 210?

Soln: Solve yourself (same as Ex 7).

Ex 9: A, B and C together earn Rs 1350 in 9 days. A and C togethe earn Rs 470 in 5 days. B and C together earn Rs 760 in 10 days Find the daily earning of C.

Soln: Daily earning of A+B+C = $\frac{\text{Rs } 1350}{9}$ = Rs 150 -----(1)

Daily earning of A+C = $\frac{\text{Rs } 470}{5}$ = Rs 94 ----- (2)

Daily earning of B+C = $\frac{\text{Rs}760}{10}$ = Rs 76 ----- (3)

From (1) and (2) daily earning of B = 150 - 94 = Rs 56 -------(4

From (3) and (4) daily earning of C = 76 - 56 = Rs 20

EXERCISES

1. Two men A and B working together complete a piece of work which it would have taken them respectively 12 and 18 days to complete if they worked separately. They received in payment Rs 149.25. Find their shares.
2. A, B and C together do a piece of work for Rs 53.50. A working alone could do it in 5 days, B working alone could do it in 6 days and C working alone could do it in 7 days. How should the money be divided among them ?
3. It the wages of 45 women amount to Rs 15525 in 48 days, how many men must work 16 days to receive Rs 5750, the daily wages of a man being double those of a woman?

4. If 3 men with 4 boys can earn Rs 210 in 7 days, and 11 men with 13 boys can earn Rs 830 in 8 days, in what time will 7 men with 9 boys earn Rs 1100 ?
5. If 12 men with 13 boys can earn Rs 326.25 in 3 days, and 5 men with 6 boys can earn Rs 237.50 in 5 days, in what time will 3 men with 4 boys earn Rs 210?

ANSWERS

1. Wages are distributed in inverse proportion of number of days. Hence the money will be divided in the ratio 18 : 12

$\therefore$ A gets $\frac{149.25}{30}\times18$ = Rs 89.55

and B gets $\frac{149.25}{30}\times12$ = Rs 59.70

2. A's share : B's share : C's share

$=6\times7:5\times7:5\times6$

$=42:35:30$

$\therefore$ A's share = $\frac{53.50}{42+35+30}\times42=\frac{53.50}{107}\times42$ = Rs 21

B's share = $\frac{53.50}{107}\times30$ = Rs 15

3. See Ex. 6.
4. See Ex. 7.
5. Same as Ex. 4.

Pipes and Cisterns

Introduction : Pipes and Cisterns problems are almost the same as those of Time and Work problems. Thus, if a pipe fills a tank in 6 hrs, then the pipe fills $\frac{1}{6}$ th of the tank in 1 hour. The only difference with Pipes and Cisterns problems is that there are outlets as well as inlets. Thus, there are agents (the outlets) which perform negative work too. The rest of the process is almost similar.

Inlet : A pipe connected with a tank (or a cistern or a reservoir) is called an **inlet**, if it fills it.

Outlet: A pipe connected with a tank is called an **outlet**, if it empties it.

FORMULAE:

(i) If a pipe can fill a tank in x hours, then the part filled in 1 hour = $\frac{1}{x}$.

(ii) If a pipe can empty a tank in y hours, then the part of the full tank emptied in 1 hour = $\frac{1}{y}$.

(iii) If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours, then the net part filled in 1 hour, when both the pipes are opened = $\left(\frac{1}{x} - \frac{1}{y}\right)$.

$\therefore$ time taken to fill the tank, when both the pipes are opened

$$= \frac{xy}{y-x}$$

(iv) If a pipe can fill a tank in x hrs and another can fill the same tank in y hrs, then the net part filled in 1 hr, when both the pipes are opened $= \left(\frac{1}{x} + \frac{1}{y}\right)$.

$\therefore$ time taken to fill the tank = $\frac{xy}{y+x}$

(v) If a pipe fills a tank in x hrs and another fills the same tank in y hrs, but a third one empties the full tank in z hrs, and all of them are opened together, the net part filled in 1 hr = $\left[\frac{1}{x} + \frac{1}{y} - \frac{1}{z}\right]$

$\therefore$ time taken to fill the tank $= \frac{xyz}{yz + xz - xy}$ hrs.

(vi) A pipe can fill a tank in x hrs. Due to a leak in the bottom it is filled in y hrs. If the tank is full, the time taken by the leak to empty the tank $= \frac{xy}{y - x}$ hrs.

Ex 1 : Two pipes A and B can fill a tank in 36 hours and 45 hours respectively. If both the pipes are opened simultaneously, how much time will be taken to fill the tank ?

Soln : Part filled by A alone in 1 hour $= \frac{1}{36}$

Part filled by B alone in 1 hour $= \frac{1}{45}$

$\therefore$ Part filled by (A+B) in 1 hour $= \left(\frac{1}{36} + \frac{1}{45}\right) = \frac{9}{180} = \frac{1}{20}$.

Hence, both the pipes together will fill the tank in 20 hours.

Direct Method : [By formula (iv)]

Time taken $= \frac{36 \times 45}{36 + 45} = 20$ hrs.

Ex 2 : A pipe can fill a tank in 15 hours. Due to a leak in the bottom, it is filled in 20 hours. If the tank is full, how much time will the leak take to empty it ?

Sol : Work done by the leak in 1 hour $= \left(\frac{1}{15} - \frac{1}{20}\right) = \frac{1}{60}$.

$\therefore$ the leak will empty the full tank in 60 hrs.

Direct Method : [By formula (vi)]

Required time $= \frac{15 \times 20}{20 - 15} = 60$ hrs.

Ex 3 : Pipe A can fill a tank in 20 hours while pipe B alone can fill it in 30 hours and pipe C can empty the full tank in 40 hours. If all the pipes are opened together, how much time will be needed to make the tank full ?

Sol : Net part filled in 1 hour $= \left(\frac{1}{20} + \frac{1}{30} - \frac{1}{40}\right) = \frac{7}{120}$.

$\therefore$ The tank will be full in $\frac{120}{7}$ i.e. $17\frac{1}{7}$ hours.

Direct Method : [(By formula (v)]

$$\frac{20 \times 30 \times 40}{30 \times 40 + 20 \times 40 - 20 \times 30} = \frac{120}{7} = 17\frac{1}{7} \text{ hrs.}$$

Ex 4 : Two pipes A and B can fill a cistern in 1 hour and 75 minutes respectively. There is also an outlet C. If all the three pipes are opened together, the tank is full in 50 minutes. How much time will be taken by C to empty the full tank ?

Soln : Work done by C in 1 min. $= \left(\frac{1}{60} + \frac{1}{75} - \frac{1}{50}\right) = \frac{3}{300} = \frac{1}{100}$.

$\therefore$ C can empty the full tank in 100 minutes.

Ex 5 : In what time would a cistern be filled by three pipes whose diameters are 1 cm, $1\frac{1}{3}$ cm, 2 cm, running together, when the largest alone will fill it in 61 minutes, the amount of water flowing in by each pipe being proportional to the square of its diameter ?

Sol : In 1 minute the pipe of 2 cm diameter fills $\frac{1}{61}$ of the cistern.

In 1 minute the pipe of 1 cm diameter fills $\frac{1}{61} \times \frac{1}{4}$ of the cistern. ---(*)

In 1 minute the pipe of $1\frac{1}{3}$ cm diameter fills $\frac{1}{61} \times \frac{4}{9}$ of the cistern. --- (**)

$\therefore$ In 1 minute $\left(\frac{1}{61} + \frac{1}{61 \times 4} + \frac{4}{61 \times 9}\right) = \frac{1}{36}$ of the cistern is filled.

$\therefore$ the whole is filled in 36 minutes. Ans.

Note :(*) We are given that amount of water flowing is proportional to the square of the diameter of the pipe. Since 2 cm diameter fills $\frac{1}{61}$ of the cistern,

1 cm diameter fills $\frac{1}{61}\left(\frac{1}{2}\right)^2 = \frac{1}{61} \times \frac{1}{4}$ of the cistern.

(**) $1\frac{1}{3} = \frac{4}{3}$ cm diameter fills $\frac{1}{61} \times \frac{1}{4}\left(\frac{4}{3}\right)^2 = \frac{1}{61} \times \frac{4}{9}$ of the cistern.

Ex 6 : There is a leak in the bottom of a cistern. When the cistern is thoroughly repaired, it would be filled in $3\frac{1}{2}$ hrs. It now takes half an hour longer. If the cistern is full, how long would the leak take to empty the cistern ?

Soln : Required time $= \frac{3.5 \times 4}{4 - 3.5} = 28$ hrs.

Ex 7 : Two pipes A and B can fill a tank in 24 minutes and 32 minutes respectively. If both the pipes are opened simultaneously, after how much time should B be closed so that the tank is full in 18 minutes ?

Soln : Let B be closed after x minutes. Then, part filled by (A+B) in x min. + part filled by A in (18-x) min. = 1.

$\therefore x\left(\frac{1}{24} + \frac{1}{32}\right) + (18\text{-}x) \times \frac{1}{24} = 1$

or $\frac{7x}{96} + \frac{18 - x}{24} = 1$ or $7x + 4(18\text{-}x) = 96$

or, $3x = 24 \quad \therefore x = 8.$

So, B should be closed after 8 min.

Direct Formula:

Pipe B should be closed after $\left(1 - \frac{18}{24}\right) \times 32 = 8$ min.

Ex. 8: Two pipes P and Q would fill a cistern in 24 hours and 32 hours respectively. If both pipes are opened together, find when the first pipe must be turned off so that the cistern may be just filled in 16 hours.

Soln: Suppose the first pipe was closed after x hrs.

Then, first's x hrs' supply + second's 16 hrs' supply = 1

or, $\frac{x}{24} + \frac{16}{32} = 1 \qquad \therefore \frac{x}{24} = 1 - \frac{1}{2} = \frac{1}{2}$

$\therefore x = 12$ hrs.

Direct Formula:

The first pipe should work for $\left(1 - \frac{16}{32}\right) \times 24$ hrs. = 12 hrs.

Ex. 9: If two pipes function simultaneously, the reservoir is filled in 12 hrs. One pipe fills the reservoir 10 hours faster than the other. How many hours does the faster pipe take to fill the reservoir?

Soln : Let the faster pipe fills the tank in x hrs.

Then the slower pipe fills the tank in x + 10 hrs.

When both of then are opened, the reservoir will be filled in

$$\frac{x(x + 10)}{x + (x + 10)} = 12$$

or, $x^2 - 14x - 120 = 0$

$\therefore x = 20, -6$

But x can't be -ve, hence the faster pipe will fill the reservoir in 20 hrs.

Ex. 10: Three pipes A, B and C can fill a cistern in 6 hrs. After working together for 2 hours, C is closed and A and B fill the cistern in 8 hrs. Then find the time in which the cistern can be filled by pipe C.

Soln : A + B + C can fill in 1 hr $= \frac{1}{6}$ of cistern.

A + B + C can fill in 2 hrs $= \frac{2}{6} = \frac{1}{3}$ of cistern.

Unfilled part $= \left(1 - \frac{1}{3} = \right)\frac{2}{3}$ is filled by A + B in 8 hrs.

$\therefore$ (A + B) can fill the cistern in $\frac{8 \times 3}{2} = 12$ hrs.

And we have (A + B + C) can fill the cistern in 6 hrs.

$\therefore$ C = (A + B + C) - (A + B) can fill the cistern in $\frac{12 \times 6}{12 - 6} = 12$ hrs.

Ex. 11: A tank has a leak which would empty it in 8 hrs. A tap is turned on which admits 6 litres a minutes into the tank, and it is now emptied in 12 hrs. How many litres does the tank hold?

Soln : The filler tap can fill the tank in $\frac{12 \times 8}{12 - 8} = 24$ hrs.

$\therefore$ Capacity of tank $= 24 \times 60 \times 6 = 8640$ litres.

Ex. 12: A tank is normally filled in 8 hours but takes 2 hrs longer to fill because of a leak in its bottom. If the cistern is full, in how many hrs will the leak empty it?

Soln : It is clear from the question that the filler pipe fills the tank in 8 hrs and if both the filler and the leak work together, the tank is filled in 8 hrs. Therefore, the leak will empty the tank in $\frac{8 \times 10}{10 - 8}$ = 40 hrs.

Ex. 13: A pipe can fill a tank in 12 minutes and another pipe in 15 minutes, but a third pipe can empty it in 6 minutes. The first two pipes are kept open for 5 minutes in the beginning and then the third pipe is also opened. In what time is the cistern emptied?

Soln : Cistern filled in 5 minutes $= 5\left(\frac{1}{12}+\frac{1}{15}\right)=\frac{3}{4}$

Net work done by 3 pipes in 1 minute

$=\left(\frac{1}{12}+\frac{1}{15}\right)-\frac{1}{6} = -\frac{1}{60}$

-ve sign shows that $\frac{1}{60}$ part is emptied in 1 minutes.

$\therefore \frac{3}{4}$ part is emptied in $60 \times \frac{3}{4} = 45$ minutes.

Ex. 14 : If three taps are opened together, a tank is filled in 12 hrs. One of the taps can fill it in 10 hrs and another in 15 hrs. How does the third tap work?

Soln : We have to find the nature of the third tap — whether it is a filler or a waste pipe.

Let it be a filler pipe which fills in x hrs.

Then, $\frac{10 \times 15 \times x}{10 \times 15 + 10x + 15x} = 12$

or, $150x = 150 \times 12 + 25x \times 12$

or, $-150x = 1800$

$\therefore x = -12$

-ve sign shows that the third pipe is a waste pipe which vacates the tank in 12 hrs.

Ex. 15: A, B and C are three pipes connected to a tank. A and B together fill the tank in 6 hrs. B and C together fill the tank in 10 hrs. A and C together fill the tank in $7\frac{1}{2}$ hrs. In how much time will A, B and C fill the tank separately?

Soln : A + B fill in 6 hrs.

B + C fill in 10 hrs.

A + C fill in $7\frac{1}{2}=\frac{15}{2}$ hrs

$\therefore 2(A+B+C)$ fill in $\frac{6 \times 10 \times \frac{15}{2}}{6 \times 10 + 6 \times \frac{15}{2} + 10 \times \frac{15}{2}}$

$= \frac{6 \times 5 \times 15}{180} = \frac{5}{2}$ hrs

$\therefore$ A + B + C fill the tank in 5 hrs.

Now, A [= (A + B + C) - (B + C)] fills in

$\frac{10 \times 5}{10-5} = 10$ hrs.

Similarly, B fills in $\frac{\frac{15}{2} \times 5}{\frac{15}{2} - 5} = 15$ hrs.

and C fills in $\frac{5 \times 6}{6-5} = 30$ hrs.

Ex. 16: Two pipes can separately fill a tank in 20 hrs and 30 hrs respectively. Both the pipes are opened to fill the tank but when the tank is $\frac{1}{3}$ full a leak develops in the tank through which $\frac{1}{3}$ of the water supplied by both the pipes leak out. What is the total time taken to fill the tank?

Soln: Time taken by the two pipes to fill the tank

$= \frac{20 \times 30}{20 + 30}$ hrs = 12 hrs.

$\therefore \frac{1}{3}$ of tank is filled in $\frac{12}{3} = 4$ hrs.

Now, $\frac{1}{3}$ of the supplied water leaks out

$\Rightarrow$ the filler pipes are only $1 - \frac{1}{3} = \frac{2}{3}$ as efficient as earlier.

$\Rightarrow$ the work of (12 - 4 =) 8 hrs will be completed now in

$8 \div \frac{2}{3} = \frac{8 \times 3}{2} = 12$ hrs $\quad \therefore$ total time = 4 + 12 = 16 hrs

OR

Since $\frac{1}{3}$ of supplied water leaks out, the leakage empties the tank in 12 × 3 = 36 hrs. Now, time taken to fill the tank by the two pipes and the leakage $= \frac{36 \times 12}{36 - 12} = 18$ hrs.

∴ time taken by the two pipes and the leakage to fill $\frac{2}{3}$ of the tank

$$= 18 \times \frac{2}{3} = 12 \text{ hrs.}$$

∴ total time = 4 hrs + 12 hrs = 16 hrs.

Ex. 17: A cistern is normally filled in 8 hrs but takes two hrs longer to fill because of a leak in its bottom. If the cistern is full, the leak will empty it in ______ hrs.

Soln: (Detailed): Suppose the leak can empty the tank in x hrs.

Then part of cistern filled in 1 hr $= \frac{1}{8} - \frac{1}{x} = \frac{x-8}{8x}$

∴ Cistern will be filled in $\frac{8x}{x-8}$ hrs.

Now, $\frac{8x}{x-8} - 8 + 2 = 10$ hrs.

or, $8x = 10x - 80$ ∴ $x = 40$ hrs.

Quicker Approach: The filler takes 2 hrs more

⇒ the leak empties in 10 hrs what the filler fills in 2 hrs.

⇒ the leak empties in 10 hrs $= \frac{2}{8} = \frac{1}{4}$ of cistern

⇒ the leak empties the full cistern in 4 × 10 = 40 hrs.

Direct formula: The leak will empty in $\frac{8 \times (8+2)}{2} = 40$ hrs

EXERCISE

1. Pipes A and B can fill a tank in 10 hours and 15 hours respectively. Both together can fill it in ___ hrs.
2. A tap can fill the cistern in 8 hours and another can empty it in 16 hours. If both the taps are opened simultaneously, the time (in hours) to fill the tank is __________.
3. A pipe can fill a tank in x hours and another can empty it in y hours. They can together fill it in (y >x) __________.
4. One tap can fill a cistern in 2 hours and another can empty the cistern in 3 hrs. How long will they take to fill the cistern if both the taps are opened ?
5. A cistern can be filled by two pipes A and B in 4 hours and 6 hours respectively. When full, the tank can be emptied by a third pipe C in 8 hours. If all the taps be turned on at the same time, the cistern will be full in __________.

6. A tank is filled by pipe A in 32 minutes and pipe B in 36 minutes. When full, it can be emptied by a pipe C in 20 minutes. If all the three pipes are opened simultaneously, half of the tank will be filled in __________ minutes.
7. If two pipes function simultaneously, the reservoir will be filled in 6 hours. One pipe fills the reservoir 5 hours faster than the other. How many hours does the faster pipe take to fill the reservoir?
8. Three pipes A, B and C can fill a cistern in 6 hours. After working at it together for 2 hours, C is closed and A and B can fill it in 7 hours. The time taken by C alone to fill the cistern is ____hrs.
9. A cistern has a leak which would empty it in 8 hours. A tap is turned on which admits 6 litres a minute into the cistern, and it is now emptied in 12 hours. How many litres does the cistern hold ?
10. Two taps can separately fill a cistern in 10 minutes and 15 minutes respectively and when the waste pipe is open, they can together fill it in 18 minutes. The waste pipe can empty the full cistern in __________ minutes.
11. A cistern has two taps which fill it in 12 min and 15 min respectively. There is also a waste pipe in the cistern. When all the pipes are opened, the empty cistern is full in 20 min. How long will the waste pipe take to empty a full cistern ?
12. A tank can be filled by one tap in 20 min. and by another in 25 min. Both the taps are kept open for 5 min. and then the second is turned off. In how many minutes more will the tank be completely filled?
13. A cistern is normally filled in 8 hours but takes two hours longer to fill because of a leak in its bottom. If the cistern is full, the leak will empty it in _________ hrs.
14. Two pipes X and Y can fill a cistern in 24 min. and 32 min. respectively. If both the pipes are opened together; then after how much time should Y be closed so that the tank is full in 18 minutes?
15. A leak in the bottom of a tank can empty the full tank in 6 hours. An inlet pipe fills water at the rate of 4 litres per minute. When the tank is full, the inlet is opened and due to the leak the tank is emptied in 8 hours. The capacity of the tank is ___________ litres.

ANSWERS

1. A + B together fill the tank in $\frac{10 \times 15}{10 + 15} = 6$ hrs.
2. $\frac{8 \times 16}{16 - 8} = 16$ hrs.

3. $\frac{xy}{y-x}$ hrs.
4. Same as Q. 2 & Q. 3.
5. A + B + C will fill the tank in

$$\frac{4 \times 6 \times 8}{6 \times 8 + 4 \times 8 - 4 \times 6} = \frac{4 \times 6 \times 8}{56} = \frac{24}{7} = 3\frac{3}{7} \text{ hrs.}$$

Note : This can also be solved in parts.

A + B fill in $\frac{4 \times 6}{4+6} = \frac{12}{5}$ hrs.

$\therefore$ A + B + C fill in $\frac{\frac{12}{5} \times 8}{8 - \frac{12}{5}} = \frac{12 \times 8}{28} = \frac{24}{7} = 3\frac{3}{7}$ hrs

6. A + B + C fill the tank in

$$\frac{32 \times 36 \times 20}{36 \times 20 + 32 \times 20 - 32 \times 36} = \frac{32 \times 36 \times 20}{208} = \frac{1440}{13} \text{ hrs.}$$

$\therefore$ A + B + C fill half the tank in $\frac{720}{13}$ hrs

$= 55\frac{5}{13}$ hrs

7. Let the faster tap fill the tank in x hrs.

$\therefore$ slower tap fills in (x + 5) hrs.

Now, both taps fill the tank in $\frac{x(x+5)}{x+x+5} = 6$ hrs.

or, $x^2 + 5x = 12x + 30$

or, $x^2 - 7x - 30 = 0$

$\therefore$ x = 10 or -3

We neglect the -ve value.

$\therefore$ the faster tap fills the tank in 10 hrs.

8. A + B + C can fill a cistern in 6 hrs --- (1)

$\therefore$ A + B + C can fill $\frac{1}{3}$ of cistern in 2 hrs.

Now, $1 - \frac{1}{3} = \frac{2}{3}$ of cistern is filled up by A + B in 7 hrs.

$\therefore$ A + B fill up the cistern in $\frac{7 \times 3}{2} = \frac{21}{2}$ hrs. --- (2)

From (1) and (2), C can fill the cistern in

$$\frac{6 \times \frac{21}{2}}{\frac{21}{2} - 6} = \frac{6 \times 21}{9} = 14 \text{ hrs.}$$

9. The filler tap can fill the tank in $\frac{12 \times 8}{12 - 8} = 24$ hrs.

$\therefore$ Capacity of tank = 6 × 60 × 24
= 8640 litres.

10. Two taps (fillers) fill the tank in $\frac{10 \times 15}{10 + 15} = 6$ minutes ---- (1)

Two fillers + a leak fill in 18 minutes (given) ---- (2)

$\therefore$ leak will empty the tank in (from (1) & (2))

$\frac{18 \times 6}{18 - 6} = 9$ minutes.

11. Same as Q. 10.

12. In 5 minutes both the pipes fill $\frac{5}{20} + \frac{5}{25} = \frac{1}{4} + \frac{1}{5} = \frac{9}{20}$ of tank.

Now, $1 - \frac{9}{20} = \frac{11}{20}$ is filled by the first tap.

The first tap can fill $\frac{11}{20}$ of tank in $20 \times \frac{11}{20} = 11$ minutes.

13. Let the leak can empty the tank in x hrs.

Then, $\frac{8 \times x}{x - 8} = 8 + 2$

or, 8x = 10x - 80

$\therefore x = \frac{80}{2} = 40$ hrs.

Quicker Approach :

The filler takes 2 hrs more.

$\Rightarrow$ The leak empties in 10 hrs what the filler fills in 8 hrs.

$\Rightarrow$ The leak empties in 10 hrs $= \frac{2}{8} = \frac{1}{4}$ (since filler fills in 8 hrs).

$\Rightarrow$ The leak empties full tank in 40 hrs.

Direct Formula :

The leak will empty in $\frac{8 \times (8 + 2)}{2} = 40$ hrs.

Note : The above formula is the same as $\frac{xy}{x-y}$. Because it can also be written as $\frac{8 \times 10}{10-8} = 40$ hrs.

14. Pipe x remains opened throughout for 18 minutes.

x fills $\frac{18}{24} = \frac{3}{4}$ of the tank in 18 minutes.

The remaining $\frac{1}{4}$ of the tank is filled by y in

$32\left(\frac{1}{4}\right) = 8$ minutes.

Hence y will be closed after 8 minutes.

15. Solve it by the same method as in Q. 9.

Time and Distance

Formulae

(i) Speed $= \dfrac{\text{Distance}}{\text{Time}}$ (ii) Time $= \dfrac{\text{Distance}}{\text{Speed}}$

(iii) Distance = Speed × Time

(iv) If the speed of a body is changed in the ratio a:b, then the ratio of the time taken changes in the ratio b:a.

(v) x km/hr $= \left(x \times \dfrac{5}{18}\right)$ m/sec,

(vi) x metres/sec $= \left(x \times \dfrac{18}{5}\right)$ km/hr.

Ex 1: Express a speed of 18 km/hr in metres per second.

Solu: 18 km/hr $= \left[18 \times \dfrac{5}{18}\right]$ m/sec = 5 metres/sec.

Ex 2: Express 10 m/sec in km/hr.

Soln : 10 m/sec $= \left[10 \times \dfrac{18}{5}\right]$ km/hr. = 36 km/hr.

Theorem : *If a certain distance is covered at x km/hr and the same distance is covered at y km/hr then the average speed during the whole journey is* $\dfrac{2xy}{x+y}$ *km/hr.*

Proof : Let the distance be A km.

Time taken to travel the distance at a speed of x km/hr $= \dfrac{A}{x}$ hrs.

Time taken to travel the distance at a speed of y km/hr $= \dfrac{A}{y}$ hrs.

Thus, we see that the total distance of 2A km is travelled in $\left(\dfrac{A}{x} + \dfrac{A}{y}\right)$ hrs.

$\therefore$ average speed $= \dfrac{2A}{\dfrac{A}{x}+\dfrac{A}{y}} = \dfrac{2Axy}{A(x+y)} = \dfrac{2xy}{x+y}$ km/hr.

Ex 3: A man covers a certain distance by car driving at 70 km/hr and he returns back to the starting point riding on a scooter at 55 km/hr. Find his average speed for the whole journey.

Soln : Average speed $= \dfrac{2\times 70\times 55}{70+55}$ km/hr = 61.6 km/hr.

Ex 4: A man covers a certain distance between his house and office on scooter. Having an average speed of 30 km/hr, he is late by 10 min. However, with a speed of 40 km/hr, he reaches his office 5 min earlier. Find the distance between his house and office.

Soln : Let the distance be x km.

Time taken to cover x km at 30 km/hr $= \dfrac{x}{30}$ hrs.

Time taken to cover x km at 40 km/hr $= \dfrac{x}{40}$ hrs.

Difference between the time taken = 15 min $= \dfrac{1}{4}$ hr.

$\therefore \quad \dfrac{x}{30} - \dfrac{x}{40} = \dfrac{1}{4}$ or $4x - 3x = 30$ or $x = 30$

Hence, the required distance is 30 km.

Direct formula:

Required distance
$= \dfrac{\text{Product of two speeds}}{\text{Difference of two speeds}} \times$ Difference between arrival times.

Thus in this case, the required distance

$$= \frac{30\times 40}{40-30} \times \frac{10+5}{60} = 30 \text{ km.}$$

Note: 10 minutes late and 5 minutes earlier make a difference of 10 + 5 = 15 minutes. As the other units are in km/hr, the difference in time should also be changed into hours.

Ex. 5 : A man walking with a speed of 5 km/hr reaches his target 5 minutes late. If he walks at a speed of 6 km/hr, he reaches on time. Find the distance of his target from his house.

Soln : This is similar to Ex. 4. Here the difference in time is 5 minutes only. Thus required distance $= \dfrac{5\times 6}{6-5} \times \dfrac{5}{60} = \dfrac{5}{2}$ km = 2.5 km.

Ex. 6 : A boy walking at a speed of 10 km/hr reaches his school 15 minutes late. Next time he increases his speed by 2 km/hr, but still he is late by 5 minutes. Find the distance of his school from his house.

Soln : Here, the difference in time = 15 - 5 = 10 minutes.

$= \dfrac{1}{6}$ hours.

His speed during next journey = 10 + 2 = 12 km/hr.

$\therefore$ required distance $= \dfrac{12\times 10}{12-10} \times \dfrac{1}{6} = 10$ km

Ex. 7 : A boy goes to school at a speed of 3 km/hr and returns to the village at a speed of 2 km/hr. If he takes 5 hrs in all, what is the distance between the village and the school?

Soln : Let the required distance be x km.

Then time taken during the first journey $= \dfrac{x}{3}$ hr.

and time taken during the second journey $= \dfrac{x}{2}$ hr.

$\therefore \quad \dfrac{x}{3} + \dfrac{x}{2} = 5 \Rightarrow \dfrac{2x+3x}{6} = 5 \Rightarrow 5x = 30.$

$\therefore \quad x = 6$

$\therefore$ required distance = 6 km

Direct formula :

Required distance
= Total time taken $\times \dfrac{\text{Product of the two speeds}}{\text{Addition of the two speeds}}$

$$= 5 \times \frac{3\times 2}{3+2} = 6 \text{ km.}$$

Ex. 8 : A motor car does a journey in 10 hrs, the first half at 21 km/hr and the second half at 24 km/hr. Find the distance.

Soln : This question is similar to Ex. 7, but we can't use the direct formula (used in Ex. 7) in this case. If we use the above formula, we get half of the distance. (But why ?) See the detailed method first.

Let the distance be x km.

Then $\frac{x}{2}$ km is travelled at a speed of 21 km/hr and $\frac{x}{2}$ km at a speed of 24 km/hr.

Then time taken to travel the whole journey

$$= \frac{x}{2 \times 21} + \frac{x}{2 \times 24} = 10 \text{ hrs}$$

$$\text{so, } x = \frac{2 \times 10 \times 21 \times 24}{(21 + 24)} = 224 \text{ km}$$

Direct Formula :

$$\text{Distance} = \frac{2 \times \text{Time} \times S_1 \times S_2}{S_1 + S_2}$$

Where, S_1 = Speed during first half and
S_2 = Speed during second half of journey

$$\therefore \text{Distance} = \frac{2 \times 10 \times 21 \times 24}{21 + 24} = 224 \text{ km}$$

Note : An absurd soln : Sometimes people think that as half of the journey was covered at a speed of 21 km/hr, so distance covered during half the journey = $21 \times (10 \div 2) = 21 \times 5 = 105$ km.
And similarly the second half distance = $24 \times 5 = 120$ km
$\therefore$ Total distance = 105 + 120 = 125 km.

But remember that half of the journey means half of the distance and not the time. Thus our above solution is absurd.

Ex. 9 : The distance between two stations, Delhi and Amritsar, is 450 km. A train starts at 4 p.m. from Delhi and moves towards Amritsar at an average speed of 60 km/hr. Another train starts from Amritsar at 3.20 p.m. and moves towards Delhi at an average speed of 80 km/hr. How far from Delhi will the two trains meet and at what time ?

Soln : Suppose the trains meet at a distance of x km from Delhi. Let the trains from Delhi and Amritsar be A and B respectively. Then,
[Time taken by B to cover (450 - x) km]
- [Time taken by A to cover x km] = $\frac{40}{60}$ -----(see note)

$$\frac{450 - x}{80} - \frac{x}{60} = \frac{40}{60}$$

$$\therefore\ 3(450 - x) - 4x = 160 \Rightarrow \quad 1190 \Rightarrow x = 170.$$

Thus, the trains meet at a distance of 170 km from Delhi.

Time taken by A to cover 170 km = $\left(\frac{170}{60}\right)$ hrs = 2hrs 50min.

So, the trains meet at 6.50 p.m.

Note : RHS = 4 : 00 p.m. - 3.20 p.m. = 40 minutes
$= \frac{40}{60}$ hr

LHS comes from the fact that the train from Amritsar took 40 minutes more to travel up to the meeting point because it had started its journey at 3.20 p.m. whereas the train from Delhi had started its journey at 4 p.m. and the meeting time is the same for both the trains.

Ex. 10 : Walking $\frac{3}{4}$ of his usual speed, a person is 10 min late to his office. Find his usual time to cover the distance.

Soln : Let the usual time be x min.

Time taken at $\frac{3}{4}$ of the usual speed = $\frac{4x}{3}$ min (from (iv) under formulae section)

$$\therefore\ \frac{4}{3}x - x = 10 \Rightarrow \frac{x}{3} = 10 \Rightarrow x = 30 \text{ min.}$$

Direct Formula :

$$\text{Usual time} = \frac{\text{Late time}}{\left(1 \div \frac{3}{4} - 1\right)} = \frac{10}{\frac{4}{3} - 1} = \frac{10}{\frac{1}{3}} = 30 \text{ minutes.}$$

Ex. 11 : Running $\frac{4}{3}$ of his usual speed, a person improves his timing by 10 minutes. Find his usual time to cover the distance.

Soln : This is similar to Ex 10, but not exactly the same. In this case, the speed is increased and hence the time is reduced. Whereas it was just opposite in Ex. 10.

You should try to solve this question by detailed method.

Direct formula for such question is slightly changed and is given as:

$$\text{Usual time} = \frac{\text{Improved time}}{1 - 1 \div \frac{4}{3}} = \frac{10}{1 - \frac{3}{4}} = 40 \text{ minutes.}$$

Note : Mark the change in the above two direct formulae.

Ex. 12 : Two men A and B start from a place P walking at 3 km and 3.5 km an hour respectively. How many km will they be apart at the end of 3 hrs

(i) if they walk in opposite directions?
(ii) if they walk in the same direction ?

What time will they take to be 16 km apart
(iii) if they walk in opposite directions?
(iv) if they walk in the same direction?

Soln : (i) When they walk in opposite directions, they will be (3 + 3.5) or 6.5 km apart in one hour.

$\therefore$ required distance = 6.5 × 3 = 19.5 km

(ii) When they walk in the same direction, they will be (3.5 - 3) or 0.5 km apart in one hour.

$\therefore$ required distance = 0.5 × 3 = 1.5 km

(iii) They are 6.5 km apart in 1 hr.

$\therefore$ required time $= \frac{16}{6.5} = 2\frac{6}{13}$ hrs.

(iv) They are 0.5 km apart in 1 hr.

$\therefore$ required time $= \frac{16}{0.5} = 32$ hrs.

Ex. 13 : A train travelling 25 km an hour leaves Delhi at 9 a.m. and another train travelling 35 km an hour starts at 2 p.m. in the same direction. How many km from Delhi will they be together ?

Soln : The first train has a start of 25 × 5 km and the second train gains (35 - 25) or 10 km per hour.

$\therefore$ the second train will gain 25 × 5 km in $\frac{25 \times 5}{10}$ or $12\frac{1}{2}$ hours.

$\therefore$ the required distance from Delhi $= 12\frac{1}{2} \times 35$ km $= 437\frac{1}{2}$ km. Ans.

Direct Formula : If you can remember the direct calculating formula, it may be more helpful.

> Meeting point's distance from starting point
> $$= \frac{S_1 \times S_2 \times \text{Difference in time}}{\text{Difference in speed}}$$
> where S_1 and S_2 are the speeds of the first and the second trains respectively.

$$\therefore \text{Reqd. distance} = \frac{25 \times 35 \times (2\text{p.m.} - 9\text{a.m.})}{35 - 25}$$

$$= \frac{25 \times 35 \times 5}{10} = 437\frac{1}{2}\text{ km}$$

Ex. 14 : Two men A and B walk from P to Q, a distance of 21 km, at 3 and 4 km an hour respectively. B reaches Q, returns immediately and meets A at R. Find the distance from P to R.

Soln : When B meets A at R, B has walked the distance $PQ + QR$ and A the distance PR. That is, both of them have together walked twice the distance from P to Q, i.e. 42 km.

Now the rates of A and B are 3 : 4 and they have walked 42 km.

Hence the distance PR travelled by $A = \frac{3}{7}$ of 42 km = 18 km Ans.

Direct Formula : When the ratio of speeds of A and B is $a : b$, then in this case:

> Distance travelled by A
> $$= 2 \times \text{Distance of two points}\left(\frac{a}{a+b}\right)$$
> and distance travelled by B
> $$= 2 \times \text{Distance of two points}\left(\frac{b}{a+b}\right)$$

Thus, distance travelled by A *(PR)*

$$= 2 \times 21\left(\frac{3}{3+4}\right) = 18\text{ km.}$$

> **Theorem :** *If two persons A and B start at the same time in opposite directions from two points and after passing each other they complete the journeys in 'a' and 'b' hrs respectively, then A's speed : B's speed* $= \sqrt{b} : \sqrt{a}$

Proof :

Let the total distance be D km

A's speed be x km/hr

B's speed be y km/hr

As they are moving in opposite directions, their relative velocity is $(x + y)$ km/hr.

Thus, they will meet after $\frac{D}{x+y}$ hrs.

Now, the distance travelled by A in $\frac{D}{x+y}$ hrs $= PO = \frac{Dx}{x+y}$ km

And the distance travelled by B in $\frac{D}{x+y}$ hrs $= QO = \frac{Dy}{x+y}$ km

Now, A passes the distance, QO, in a hrs.

Therefore, his speed $= \frac{Dy}{(x+y)a}$

Similarly, B passes the distance, PO, in b hrs.

Therefore, his speed $= \frac{Dx}{(x+y)b}$

Now, ratio of their speeds =

$$x : y = \frac{Dy}{(x+y)a} : \frac{Dx}{(x+y)b}$$

or, $\frac{x}{y} = \frac{Dy}{(x+y)a} \div \frac{Dx}{(x+y)b}$

or, $\frac{x}{y} = \frac{Dy}{(x+y)a} \times \frac{(x+y)b}{Dx}$

or, $\frac{x}{y} = \frac{y}{x} \times \frac{b}{a}$

or, $\frac{x^2}{y^2} = \frac{b}{a}$ $\therefore$ $\frac{x}{y} = \sqrt{\frac{b}{a}}$ $\therefore$ $x : y = \sqrt{b} : \sqrt{a}$

This proves the theorem.

Ex. 15 : A man sets out to cycle from Delhi to Rohtak, and at the same time another man starts from Rohtak to cycle to Delhi. After passing each other they complete their journeys in $3\frac{1}{3}$ and $4\frac{4}{5}$ hours respectively. At what rate does the second man cycle if the first cycles at 8 km per hour ?

Soln : If two persons (or train) A and B start at the same time in opposite directions from two points, and arrive at the point a and b hrs respectively after having met, then

A's rate : B's rate $= \sqrt{b} : \sqrt{a}$ (from the theorem)

Thus in the above case

$$\frac{\text{1st man's rate}}{\text{2nd man's rate}} = \frac{\sqrt{4\frac{4}{5}}}{\sqrt{3\frac{1}{3}}} = \frac{6}{5}$$

$\therefore$ 2nd man's rate $= \frac{5}{6} \times 8 = 6\frac{2}{3}$ km/hr.

Report of guns

Ex. 16. Two guns were fired from the same place at an interval of 13 minutes but a person in a train approaching the place hears the second report 12 minutes 30 seconds after the first. Find the speed of the train, supposing that sound travels at 330 metres per second.

Soln. It is easy to see that the distance travelled by the train in 12 min. 30 seconds could be travelled by sound in (13 min. -12 min. 30 seconds =) 30 seconds.

$\therefore$ the train travels 330×30 metres in $12\frac{1}{2}$ min.

$\therefore$ the speed of the train per hour

$$\frac{330 \times 30 \times 2 \times 60}{25 \times 1000} = \frac{1188}{25} \text{ or } 47\frac{13}{25} \text{ km. Ans.}$$

Carriage driving in a fog

Ex. 17. A carriage driving in a fog passed a man who was walking at the rate of 3 km. an hour in the same direction. He could see the carriage for 4 minutes and it was visible to him upto a distance of 100m. What was the speed of the carriage?

Soln. The distance travelled by the man in 4 minutes $= \frac{3 \times 1000}{60} \times 4$

$= 200$ metres.

$\therefore$ distance travelled by the carriage in 4 minutes

$= (200 + 100) = 300$ metres.

$\therefore$ speed of carriage $= \frac{300}{4} \times \frac{60}{1000}$ km per hour

$= 4\frac{1}{2}$ km per hour Ans.

Ex. 18 : A monkey tries to ascend a greased pole 14 metres high. He ascends 2 metres in first minute and slips down 1 metre in the alternate minute. If [illegible] continues to ascend in this fashion, how long does he take [illegible]ach the top?

Soln : In every 2 minutes he is able to ascend 2 - 1 =1 metre. This way he ascends upto 12 metres because when he reaches at the top, he does not slip down. Thus, upto 12 metres he takes $12 \times 2 = 24$ minutes and for the last 2 metres he takes 1 minute. Therefore, he takes $24 + 1 = 25$ minutes to reach the top. That is, in 26th minute he reaches the top.

Ex. 19 : Two runners cover the same distance at the rate of 15 km and 16 km per hour respectively. Find the distance travelled when one takes 16 minutes longer than the other.

Soln : Let the distance be x km.

Time taken by the first runner $= \frac{x}{15}$ hrs

Time take by the second runner $= \frac{x}{16}$ hrs

Now, $\frac{x}{15} - \frac{x}{16} = \frac{16}{60}$

or, $\frac{x(16-15)}{15 \times 16} = \frac{16}{60}$

$\therefore x = \frac{16}{60} \times 15 \times 16 = 64$ km

Direct Formula :

Distance

$= \frac{\text{Multiplication of speeds}}{\text{Difference of Speeds}} \times$ Difference in time to cover the distance

$= \frac{15 \times 16}{16-15} \times \frac{16}{60} = 64$ km.

Ex. 20 : Two cars run to a place at the speeds of 45 km/hr and 60 km/hr respectively. If the second car takes 5 hrs less than the first for the journey, find the length of the journey.

Soln : This example is similar to Ex. 19. The only difference is that in Ex. 19, one takes longer time than the other but in Ex. 20, one takes shorter time than the other. It hardly matters because both are different forms of the same statement. "One takes 5 hrs less than the other" means the second takes 5 hrs more than the first to reach the destination. So the above **direct formula** works in this case also.

$\therefore$ distance $= \frac{45 \times 60}{60-45} \times 5 = 900$ km

Ex. 21 : A man takes 8 hours to walk to a certain place and ride back. However, he could have gained 2 hrs, if he had covered both ways by riding. How long would he have taken to walk both ways?

Soln : Walking time + Riding time = 8 hrs ----- (1)

2 Riding time = 8 - 2 = 6 hrs ------- (2)

2 × (1) - (2) gives the result

2 × walking time = 2 × 8 - 6 = 10 hrs.

$\therefore$ both ways walking will take 10 hrs.

Quicker Approach : Two ways riding saves a time of 2 hrs. It simply means that one way riding takes 2 hrs less than one way walking. It further means that one way walking takes 2 hrs more than one way riding. Thus, both way walking will take 8 + 2 = 10 hrs.

Therefore, **direct formula :**

Both way walking = One way walking and one way riding time + Gain in time = 8 + 2 = 10 hrs.

Ex. 22 : A man takes 12 hrs to walk to a certain place and ride back. However, if he walks both the ways he needs 3 hrs more. How long would he have taken to ride both ways?

Soln : Quicker Method :

Required time = 12 - 3 = 9 hrs.

Note : The approach for the quicker method is similar to that of Ex. 21. Try to define it.

Ex. 23 : Two trains for Patna leave Delhi at 10 a.m. and 10.30 a.m. and travel at 60 km/hr and 75 km/hr respectively. How many kilometres from Delhi will the two trains be together?

Soln : Use the **direct formula** used in Ex. 13.

Meeting point's distance $= \frac{60 \times 75}{75-60}$ (10.30 am - 10 am)

$= \frac{60 \times 75}{15}\left(\frac{30}{60}\right) = 150$ km.

Note : Ex. 23, Ex 19 and Ex 20 are similar. Do you agree? Yes, you must agree with it. Ex. 23 can be rewritten as:

"Two trains leaves Delhi at speeds of 60 km/hr and 75 km/hr respectively. The faster train takes 30 minutes (10.30 - 10) less to meet the slower one. Find the distance travelled by them to meet each other."

This form of the above example is the same as Ex. 20 or Ex. 19. That is why we have used the same direct formulae in all the cases of Ex. 19, Ex 20 and Ex 23.

Ex. 24 : A man leaves a point P at 6 a.m. and reaches the point Q at 10 a.m. Another man leaves the point Q at 8 a.m. and reaches the point P at 12 noon. At what time do they meet?

Soln : Let the distance PQ = A km.

And they meet x hrs after the first man starts.

Average speed of first man $= \frac{A}{10-6} = \frac{A}{4}$ km/hr.

Average speed of second man $= \frac{A}{12-8} = \frac{A}{4}$ km/hr.

Distance travelled by first man $= \frac{Ax}{4}$ km

They meet x hrs after the first man starts. The second man, as he starts 2 hrs late, meets after $(x - 2)$ hrs from his start. Therefore, the distance travelled by the second man $= \frac{A(x-2)}{4}$ km

Now, $\frac{Ax}{4} + \frac{A(x-2)}{4}$ km $= A$

or, $2x - 2 = 4$

$\therefore x = 3$ hrs.

$\therefore$ They meet at 6 a.m. + 3 hrs = 9 a.m.

Quicker Approach : Since both the persons take equal time of 4 hrs to cover the distance, their meeting time will be exactly in the middle of 6 a.m. and 12 noon, ie, at 9 a.m.

But what happens when they take different times? In that case, the following formula works good.

> They will meet at
> = First's starting time
> $+ \frac{\text{(Time taken by first) (2nd's arrival time − 1st's starting time)}}{\text{Sum of time taken by both}}$

$= 6 \text{ a.m.} + \frac{(10.00 - 6.00)(12.00 - 6.00)}{(10.00 - 6.00) + (12.00 - 8.00)}$

$= 6 \text{ a.m.} + \frac{4 \times 6}{4 + 4} = 9$ a.m.

This formula is more useful in the following example.

Ex. 25 : A train leaves Patna at 5 a.m. and reaches Delhi at 9 a.m. Another train leaves Delhi at 6.30 a.m. and reaches Patna at 10 a.m. At what time do the two trains meet?

Soln : By Direct Formula :

They will meet at $= 5 \text{ a.m.} + \frac{(9.00 - 5.00)(10.00 - 5.00)}{(9.00 - 5.00) + (10.00 - 6.30)}$

$= 5 \text{ a.m.} + \frac{4 \times 5}{7.5}$ hrs $= 5$ a.m. $+ 2\frac{2}{3}$ hrs. $= 7.40$ a.m.

Note : $(10.00 - 6.30) = 3.30 = 3\frac{1}{2}$ hrs $= 3.5$ hrs

Ex. 26 : A person has to cover a distance of 80 km in 10 hrs. If he covers half of the journey in $\frac{3}{5}$ time, what should be his speed to cover the remaining distance in the time left?

Soln : Left distance $= 80\left(1 - \frac{1}{2}\right) = 40$ km.

Time left $= 10\left(1 - \frac{3}{5}\right) = 4$ hrs.

$\therefore$ required speed $= \frac{40}{4} = 10$ km/hr.

Ex. 27 : A person covers a distance in 40 minutes if he runs at a speed of 45 km per hour on an average. Find the speed at which he must run to reduce the time of journey to 30 minutes.

Soln :

> **Theorem :** *Speed and time taken are inversely proportional.*
> *Therefore,* $S_1 T_1 = S_2 T_2 = S_3 T_3$ .....
> *Where* S_1, S_2, S_3 .... *are the speeds*
> *and* T_1, T_2, T_3 .... *are the time taken to travel the same distance.*

Thus in this case;

$45 \times 40 = S_2 \times 30 \quad \therefore S_2 = \frac{45 \times 40}{30} = 60$ km/hr.

Ex. 28 : Without any stoppage a person travels a certain distance at an average speed of 80 kmph, and with stoppages he covers the same distance at an average speed of 60 kmph. How many minutes per hour does he stop?

Soln : Let the total distance be x km.

Time taken at the speed of 80 km/hr $= \frac{x}{80}$ hrs.

Time taken at the speed of 60 km/hr $= \frac{x}{60}$ hrs.

$\therefore$ he rested for $\left(\frac{x}{60} - \frac{x}{80}\right)$ hrs $= \frac{20x}{60 \times 80} = \frac{x}{240}$ hrs.

$\therefore$ his rest per hour $= \frac{x}{240} \div \frac{x}{60} = \frac{x}{240} \times \frac{60}{x} = \frac{1}{4}$ hrs. = 15 minutes.

By Direct Formula :

$$\boxed{\text{Time of rest per hour} = \frac{\text{Difference of speed}}{\text{Speed without stoppage}}}$$

$$= \frac{80 - 60}{80} = \frac{1}{4} \text{ hr} = 15 \text{ minutes.}$$

Ex. 29 : A man travels 360 km in 4 hrs, partly by air and partly by train. If he had travelled all the way by air, he would have saved $\frac{4}{5}$ of the time he was in train and would have arrived at his destination 2 hours early. Find the distance he travelled by air and train.

Soln : $\frac{4}{5}$ of total time in train = 2 hours.

$\therefore$ Total time in train $= \frac{2 \times 5}{4} = \frac{5}{2}$ hrs.

$\therefore$ Total time spent in air $= 4 - \frac{5}{2} = \frac{3}{2}$ hrs.

By the given hypothesis, if 360 km is covered by air, then time taken is (4 - 2 =) 2 hrs.

$\therefore$ when $\frac{3}{2}$ hrs is spent in air, distance covered

$= \frac{360}{2} \times \frac{3}{2} = 270$ km

$\therefore$ Distance covered by train = 360 - 270 = 90 km.

Ex. 30 : A man rode out a certain distance by train at the rate of 25 km an hour and walked back at the rate of 4 km per hour. The whole journey took 5 hours and 48 minutes. What distance did he ride?

Soln : Let the distance be x km.

Then time spent in journey by train $= \frac{x}{25}$ hrs.

And time spent in journey by walking $= \frac{x}{4}$ hrs.

Therefore, $\frac{x}{25} + \frac{x}{4} = 5$ hrs 48 minutes.

or, $\frac{29x}{100} = 5\frac{48}{60} = \frac{29}{5} \quad \therefore x = \frac{100}{5} = 20$ km

Direct Formula :

$$\boxed{\text{Distance} = \text{Total time} \times \frac{\text{Multiplication of two speeds}}{\text{Sum of speeds}}}$$

$$= 5\frac{48}{60} \times \frac{25 \times 4}{25 + 4} = \frac{29}{5} \times \frac{25 \times 4}{29} = 20 \text{ km}$$

Note : This example is different from Ex 19, Ex 20 and Ex 23 because here **total time** during both types of journey is given whereas in the previous examples the **difference in time** between both types of journey were given. And accordingly the denominator in direct formula is changed. Mark the difference carefully and try to understand the reason. Otherwise, it will create a confusion during practice time.

Ex. 31: One aeroplane started 30 minutes later than the scheduled time from a place 1500 km away from its destination. To reach the destination at the scheduled time the pilot had to increase the speed by 250 km/hr. What was the speed of the aeroplane per hour during the journey?

Soln: Detail Method: Let it take x hrs in second case.

Then speed $= \frac{1500}{x} = \frac{1500}{x + \frac{1}{2}} + 250$

or, $\frac{1500\left(x + \frac{1}{2}\right) - 1500x}{x\left(x + \frac{1}{2}\right)} = 250$

or, $750 = 250x\left(x + \frac{1}{2}\right)$

or, $x^2 + \frac{x}{2} - 3 = 0$

or, $2x^2 + x - 6 = 0$

or, $2x^2 + 4x - 3x - 6 = 0$

or, $x(2x - 3) + 2(2x-3) = 0$

or, $(x+2)(2x-3) = 0$

$\therefore \quad x = -2, \frac{3}{2}$

Therefore, the plane takes $\frac{3}{2}$ hrs in second case, i.e. $\frac{3}{2}+\frac{1}{2}=2$ hrs in normal case. Thus, normal speed $=\frac{1500}{2}=750$ km/hr

Quicker Maths:

Lesser time	:	Increase in Speed
$\frac{1}{2}$	:	250
1	:	500
$\frac{3}{2}$	:	750
2	:	1000
$\frac{5}{2}$	:	1250

We arrange the given information in two columns as given above. The ratio is continued until we get the two ratios such that their cross-products give the distance between the points.

Thus, we find our answer as:

With the speed of 1000 km/hr the plane takes $\frac{3}{2}$ hrs and with the speed of 750 km/hr the plane takes 2 hrs.

Therefore, normal speed is 750 km/hr.

Ex. 32: An aeroplane started one hour later than its scheduled time from a place 1200 km away from its destination. To reach the destination at the scheduled time the pilot had to increase the speed by 200 km/hr. What was the speed of the aeroplane per hour in normal case?

Soln:

Lesser time	:	Increase in Speed
1	:	200
2	:	400
3	:	600

Thus, the normal time is 3 hrs and normal speed is 400 km/hr.

Ex. 33: A train was late by 6 minutes. The driver increased its speed by 4 km/hr. At the next station, 36 km away, the train reached on time. Find the original speed of the train.

Soln: If you solve this question by detail method, you will get a quadratic equation. But with the help of the above-discussed method it becomes very easy to solve the question.

Lesser time	:	Increase in speed
$\frac{1}{10}$ hr	:	4 km/hr
$\frac{2}{10}$	:	8
$\frac{3}{10}$	:	12
$\frac{9}{10}$	:	36
1	:	40

Thus, the normal speed is 36 km/hr.

Ex. 34: When a man travels equal distance at speeds V_1 and V_2 km/hr, his average speed is 4 km/hr. But when he travels at these speeds for equal times his average speed is 4.5 km/hr. Find the difference of the two speeds.

Soln : Detail method : Suppose the equal distance = D km

Then time taken with V_1 and V_2 speeds are

$\frac{D}{V_1}$ hrs & $\frac{D}{V_2}$ hrs respectively.

$$\therefore \text{ average speed} = \frac{\text{Total distance}}{\text{Total time}} = \frac{2D}{\frac{D}{V_1}+\frac{D}{V_2}}$$

$$= \frac{2V_1 V_2}{V_1+V_2} = 4 \text{ km/hr}$$

In second case, average speed $=\frac{V_1+V_2}{2}=4.5$ km/hr

That is; $V_1+V_2=9$ and $V_1V_2=18$

Now, $(V_1-V_2)^2=(V_1+V_2)^2-4V_1V_2$

$=81-72=9$

$\therefore V_1-V_2=3$ km/hr

Direct Formula :

$V_1-V_2=\sqrt{4(4.5)(4.5-4)}=3$ km/hr

Note : You may be asked to find the two speeds. Find them.

Ex. 35: A person travels for 3 hrs at the speed of 40 km/hr and for 4.5 hrs at the speed of 60 km/hr. At the end of it, he finds that he has

covered $\frac{3}{5}$ of the total distance. At what average speed should he travel to cover the remaining distance in 4 hrs?

Soln : Total distance covered in (3 + 4.5) hrs
$= 3 \times 40 + 4.5 \times 60 = 390$ km.

Now, since $\frac{3}{5}$ of the distance = 390

$\therefore \frac{2}{5}$ of the distance $= 390 \times \frac{5}{3} \times \frac{2}{5} = 260$ km.

$\therefore$ average speed for the remaining distance $= \frac{260}{4} = 65$ km/hr.

Direct formula: The average speed for the remaining distance

$$= \frac{(R_1T_1 + R_2T_2)\left(\frac{1}{f} - 1\right)}{T} = \frac{(40 \times 3 + 60 \times 4.5)\left(\frac{5}{3} - 1\right)}{4}$$

$$= \frac{390 \times 2}{4 \times 2} = 65 \text{ km/hr.}$$

Ex. 36: Hari took 20 minutes to walk 3 km. If Shyam is 20% faster than Hari, how much time will he take to cover the same distance?

Soln : Before going for the solution we should discuss a generalised formula which may be useful for some more similar cases.

Ram, walking at the rate of S_1 km/hr, takes t_1 hours to cover a distance. Shyam, walking at the rate of S_2 km/hr, takes t_2 hours to cover the same distance. Then,

$S_1t_1 = S_2t_2 = $ Distance

or, $S_1t_1 = S_2t_2 = $ Constant

Thus we see that both speed and time are inversely proportional to each other. That is, if the speed increases to 4 times, the time will decrease to $\frac{1}{4}$ times. See the following cases.

(a) If 'A' takes 8 hrs to cover a distance and 'B' is four times faster than

A, then what time will 'B' take to cover the same distance?

We have $S_1t_1 = S_2t_2$

$\Rightarrow S_1t_1 = 4S_1t_2 \Rightarrow t_2 = \frac{t_1}{4} = \frac{8}{4} = 2$ hrs

(b) If A takes 8 hrs to cover a distance and he is 4 times faster than 'B', then what time will 'B' take to cover the same distance?

We have,

$S_1t_1 = S_2t_2 \Rightarrow 4S_2 \times 8 = S_2 \times t_2$

$\therefore t_2 = 32$ hrs.

Note : In case (b), it is clear that B is 4 times slower, so he will take 4 times the time taken by A.

(c) If 'B' is 20% faster than 'A', then what time will he take to travel the distance which 'A' travels in 20 minutes?

$S_1t_1 = S_2t_2$

$S_1 \times 20 = \frac{120}{100} S_1 \times t_2 \quad \therefore t_2 = \frac{20 \times 100}{120} = \frac{50}{3} = 16\frac{2}{3}$ min.

Note: (1) Since S_2 is 20% more than S_1, then $S_2 = S_1\left(\frac{120}{100}\right)$.

(2) This is the same question as asked by the student.

(d) 'B' takes 30% less time than 'A' to cover the same distance. What should be the speed of 'B' if A walks at a rate of 7 km/hr ?

Again, $7 \times t_1 = S_2\left(\frac{100 - 30}{100}\right)t_1 \quad \therefore S_2 = \frac{7 \times 100}{70} = 10$ km/hr.

Ex. 37: A person travelled 120 km by steamer, 450 km by train and 60 km by horse. It took 13 hours 30 minutes. If the rate of the train is 3 times that of the horse and 1.5 times that of the steamer, find the rate of the train per hour.

Soln : Suppose the speed of horse = x km/hr. Then speed of the train = 3x km/hr and speed of the steamer = 2x km/hr

Now, $\frac{120}{2x} + \frac{450}{3x} + \frac{60}{x} = 13.5$ hours

(Since 13 hrs 30 minutes = 13.5 hrs)

or, $\frac{360 + 900 + 360}{6x} = 13.5 \quad \therefore x = \frac{1620}{6 \times 13.5} = 20$

$\therefore$ Speed of train $= 3x = 3 \times 20 = 60$ km/hr

Quicker Method : Arrange the informations like:

	Train	Steamer	Horse
Distance	450 km	120 km	60 km
Speed	3	2	1

Total time = 13.5 hrs

$$\text{Speed of train} = \frac{450 \times 2 \times 1 + 120 \times 3 \times 1 + 60 \times 3 \times 2}{13.5 \times 2 \times 1}$$

$$= \frac{1620}{27} = 60 \text{ km/hr}$$

And speed of steamer = $\frac{1620}{40.5}$ = 40 km/hr

Similarly, we can find the speed of the horse directly.

Note : The above formula is easier to remember because we find that

(1) In numerator, the distance covered by the train (450) is multiplied by speeds of steamer and horse. We deal with the other two in a similar manner.

(2) In denominator for speed of train, total time (13.5 hrs) is multiplied by speed of steam and horse.

(3) Now, you can write down the formula for the speed of the horse very easily.

Ex. 38: A man covers a certain distance on scooter. Had he moved 3 kmph faster, he would have taken 40 minutes less. If he had moved 2 kmph slower, he would have taken 40 minutes more. Find the distance (in km) and original speed.

Soln: (Detailed): Suppose the distance is D km and the initial speed is x km/hr.

Then, we have $\frac{D}{x+3} = \frac{D}{x} - \frac{40}{60}$ and $\frac{D}{x-2} = \frac{D}{x} + \frac{40}{60}$

or, $\frac{D}{x} - \frac{D}{x+3} = \frac{2}{3}$ or, $\frac{3D}{x(x+3)} = \frac{2}{3}$ .... (1)

and $\frac{D}{x-2} - \frac{D}{x} = \frac{2}{3}$ or, $\frac{2D}{x(x-2)} = \frac{2}{3}$ .... (2)

From (1) and (2) we have $\frac{3D}{x(x+3)} = \frac{2D}{x(x-2)}$

or, $3(x-2) = 2(x+3)$

or, $3x - 6 = 2x + 6$ $\therefore$ x = 12 km/hr.

Now, if we put this value in (1) we get

$D = \frac{2}{3} \times \frac{12 \times 15}{3} = 40$ km.

Quicker Method: In the above question when time reduced in arrival (40 minutes) is equal to the time increased in arrival (40 minutes) then

$$\text{Speed} = \frac{2 \times (\text{Increase in speed} \times \text{Decrease in speed})}{\text{Difference in increase and decrease in speeds}}$$

$$= \frac{2 \times (3 \times 2)}{(3-2)} = 12 \text{ km/hr.}$$

$$\text{Now, Distance} = \frac{(12+3) \times (12-2)}{(12+3) - (12-2)} \times \text{Diff between arrival time}$$

$$= \frac{15 \times 10}{5} \times \frac{40+40}{60} = 40 \text{ km.}$$

Note: 40 minutes late and 40 minutes earlier make a difference of

$40 + 40 = 80$ minutes $= \frac{80}{60}$ hrs.

Ex. 39: A man takes 8 hrs to walk to a certain place and ride back. However, he could have gained 2 hrs, if he had covered both ways by riding. How long would he take to walk both ways?

Soln. Walking time + Riding time = 8 hrs. .... (1)

2 × riding time = 8 - 2 = 6 hrs. ..... (2)

Performing 2 × (1) - (2) gives the result

2 × walking time = 2 × 8 - 6 = 10 hrs.

$\therefore$ both ways walking will take 10 hrs.

Direct formula: Both ways walking

= One way walking and one way riding time + Gain in time

= 8 + 2 = 10 hrs.

EXERCISES

1. A train runs at the rate of 45 km an hour. What is its speed in metres per second?
2. A motor car takes 50 seconds to travel 500 metres. What is its speed in km per hour?
3. How many km per hour does a man walk who passes through a street 600m long in 5 minutes?
4. Compare the rates of two trains, one travelling at 45 km an hour and the other at 10 m a second.
5. The wheel of an engine $4\frac{2}{7}$ metres in circumference makes seven revolutions in 4 seconds. Find the speed of the train in km per hour.
6. What is the length of the bridge which a man riding 15 km an hour can cross in 5 minutes?
7. A man takes 6 hrs. 30 min. in walking to a certain place and riding back. He would have gained 2 hrs. 10 min. by riding both ways. How long would he take to walk both ways?
8. I walk a certain distance and ride back and take $6\frac{1}{2}$ hours altogether.

I could walk both ways in $7\frac{3}{4}$ hours. How long would it take me to ride both ways ?

9. At what distance from Delhi will a train, which leaves Delhi for Amritsar at 2.45 p.m., and goes at the rate of 50 km an hour, meet a train which leaves Amritsar for Delhi at 1.35 p.m. and goes at the rate of 60 km per hour, the distance between the two towns being 510 km?
10. A train which travels at the uniform rate of 10 m a second leaves Madras for Arconum at 7 a.m. At what distance from Madras will it meet a train which leaves Arconum for Madras at 7.20 a.m., and travels one-third faster than the former does, the distance from Madras to Arconum being 68 km ?
11. Two boys begin together to write out a booklet containing 8190 lines. The first boy starts with the first line, writing at the rate of 200 lines an hour; and the second boy starts with the last line, then writes 8189th line and so on, proceeding backward at the rate of 150 lines an hour. At what line will they meet ?
12. A, B and C can walk at the rates of 3, 4 and 5 km an hour respectively. They start from Poona at 1, 2, 3 o'clock respectively. When B catches A, B sends him back with a message to C. When will C get the message?
13. A thief is spotted by a policeman from a distance of 200 metres. When the policeman starts the chase, the thief also starts running. Assuming the speed of the thief 10 kilometres an hour, and that of the policeman 12 kilometres an hour, how far will have the thief run before he is overtaken ?
14. Two men start together to walk a certain distance, one at $3\frac{3}{4}$ km an hour and the other at 3 km an hour. The former arrives half an hour before the latter. Find the distance.
15. Two bicyclists do the same journey by travelling respectively at the rates of 9 and 10 km an hour. Find the length of the journey when one takes 32 minutes longer than the other.
16. A motor car does a journey in 10 hours, the first half at 21 km per hour, and the rest at 24 km per hour. Find the distance.
17. A man walks from A to B and back in a certain time at the rate of $3\frac{1}{2}$ km per hour. But if he had walked from A to B at the rate of 3 km an hour and back from B to A at the rate of 4 km an hour, he would have taken 5 minutes longer. Find the distance between A and B.
18. A man rode out a certain distance by train at the rate of 25 km per hour and walked back at the rate of 4 km per hour. The whole journey took 5 hours 48 minutes. What distance did he ride ?

19. Two trains start at the same time from two stations and proceed towards each other at the rates of 20 km and 25 km per hour respectively. When they meet, it is found that one train has travelled 80 km more than the other. Find the distance between the two stations.
20. I have to be at a certain place at a certain time and I find that I shall be 15 minutes too late, if I walk at 4 km an hour; and 10 minutes too soon, if I walk at 6 km an hour. How far have I to walk ?
21. A person going from Pondicherry to Ootacamond travels 120 km by steamer, 450 km by rail and 60 km by horse transit. The journey occupies 13 hours 30 minutes, and the rate of the train is three times that of the horse transit and $1\frac{1}{2}$ times that of the steamer. Find the rate of the train.
22. Supposing that telegraph poles on a railroad are 50 metres apart, how many will be passed by a car in 4 hours if the speed of the train is 45 km an hour?

ANSWERS

1. $45 \times \frac{5}{18} = \frac{25}{2} = 12.5$ m/s

2. Speed in m/s $= \frac{500}{50} = 10$

$\therefore$ Speed in km/hr $= 10 \times \frac{18}{5} = 36$ km/hr

3. Speed in m/s $= \frac{600}{5 \times 60} = 2$

$\therefore$ Speed in km/hr $= 2 \times \frac{18}{5} = \frac{36}{5} = 7\frac{1}{5}$ km/hr

4. 10 m/s $= 10 \times \frac{18}{5} = 36$ km/hr

$\therefore$ ratio $= 45 : 36 = 5 : 4$

5. Distance covered in 4 seconds $= \frac{30}{7} \times 7$ metres

$\therefore$ Speed in m/s $= \frac{30}{4}$

$\therefore$ Speed in km/hr $= \frac{30}{4} \times \frac{18}{5} = 27$ km/hr

6. Distance covered in 5 minutes = $\frac{15}{60} \times 5 = \frac{15}{12} = \frac{5}{4}$ km = 1250 metres.

7. Walking + Riding = 6hrs 30 min. ----- (1)
2 Riding = 6 hrs 30 min - 2 hrs 10 min = 4 hrs 20 min ---- (2)
Solving the above two relations (equations);
2 × (1) - (2) gives
2 walking = 13 hrs - 4 hrs 20 min = 8 hrs 40 minutes.

Quicker Approach : Instead of walking and riding if he covered both the ways by riding he saves 2 hrs 10 min.
⇒ One way walking takes 2 hrs 10 min more than one way riding. Therefore, if one way walking + one way riding takes 6 hrs 30 min then both ways walking will take 6 hrs 30 min + 2 hrs 10 min = 8 hrs 40 min.

8. Walking + Riding = $\frac{13}{2}$ hrs --- (1)

Walking + Walking = $7\frac{3}{4}$ hrs

or, 2 walking = $\frac{31}{4}$ hrs ---- (4)

2 × (1) - (2) gives, 2 Riding = 13 - $\frac{31}{4} = \frac{21}{4}$ = 5 hrs 15 min.

Quicker Approach :
Both ways walking takes 7 hrs 45 min and one way walking + one way riding takes 6 hrs 30 min.
⇒ One way walking takes (7 hrs 45 min) - (6 hrs 30 min)
= 1 hr 15 min more time than one way riding.
Therefore, if one way riding +one way walking takes 6 hrs 30 min then both ways riding will take 6 hrs 30 min - 1 hr 15 min = 5 hr 15 min.

9. Let them meet at x km from Delhi.

Then, 2.45 PM + $\frac{x}{50}$ = 1.35 PM + $\frac{510-x}{60}$ = Their meeting time.

or, (2.45 PM - 1.35 PM) + $\left(\frac{x}{50} + \frac{x}{60}\right) = \frac{510}{60} = \frac{17}{2}$

or, $\left(1\frac{1}{6}\text{ hr}\right) + x\left(\frac{50+60}{50 \times 60}\right) = \frac{17}{2}$

or, $x\left(\frac{110}{3000}\right) = \frac{17}{2} - \frac{7}{6} = \frac{51-7}{6} = \frac{44}{6}$

$\therefore x = \frac{44}{6} \times \frac{3000}{110}$ = 200 km.

Quicker Approach : The second (leaving Amritsar) train starts its journey earlier. It covers 60 × (2.45 PM - 1.35 PM) = 60 × $1\frac{1}{6}$ hr = 70 km when the first (leaving Delhi) starts its journey.

Now, both the trains cover 510 - 70 = 440 km with relative speed of 50 + 60 = 110 km/hr

Thus, they meet after $\frac{440}{110}$ = 4 hrs after the first train starts at 2.45 PM.

Now, the first train covers 4 × 50 = 200 km to meet the second train.

Direct Formula : With the help of the above approach, a direct formula can be derived as :

The meeting point from Delhi (first train's starting point) =

$$S_1\left[\frac{\text{Total distance} - S_2 \times (T_1 - T_2)}{S_1 + S_2}\right]$$

Where, S_1 is the speed of first train
S_2 is the speed of 2nd train.
T_1 is starting time of 1st train.
T_2 is starting time of 2nd train.

$$= 50\left[\frac{510 - 60 \times \frac{7}{6}}{60 + 50}\right] = 50\left[\frac{440}{110}\right] = 200 \text{ km.}$$

10. **Quicker Method :** By the direct formula used in Q. 9.

$S_1 = 10 \times \frac{18}{5}$ = 36 km/hr

$S_2 = 36 + \frac{36}{3}$ = 48 km/hr

Difference in time = $T_1 - T_2$ = 7 am - 7.20 am = $-\frac{1}{3}$ hr

$\therefore$ Distance of meeting point from Madras

$$= 36\left[\frac{68 - 48\left(-\frac{1}{3}\right)}{36 + 48}\right] = 36\left[\frac{68 + 16}{36 + 48}\right] = 36 \text{ km}$$

11. Their relative speed = 200 + 150 = 350 lines/hr.

So, they will meet after $= \frac{8190}{350} = \frac{117}{5}$ hrs

$\therefore$ they will meet at $\frac{117}{5} \times 200 = 4680$th line from the beginning.

12.

	Speed	Starting time
A	3 km	1 o'clock
B	4 km	2 o'clock
C	5 km	3 o'clock

A takes a lead of 3 km from B.

Relative speed of A and B = 4 - 3 = 1 km/hr.

$\therefore$ B catches A after $\frac{3}{1} = 3$ hrs, ie, at 2 + 3 = 5 o'clock.

$\therefore$ A returns at 5 o'clock and from a distance of $3 \times 4 = 12$ km from Poona.

In the mean time C covers a distance of $5 \times 2 = 10$ km from Poona.

Thus, A and C are 12 - 10 = 2 km apart at 5 o'clock.

Relative speed of A and C = 3 + 5 = 8 km/hr.

Thus, they meet after $\frac{2}{8} = \frac{1}{4}$ hr = 15 min.

Thus, C will get the message at 5.15 o'clock.

13. Relative speed = 12 - 10 = 2 km/hr.

$\therefore$ the thief will be caught after $= \frac{0.2}{2} = \frac{1}{10}$ hr.

$\therefore$ distance covered by the thief before he gets caught

$$= 10 \times \frac{1}{10} = 1 \text{ km.}$$

Quicker Maths (Direct formula) :

The distance covered by the thief before he gets caught

$$= \frac{\text{Lead of distance}}{\text{Relative speed}} \times \text{Speed of thief}$$

$$= \frac{0.2 \text{ km}}{2 \text{ km/hr}} \times 10 \text{ km/hr} = 1 \text{ km}$$

14. **Quicker Maths (Direct formula) :**

$$\text{Distance} = \frac{\text{Time difference} \times S_1 \times S_2}{S_1 - S_2}$$

Where, S_1 and S_2 are the speeds of the two persons.

$$\therefore \text{ distance} = \frac{\frac{1}{2} \times 3 \times \frac{15}{4}}{\frac{15}{4} - 3} = \frac{15}{2} = 7.5 \text{ km.}$$

Note : For the detail method:

Let the distance be x km.

Then, $\frac{x}{3} - \frac{x}{\frac{15}{4}} = \frac{1}{2} \Rightarrow x = 7.5$ km.

15. **Quicker Method (Same as in Q. 14)**

$$\text{Ans} = \frac{\frac{32}{60} \times 9 \times 10}{10 - 9} = 48 \text{ km.}$$

16. Let the distance = x km.

Then, total time $= \frac{x}{2 \times 21} + \frac{x}{2 \times 24} = 10$ hrs.

or, $\frac{x(24 + 21)}{2 \times 21 \times 24} = 10 \quad \therefore x = \frac{10 \times 2 \times 21 \times 24}{45} = 224$ km

Quicker Maths (Direct Formula) :

$$\text{Distance} = \frac{2 \times \text{Total time} \times \text{Product of speeds}}{\text{Sum of Speeds}}$$

$$= \frac{2 \times 10 \times 21 \times 24}{21 + 24} = 224 \text{ km.}$$

17. Let the distance be x km

Then, time taken in up and down journey

$$= \frac{x}{3.5} + \frac{x}{3.5} = \frac{x}{3} + \frac{x}{4} - \frac{5}{60}$$

or, $\frac{2x}{3.5} = \frac{7x}{12} - \frac{5}{60}$

or, $\frac{x(24.5 - 24)}{3.5 \times 12} = \frac{5}{60} \qquad$ or, $x = \frac{5}{60} \times \frac{3.5 \times 12}{0.5} = 7$ km.

18. **Quicker Maths (Direct formula) :**

$$\text{Distance} = \text{Total time} \left(\frac{\text{Product of speeds}}{\text{Sum of speeds}}\right)$$

$$= 5\frac{48}{60}\left(\frac{25 \times 4}{25 + 4}\right) = 5\frac{4}{5}\left(\frac{100}{29}\right) = \frac{29}{5}\left(\frac{100}{29}\right) = 20 \text{ km.}$$

Note : For detail method let the distance be x km.

Then, $\frac{x}{4}+\frac{x}{25} = 5\frac{48}{60} = \frac{29x}{100} = \frac{29}{5}$ $\quad \therefore x = 20$ km.

19. **Quicker Maths (Direct formula) :**

$$\text{Distance} = \frac{80(25+20)}{(25-20)} = 720 \text{ km.}$$

20. **Quicker Maths (Direct formula) :**

$$\text{Distance} = \frac{\text{Product of two speeds}}{\text{Diff. of two speeds}} \times \text{Diff. in time}$$

$$= \frac{4\times 6}{6-4} \times \frac{15+10}{60} = \frac{24}{2} \times \frac{25}{60} = 5 \text{ km.}$$

Note : 15 minutes late and 10 minutes early means the difference in arrival time is 15 + 10 = 25 minutes.

21. Let the speed of train be x km/hr

Then, speed of horse transit $= \frac{x}{3}$ km/hr

and speed of steamer $= \frac{2x}{3}$ km/hr

Now, total time $= \frac{120\times 3}{2x} + \frac{450}{x} + \frac{60\times 3}{x} = 13\frac{1}{2}$

or, $\frac{1}{x} = \frac{27}{2\times 810} = \frac{1}{60}$ $\quad \therefore x = 60$ km/hr.

22. Total distance covered by the train in 4 hours = 4 × 45 = 180 km.

$\therefore$ In the distance of 180 km, there will be

$\frac{180000}{50} + 1 = 3601$ poles.

Trains

This chapter is the same as the previous chapter (Time & Distance). The only difference is that the length of the moving object (train) is also considered in this chapter.

Some important things to be noticed in this chapter are:

(1) When two trains are moving in **opposite** directions their speeds should be **added** to find the relative speed.

(2) When they are moving in the **same** direction the relative speed is the **difference** of their speeds.

(3) When a train passes a platform it should travel the length equal to the sum of the lengths of **train & platform both.**

Trains passing a telegraph post or a stationary man

Ex 1 : How many seconds will a train 100 metres long running at the rate of 36 km an hour take to pass a certain telegraph post ?

Soln : In passing the post the train must travel its own length.

Now, 36 km/hr $= 36 \times \frac{5}{18} = 10$ m/sec.

$\therefore$ required time $= \frac{100}{10} = 10$ seconds.

Trains crossing a bridge or passing a railway station

Ex 2: How long does a train 110 metres long running at the rate of 36 km/hr take to cross a bridge 132 metres in length?

Soln : In crossing the bridge the train must travel its own length plus the length of the bridge.

Now, 36 km/hr $= 36 \times \frac{5}{18} = 10$ m/sec.

$\therefore$ required time $= \frac{242}{10} = 24.2$ seconds.

Trains running in opposite direction

Ex 3: Two trains 121 metres and 99 metres in length respectively are running in opposite directions, one at the rate of 40 km and the other at the rate of 32 km an hour. In what time will they be completely clear of each other from the moment they meet ?

Soln : As the two trains are moving in opposite directions their relative speed = 40+32=72 km/hr, i.e. they are approaching each other at 72 km/hr or 20 m/sec.

$\therefore$ the required time = $\frac{\text{Total length}}{\text{Relative speed}} = \frac{121+99}{20} = 11$ secs.

Trains running in the same direction

Ex 4: In Ex 3 if the trains were running in the same direction, in what time will they be clear of each other ?

Soln : Relative speed = 40-32 = 8 km/hr = $\frac{20}{9}$ m/sec

Total length = 121 + 99 = 220 m.

$\therefore$ required time = $\frac{\text{Total length}}{\text{Relative speed}} = \frac{220}{20} \times 9 = 99$ sec.

Train passing a man who is walking

Ex 5 : A train 110 metres in length travels at 60 km/hr. In what time will it pass a man who is walking at 6 km an hour (i) against it; (ii) in the same direction ?

Soln : This question is to be solved like the above examples 3 and 4, the only difference being that the length of the man is zero.

(i) Relative speed = 60+6 = 66 km/hr = $\frac{55}{3}$ m/sec.

$\therefore$ required time = $\frac{110}{55} \times 3 = 6$ seconds.

(ii) Relative speed = 60 - 6 = 54 km/hr = 15 m/sec.

$\therefore$ required time = $\frac{110}{15} = 7\frac{1}{3}$ seconds.

Ex 6 : Two trains are moving in the same direction at 50 km/hr and 30 km/hr. The faster train crosses a man in the slower train in 18 seconds. Find the length of the faster train.

Soln : Relative speed = (50 - 30) km/hr = $\left(20 \times \frac{5}{18}\right)$ m/sec

$= \frac{50}{9}$ m/sec

Distance covered in 18 sec at this speed = $18 \times \frac{50}{9} = 100$ m

$\therefore$ length of the faster train = 100 m

Ex 7 : A train running at 25 km/hr takes 18 seconds to pass a platform. Next, it takes 12 seconds to pass a man walking at 5 km/hr in the opposite direction. Find the length of the train and that of the platform.

Soln : Speed of the train relative to man = 25 + 5 = 30 km/hr

$= 30 \times \frac{5}{18} = \frac{25}{3}$ m/sec.

Distance travelled in 12 seconds at this speed = $\frac{25}{3} \times 12 = 100$ m.

$\therefore$ Length of the train = 100m.

Speed of train = 25 km/hr = $25 \times \frac{5}{18} = \frac{125}{18}$ m/sec.

Distance travelled in 18 secs at this speed = $\frac{125}{18} \times 18 = 125$ m.

$\therefore$ length of train + length of platform = 125 m.

$\therefore$ length of platform = 125-100 = 25 m.

Miscellaneous

Ex. 8 : Two trains start at the same time from Hyderabad and Delhi and proceed toward each other at the rate of 80 km and 95 km per hour respectively. When they meet, it is found that one train has travelled 180 km more than the other. Find the distance between Delhi and Hyderabad.

Soln : Faster train moves 95 - 80 = 15 km more in 1 hr.

$\therefore$ faster train moves 180 km more in $\frac{1}{15} \times 180 = 12$ hrs.

Since, they are moving in opposite directions, they cover a distance of $80 + 95 = 175$ km in 1 hr.

$\therefore$ in 12 hrs they cover a distance = $175 \times 12 = 2100$ km

$\therefore$ distance = 2100 km.

Direct Formula: $\boxed{\text{Distance} = \text{Difference in distance} \times \frac{\text{Sum of speed}}{\text{Diff in speed}}}$

$= 180 \times \frac{175}{15} = 2100$ km.

Ex. 9 : Two trains for Delhi leave Jaipur at 8.30 a.m. and 9.00 a.m. and travel at 60 and 75 km/hr respectively. How many km from Jaipur will the two trains meet?

Soln : Use the **direct formula** given for Ex 23 in previous chapter (*Time and Distance*)

Required distance = $(9.00 - 8.30) \times \left(\frac{60 \times 75}{75 - 60}\right)$

$= \frac{1}{2}\left(\frac{60 \times 75}{15}\right) = 150$ km.

Ex. 10 : Without stoppage a train travels at an average speed of 75 km per hour and with stoppages it covers the same distance at an average speed of 60 km/hr. How many minutes per hour does the train stop?

Soln : Use the **Direct Formula** given in Ex. 28 in previous chapter (*Time and Distance*)

$$\text{Time of rest per hour} = \frac{\text{Difference in average speed}}{\text{Speed without stoppage}}$$

$$= \frac{75-60}{75} = \frac{1}{5}\text{ hr} = 12 \text{ minutes.}$$

Ex. 11 : A train passes by a stationary man standing on the platform in 7 seconds and passes by the platform completely in 28 seconds. If the length of the platform is 330 metres, what is the length of the train?

Soln : Let the length of the train be x m.

Then speed of the train $= \frac{x}{7}$ m per sec.

And also the speed of the train $= \frac{x+330}{28}$ m per sec.

Both the speeds should be equal, ie, $\frac{x}{7} = \frac{x+330}{28}$

or, $28x - 7x = 7 \times 330$

$\therefore x = \frac{7 \times 330}{21} = 110$ m.

Quicker Approach : The train covers its length in 7 seconds and covers its length plus length of platform in 28 seconds. That is, it covers the length of the platform in 28 - 7 = 21 seconds.

Now, since it covers 330 m in 21 seconds

$\therefore$ Distance covered in 7 seconds $= \frac{330}{21} \times 7 = 110$ m

Thus we get a **direct formula** as :

$$\boxed{\text{Length of train} = \frac{\text{Length of platform}}{\text{Difference in time}} \times \text{Time taken to cross a stationary pole or man}}$$

$$= \frac{330}{21} \times 7 = 110 \text{ m}$$

Ex. 12 : Two stations A and B are 110 km apart on a straight line. One train starts from A at 8 a.m. and travels towards B at 40 km per hour. Another train starts from B at 10 a.m. and travels towards A at 50 km per hour. At what time will they meet?

Soln : Let the first train meet the second x hrs after it starts, then

$40x + (x - 2) \times 50 = 110$ ------ (see note)

or, $90x = 110 + 100 = 210$

$\therefore x = \frac{210}{90}$ hrs $= \frac{7}{3}$ hrs $= 2\frac{1}{3}$ hrs = 2 hrs 20 minutes

= 10.20 a.m.

Direct Formula : They will meet at

$$8 \text{ a.m.} + \frac{110 + (10 \text{ a.m.} - 8 \text{ a.m.}) \times 50}{40 + 50}$$

$$= 8 \text{ a.m.} + \frac{210}{90} = 10.20 \text{ a.m.}$$

Note : Distance covered by the first train = 40x km

The second train starts 2 hrs after the first starts its journey, so the distance covered by the second train = 50 (x - 2)

$\therefore$ Total distance = 110 km = 40x + 50 (x - 2)

Ex. 13 : A train 105 metres long moving at a speed of 54 km per hour crosses another train in 6 seconds. Then which of the following is true?

(1) Trains are moving in the same direction.
(2) Trains are moving in opposite directions.
(3) The other train is not moving.

Soln : Speed of the first train = 54 km/hr

$$= 54 \times \frac{5}{18} = 15 \text{ m/sec.}$$

Time taken by it to cover its own length $= \frac{105}{15} = 7$ seconds.

Since, the time (6 sec) taken to cross the other train is less than 7 seconds, it is clear that the other train is moving in the opposite direction.

Ex. 14 : Two trains of the same length but with different speeds pass a static pole in 4 seconds and 5 seconds respectively. In what time will they cross each other when they are moving in

(1) the same direction
(2) opposite directions.

Soln : Let the length of the trains be x m.

The speeds of the two trains $= \frac{x}{4}$ m/s & $\frac{x}{5}$ m/s.

Total distance to be travelled = 2x m.

(1) Relative speed when they are moving in the same direction

$$= \frac{x}{4} - \frac{x}{5} = \frac{x}{20} \text{ m/sec.}$$

$\therefore$ required time $= 2x \div \frac{x}{20} = 40$ seconds

(2) Relative speed when they are moving in opposite directions

$$= \frac{x}{4} + \frac{x}{5} = \frac{9x}{20} \text{ m/sec.}$$

$\therefore$ required time $= 2x \div \frac{9x}{20} = \frac{40}{9} = 4\frac{4}{9}$ seconds.

Direct Formula : (1) When they are moving in the same direction:

$$\text{Time} = \frac{2(4 \times 5)}{5 - 4} = 40 \text{ seconds.}$$

(2) When they are moving in opposite directions:

$$\text{Time} = \frac{2(4 \times 5)}{5 + 4} = \frac{40}{9} = 4\frac{4}{9} \text{ seconds.}$$

Ex. 15 : In the above example, if it is given that the trains are of different length but moving with the same speed, discuss the case.

Soln : Let the lengths of the trains be 4x m & 5x m respectively.

(1) When they are moving in the same direction,
relative velocity = x - x = 0 m/s
Thus, they can't pass each other.

(2) When they are moving in opposite directions, relative velocity = x + x = 2x m/s

$$\therefore \text{time} = \frac{\text{Total length}}{\text{Speed}} = \frac{4x + 5x}{2x} = \frac{4 + 5}{2} = 4.5 \text{ sec}$$

Thus we see that in this case the direct formula is

Required Time = Average of the two times

$$= \frac{4 + 5}{2} = 4.5 \text{ sec.}$$

Ex. 16 : Two trains of length 100 m and 80 m respectively run on parallel lines of rails. When running in the same direction the faster train passes the slower one in 18 seconds, but when they are running in opposite directions with the same speeds as earlier, they pass each other in 9 seconds. Find the speed of each train.

Soln : Let the speeds of the trains be x m/s and y m/s.
When they are moving in the same direction,
the relative speed = (x - y) m/s

$$\therefore x - y = \frac{100 + 80}{18} = 10$$

Similarly, $x + y = \frac{100 + 80}{9} = 20$

Solving the two equations
x = 15 m/s and y = 5 m/s

Direct Formula :

$$\text{Speed of the faster train} = \frac{100 + 80}{2}\left(\frac{18 + 9}{18 \times 9}\right)$$

$$= 90\left(\frac{27}{18 \times 9}\right) = 15 \text{ m/s}$$

$$\text{Speed of the slower train} = \frac{100 + 80}{2}\left(\frac{18 - 9}{18 \times 9}\right) = 5 \text{ m/s}$$

Thus a general formula for the speed is given as:

Average length of two trains ×

$$\left[\frac{1}{\text{Opposite direction's time}} \pm \frac{1}{\text{Same direction's time}}\right]$$

Ex. 17 : Two trains, each of 80 m long, pass each other on parallel lines. If they are moving in the same direction, the faster one takes one minute to pass the slower one completely. If they are moving in opposite directions, they completely pass each other in 3 seconds. Find the speed of the trains in metre per second.

Soln : This is a special case of Ex. 16. Here both the trains have the same length. Let the speeds of the trains be x m/s & y m/s

When they are moving in the same direction, relative speed

$$= (x - y) \text{ m/s}$$

$$\therefore x - y = \frac{80 + 80}{60}$$

$$\text{or, } x - y = \frac{8}{3} \text{ ------ (1)}$$

When they are moving in the opposite directions,
relative speed = x + y m/s

$\therefore x+y=\frac{80+80}{3}=\frac{160}{3}$ ---- (2)

Adding (1) and (2) we have; $2x=\frac{8}{3}+\frac{160}{3}=\frac{168}{3}=56$

$\therefore x=28$ m/s

And from (1) $y=x-\frac{8}{3}$

$$=28-\frac{8}{3}=\frac{84-8}{3}=\frac{76}{3}=25\frac{1}{3}$$

Direct Formula : The same direct formula as in Ex. 16 works in this case also.

Speed of faster train $=\frac{80+80}{2}\left(\frac{60+3}{60\times 3}\right)=28$ m/s

Speed of slower train $=\frac{80+80}{2}\left(\frac{60-3}{60\times 3}\right)$

$$=\frac{76}{3}=25\frac{1}{3}\text{ m/s}$$

Note : The general formula for the above question gives speed of trains

$$=80\left[\frac{1}{3}\pm\frac{1}{60}\right]\text{m/s}=80\left[\frac{60\pm 3}{180}\right]$$

$=28$ m/s and $25\frac{1}{3}$ m/s

Ex. 18 : Two trains can run at the speed of 54 km/hr and 36 km/hr respectively on parallel tracks. When they are running in opposite directions they pass each other in 10 seconds. When they are running in the same direction, a person sitting in the faster train observes that he passes the other train in 30 seconds. Find the length of the trains.

Soln : Speeds of trains in metres per second is 15 m/s and 10 m/s respectively. Let the length of faster & slower trains be x m and y m respectively. When they are running in opposite directions :
Relative speed = 15 + 10 = 25 m/s.
Total length = (x + y) m.

$\therefore$ time to cross each other $=\frac{x+y}{25}=10$

$\therefore x+y=250$ ----- (1)

In the second case, the man passes the length of the slower train (y) with a speed of (15 - 10) m/s = 5 m/s

Then time $=\frac{y}{5}=30$

$\therefore y=150$ m.

$\therefore$ length of slower train = 150 m.

And from (1), x = 100 m.

$\therefore$ Length of faster train = 100 m.

Note : This example needs no quicker method. If you are clear with the second case you can get the result very quickly.

Ex. 19 : A train overtakes two persons who are walking in the same direction as the train is moving, at the rate of 2 km/hr and 4 km/hr and passes them completely in 9 and 10 seconds respectively. Find the speed and the length of the train.

Soln : Speeds of two men are :

2 km/hr $=2\times\frac{5}{18}=\frac{5}{9}$ m/s

and 4 km/hr $=4\times\frac{5}{18}=\frac{10}{9}$ m/s

Let the speed of the train be x m/s. Then relative speeds are $\left(x-\frac{5}{9}\right)$ m/s and $\left(x-\frac{10}{9}\right)$ m/s

Now, length of train = Relative Speed × Time taken to pass a man

$\therefore$ length of train $=\left(x-\frac{5}{9}\right)\times 9=\left(x-\frac{10}{9}\right)\times 10$ ---- (*)

$\therefore x=\frac{100}{9}-\frac{45}{9}=\frac{55}{9}$ m/s

$\therefore$ Speed of train $=\frac{55}{9}\times\frac{18}{5}=22$ km/hr

and length of train $=\left(x-\frac{5}{9}\right)9=\left(\frac{55}{9}-\frac{5}{9}\right)9=50$ m.

Note : During calculation (*) should be your first step.

Quicker Method (Direct Formula) :

$$\boxed{\text{Length of the train}=\frac{\text{Diff in Speed of two men}\times T_1\times T_2}{(T_2-T_1)}}$$

where T_1 and T_2 are times taken by the train to pass the two men, all in the same direction.

Thus in this case $= \dfrac{\left(\frac{10}{9}-\frac{5}{9}\right)\times 9\times 10}{10-9} = 50$ m.

Once you get the length of the train it becomes easy to find its speed. Try it.

What happens when the train and the men are moving in opposite directions? See the following example.

Ex. 20 : A train passes two persons who are walking in the direction opposite which the train is moving, at the rate of 5 m/s and 10 m/s in 6 seconds and 5 seconds respectively. Find the length of the train and speed of the train.

Soln : Let the speed of the train be x m/s. Then, as in previous example,
Length of the train = (x + 5) 6 = (x + 10) 5
or, (x + 5) 6 = (x + 10) 5
$\therefore$ x = 20 m/s
and length of the train = (20 + 5) × 6 = 150 m.

Quicker Method (Direct Formula) : If we look at the question carefully we find that the slower person takes more time than the faster person to cross the train. But it was just opposite in the previous case. Thus our direct formula in this case is:

$$\boxed{\text{Length of the train} = \frac{\text{Difference in speed} \times T_1 \times T_2}{T_1 - T_2}}$$

$$= \frac{(10-5)\times 5\times 6}{6-5} = 150 \text{ m.}$$

Note : The two direct formulae differ in denominator only. And this difference may be finished when we write the two formulae in the form :
Length of the train

$$= \frac{\text{Difference in speed} \times \text{Multiplication of times}}{\text{Difference in time}}$$

Ex. 21 : A train passes a pole in 15 seconds and passes a platform 100m long in 25 seconds. Find its length.

Soln : Let the length of the train = x m; then, equating the speeds,

$$\frac{x}{15} = \frac{x+100}{25}$$

or, 25x = 15x + 100 × 15
or, 10x = 1500
$\therefore$ x = 150 m
$\therefore$ Length of the train = 150 m

Quicker Method (Direct Formula) :

$$\text{Length of the train} = \frac{\text{Time to pass a pole} \times \text{Length of platform}}{\text{Difference in time to cross a pole and platform}}$$

$\therefore$ length of the train $= \dfrac{15 \times 100}{25-15} = 150$m

Ex. 22 : A train 100 metres in length passes a pole in 10 seconds and another train of the same length travelling in opposite direction in 8 seconds. Find the speed of the second train.

Soln : Speed of the first train $= \dfrac{100}{10} = 10$ m/s

Relative speed in second case $= \dfrac{100+100}{8} = 25$ m/s

$\therefore$ Speed of the second train = 25 - 10 = 15 m/s

or, $15 \times \dfrac{18}{5} = 54$ km/hr.

Ex. 23: A goods train and a passenger train are running on parallel tracks in the same direction. The driver of the goods train observes that the passenger train coming from behind overtakes and crosses his train completely in 60 seconds. Whereas a passenger on the passenger train marks that he crosses the goods train in 40 seconds. If the speeds of the trains be in the ratio of 1 : 2, find the ratio of their lengths.

Soln: Suppose the speeds of the two trains are x m/s and 2x m/s respectively. Also, suppose that the lengths of the two trains are A m and B m respectively.

Then, $\dfrac{A+B}{2x-x} = 60$ ------ (1)

and $\dfrac{A}{2x-x} = 40$ ------ (2)

Dividing (1) by (2) we have

$$\frac{A+B}{A} = \frac{60}{40}$$

or, $\frac{B}{A}+1=\frac{3}{2}$ or, $\frac{B}{A}=\frac{1}{2}$

$\therefore$ A : B = 2 : 1

Quicker Approach : The man in the passenger train crosses the goods train in 40 seconds. This implies that the man in the goods train can observe that the passenger train passes his in 60-40 = 20 seconds. (This is only because relative velocity for both the persons are the same.)

Therefore, we may conclude that a person takes double the time to cross the goods train than to cross the passenger train. Thus ratio of their lengths = 40 : 20 = 2 : 1.

Ex. 24: A train after travelling 50 km meets with an accident and then proceeds at $\frac{3}{4}$ of its former speed and arrives at its destination 35 minutes late. Had the accident occurred 24 km further, it would have reached the destination only 25 minutes late. The speed of the train is ______.

Soln : Quicker Approach : If we think carefully, we may conclude that the speeds of the train upto 50 km are the same in both the cases. And also, the speeds after (50 + 24 =) 74 km are the same in both the cases. Thus the difference in time (35 min - 25 min = 10 min) is only due to the difference in speeds for the 24-km journey.

Now, if the speed of the train is x km/hr then $\frac{24}{\frac{3x}{4}}-\frac{24}{x}=\frac{10}{60}$

or, $\frac{32-24}{x}=\frac{10}{60}$ $\quad\therefore x=\frac{8\times 60}{10}=48$ km/hr

Direct Formula :

$$\text{Speed of train}=\frac{24\left(1-\frac{3}{4}\right)}{\frac{3}{4}\left(\frac{35-25}{60}\right)}=\frac{6\times 4\times 6}{3}=48\text{ km/hr}$$

Note : In the above formula, the numerator is clear. $\left(1-\frac{3}{4}\right)$ shows the fractional change in speed. Initially, it was 1 (suppose). After accident it was reduced to $\frac{3}{4}$. The denominator has two parts $\frac{3}{4}$ and $\left(\frac{35-25}{60}\right)$. $\frac{3}{4}$ is the changed fractional speed. $\frac{35-25}{60}$ is the difference (in hour) in arrival times.

Ex. 25: A train leaves Delhi for Amritsar at 2 : 45 pm and goes at the rate of 50 km an hour. Another train leaves Amritsar for Delhi at 1 : 35 pm and goes at the rate of 60 km per hour. If the distance between Delhi and Amritsar is 510 km, at what distance from Delhi will the two trains meet?

Soln: Let the two trains meet at x km from Delhi. Then their meeting time

$=2.45\text{ pm}+\frac{x}{50}=1.35\text{ pm}+\frac{510-x}{60}$

or, $(2.45\text{ pm - }1.35\text{ pm})+\left(\frac{x}{50}+\frac{x}{60}\right)=\frac{510}{60}=\frac{17}{2}$

or, $\left(1\frac{1}{6}\text{hr}\right)+x\left(\frac{50+60}{50\times 60}\right)=\frac{17}{2}$

or, $x\left(\frac{110}{3000}\right)=\frac{17}{2}-\frac{7}{6}=\frac{44}{6}$

$\therefore\ x=\frac{44}{6}\times\frac{3000}{110}=200$ km

Quicker Approach: The second train (from Amritsar) starts its journey earlier. It covers $60\times(2.45\text{ pm}-1.35\text{ pm})=60\times 1\frac{1}{6}\text{ hr}=70$ km when the first (from Delhi) train starts its journey.

Now, both the trains cover 510 - 70 = 440 km with relative speed of 50 + 60 = 110 km/hr

Thus they meet after $\frac{440}{110}=4$ hrs after the first train starts at 2.45 pm.

Now, the first train covers 4 × 50 = 200 km to meet the second train.

Direct Formula: With the help of the above approach a direct formula can be derived as:

The meeting point from Delhi (first train's starting point)

$$=S_1\left[\frac{\text{Total Dist}-S_2\times(T_1-T_2)}{S_1+S_2}\right]$$

Where, S_1 is the speed of the first train

S_2 is the speed of the second train

T_1 is the starting time of the 1st train

T_2 is the starting time of the 2nd train

$$= 50\left[\frac{510 - 60 \times 7}{60 + 50}\right] = 50\left[\frac{440}{110}\right] = 200 \text{ km}$$

Ex. 26: A train covers a distance between stations A and B in 45 minutes. If the speed is reduced by 5 km/hr, it will cover the same distance in 48 minutes. What is the distance between the two stations A and B (in km)? Also, find the speed of the train.

Soln : Suppose the distance is x km and the speed of the train is y km/hr. Thus we have two relationships:

(1) $\frac{x}{y} = \frac{45}{60} = \frac{3}{4} \Rightarrow x = \frac{3}{4}y$

(2) $\frac{x}{y-5} = \frac{48}{60} = \frac{4}{5} \Rightarrow x = \frac{4}{5}(y - 5)$

From (1) and (2)

$\frac{3}{4}y = \frac{4}{5}(y - 5)$ or, $y\left(\frac{4}{5} - \frac{3}{4}\right) = 4$

or, $y = \frac{4 \times 20}{16 - 15} = 80$ km/hr

Therefore speed = 80 km/hr and distance

$x = \frac{3}{4} \times 80 = 60$ km

Direct formula :

Speed of the train $= \frac{48}{48 - 45} \times 5 = 80$ km/hr

and distance $= 5\left[\frac{45 \times 48}{48 - 45}\right]\frac{1}{60} = 60$ km.

Note : (1) In the formula for distance we have used $\frac{1}{60}$ to change minutes into hours.

(2) We don't need to remember the formula for distance. Once we find the speed, we may use the first information to find the distance.

Thus, distance $= 80 \times \frac{45}{60} = 60$ km

Ex. 27: Two places P and Q are 162 km apart. A train leaves P for Q and at the same time another train leaves Q for P. Both the trains meet 6 hrs after they start moving. If the train travelling from P to Q travels 8 km/hr faster than the other train, find the speed of the two trains.

Soln : Suppose the speeds of the two trains are p km/hr and q km/hr respectively. Thus

$p + q = \frac{162}{6} = 27$ ..... (i) and $p - q = 8$ ..... (ii)

(i) + (ii) implies that

$2p = 35 \quad \therefore p = 17.5$ km/hr

and (i) - (ii) implies that

$2q = 19 \quad \therefore q = 9.5$ km/hr

Direct Formula:

Speeds of the trains $= \frac{162 + 6 \times 8}{2 \times 6}$ and $\frac{162 - 6 \times 8}{2 \times 6}$

Ex. 28: Two trains A and B start from Delhi and Patna towards Patna and Delhi respectively. After passing each other they take 4 hours 48 minutes and 3 hours and 20 minutes to reach Patna and Delhi respectively. If the train from Delhi is moving at 45 km/hr then find the speed of the other train.

Soln: Detailed:

A | B
Delhi (45 km) M x km/hr Patna

Suppose the speed of train B is x km/hr and they meet at M. Now, distance MB = 45 × (4 hrs + 48 minutes)

$$= 45 \times \left(4\frac{4}{5}\right) = 45 \times \frac{24}{5} = 216 \text{ km}.$$

And the distance AM = x × (3 hrs + 20 minutes)

$$= x\left(3\frac{1}{3}\right) = \frac{10x}{3} \text{ km}.$$

Now, the time to reach the train from Patna to M = the time to reach the train from Delhi to M.

or, $\frac{MB}{x} = \frac{AM}{45}$ or, $\frac{216}{x} = \frac{10x}{3 \times 45}$

or, $10x^2 = 216 \times 3 \times 45$ or, $x^2 = 2916 \quad \therefore x = 54$ km/hr.

Quicker Method (Direct Formula): Speed of the other train

= Speed of first train

$$\times \sqrt{\frac{\text{Time taken first train after meeting}}{\text{Time taken by second train after meeting}}}$$

$$= 45\sqrt{\frac{4\frac{4}{5}}{3\frac{1}{3}}} = 45\sqrt{\frac{24}{5}\times\frac{3}{10}} = 45\sqrt{\frac{36}{25}} = 45\times\frac{6}{5} = 54 \text{ km/hr.}$$

Ex. 29: The speeds of two trains are in the ratio 3 : 4. They are going in opposite directions along parallel tracks. If each takes 3 seconds to cross a telegraph post, find the time taken by the trains to cross each other completely?

Soln: Since both the trains cross a telegraph pole in equal time, the ratio of their speeds should be equal to the ratio of their lengths. That is, the lengths of the two trains are in the ratio of 3 : 4.

Suppose the lengths of the two trains be 3x and 4x metres respectively. Since each of them takes 3 seconds to cross a telegraph pole, speed of the first train $= \frac{3x}{3} = x$ m/s and speed of the second train $= \frac{4x}{3}$ m/s

Since they are moving in opposite directions their relative speed $= x + \frac{4x}{3} = \frac{7x}{3}$ m/s

Sum of their lengths = 3x + 4x = 7x m

$\therefore$ time taken to cross each other $= \frac{7x}{\left(\frac{7x}{3}\right)} = 3$ seconds

Quicker Approach: In the above case where each train takes the same time to cross a telegraph pole, they will take the same time to cross each other, ie, 3 seconds, whatever be the ratio of their speeds.

Thus you don't need to do any calculation. The answer is 3 seconds.

Note: (1) You must verify that the answer remains the same (ie, 3 secs) if the ratio of their speeds are different from the ratio 3 : 4.

Suppose the ratio of speeds be a : b.

Then again, the ratio of their lengths = a : b

Let the lengths be ax and bx metres.

Then speed of the first train $= \frac{ax}{3}$ m/s

and speed of the second train $= \frac{bx}{3}$ m/s

Since they are moving in opposite directions, their relative speed

$= \frac{ax}{3} + \frac{bx}{3} = \frac{x(a+b)}{3}$ m/s

Sum of their lengths = ax + bx = x (a + b) m

$\therefore$ time taken to cross each other

$$= \frac{\text{Total length}}{\text{Relative speed}} = \frac{x(a+b)}{\frac{x(a+b)}{3}} = 3 \text{ sec}$$

Thus we see that the result does not depend on the ratio of speeds.

(2) What happens when both the trains take different times to cross a telegraph pole?

See the Ex. 30: A most generalised form of the question.

Ex. 30: The speed of two trains are in the ratio x : y. They are moving in the opposite directions on parallel tracks. The first train crosses a telegraph pole in 'a' seconds where as the second train crosses a telegraph pole in 'b' seconds. Find the time taken by the trains to cross each other completely.

Soln: Suppose the speeds are Ax m/s and Ay m/s.

Then length of first train = Ax × a = Axa metres

and length of second train = Ay × b = Ayb metres

Time to cross each other $= \frac{\text{Sum of lengths}}{\text{Sum of speeds}}$

$$= \frac{Axa + Ayb}{Ax + Ay} = \frac{ax + by}{x + y} \text{ seconds.}$$

The above general formula can also be used in Ex. 29.

Here x : y = 3 : 4 and a = b = 3 seconds

Thus, time to cross each other $= \frac{3\times 3 + 3\times 4}{3+4} = \frac{21}{7} = 3$ seconds.

As in **Ex. 29**, if a = b then the general formula becomes:

Reqd time to cross each other $= \frac{a(x+y)}{(x+y)} = a$ seconds.

Ex. 31: The speeds of two trains are in the ratio of 7 : 9. They are moving on the opposite directions on parallel tracks. The first train crosses a telegraph pole in 4 seconds whereas the second train crosses the pole in 6 seconds. Find the time taken by the trains to cross each other completely.

Soln: Using the generalised formula of Ex. 30:

The required time $= \frac{7 \times 4 + 9 \times 6}{7 + 9} = \frac{82}{16} = 5\frac{1}{8}$ seconds.

Note: Verify the above answer with detail method.

Ex. 32: Two trains are moving in the opposite directions on parallel tracks at the speeds of 64 km/hr and 96 km/hr respectively. The first train passes a telegraph post in 5 seconds whereas the second train passes the post in 6 seconds. Find the time taken by the trains to cross each other completely.

Detailed solution:

Length of the first train $= 64\left(\frac{5}{18}\right) \times 5$ metres.

Length of the second train $= 96\left(\frac{5}{18}\right) \times 6$ metres.

Relative speed $= \left[64\left(\frac{5}{18}\right) + 96\left(\frac{5}{18}\right)\right]$ m/s.

$\therefore$ required time to cross each other

$$= \frac{\text{Total length of two trains}}{\text{Relative speed}} = \frac{64\left(\frac{5}{18}\right) \times 5 + 96\left(\frac{5}{18}\right) \times 6}{64\left(\frac{5}{18}\right) + 96\left(\frac{5}{18}\right)}$$

$$= \frac{64 \times 5 + 96 \times 6}{64 + 96} = \frac{320 + 576}{160} = \frac{896}{160} = \frac{28}{5} = 5\frac{3}{5} \text{ sec}$$

Quicker Method: Ratio of speeds = 64 : 96 = 2 : 3

Times to cross a telegraph post are 5 sec & 6 sec.

Now, we can use the general formula given in **Ex. 30**

Required time to cross each other

$$= \frac{2 \times 5 + 3 \times 6}{2 + 3} = \frac{10 + 18}{5} = \frac{28}{5} = 5\frac{3}{5} \text{ sec.}$$

EXERCISE

1. How long will a train 130 m long travelling at 40 km an hour take to pass a kilometre stone ?
2. How long will a train 60 m long travelling at 40 km an hour take to pass through a station whose platform is 90 m long ?
3. A train travelling at 30 km an hour took $13\frac{1}{2}$ sec in passing a certain point. Find the length of the train.

4. Find the length of a bridge which a train 130 m long, travelling at 45 km an hour, can cross in 30 secs.
5. The length of the train that takes 8 seconds to pass a pole when it runs at a speed of 36 km/hr is ________ metres.
6. A train 50 metres long passes a platform 100 metres long in 10 seconds. The speed of the train is ________ km/hr.
7. How many seconds will a train 60 m in length, travelling at the rate of 42 km an hour, take to pass another train 84 m long, proceeding in the same direction at the rate of 30 km an hour?
8. A train 75 metres long overtook a person who was walking at the rate of 6 km an hour, and passed him in $7\frac{1}{2}$ seconds. Subsequently it overtook a second person, and passed him in $6\frac{3}{4}$ seconds. At what rate was the second person travelling ?
9. Two trains running at the rates of 45 and 36 km an hour respectively, on parallel rails in opposite directions, are observed to pass each other in 8 seconds, and when they are running in the same direction at the same rate as before, a person sitting in the faster train observes that he passes the other in 30 seconds. Find the lengths of the trains.
10. Two trains measuring 100 and 80 m respectively, run on parallel lines of rails. When travelling in opposite directions they are observed to pass each other in 9 seconds, but when they are running in the same direction at the same rates as before, the faster train passes the other in 18 seconds. Find the speed of the two trains in km per hour.
11. Two trains, each 80 m long, pass each other on parallel lines. If they are going in the same direction, the faster one takes one minute to pass the other completely. If they are going in different directions, they completely pass each other in 3 seconds. Find the rate of each train in m per second.
12. A train takes 5 seconds to pass an electric pole. If the length of the train is 120 metres, the time taken by it to cross a railway platform 180 metres long is ________ seconds.
13. A train is running at the rate of 40 kmph. A man is also going in the same direction parallel to the train at the speed of 25 kmph. If the train crosses the man in 48 seconds, the length of the train is ________ metres.
14. A train speeds past a pole in 15 seconds and speeds past a platform 100 metres long in 25 seconds. Its length in metres is ______.
15. A train 100 metres in length passes a milestone in 10 seconds and

another train of the same length travelling in opposite direction in 8 seconds. The speed of the second train is ______kmph.

16. Two trains are running in opposite directions with speeds of 62 kmph and 40 kmph respectively. If the length of one train is 250 metres and they cross each other in 18 seconds, the length of the other train is _________ metres.

17. Two trains running in the same direction at 40 kmph and 22 kmph completely pass one another in 1 minute. If the length of the first train is 125 metres, the length of the second train is __metres.

18. A train 100 metres long moving at a speed of 50 kmph crosses a train 120 metres long coming from opposite direction in 6 seconds. The speed of the second train is _________ kmph.

19. Two stations A and B are 110 km apart on a straight line. One train starts from A at 7 a.m. and travels towards B at 20 km per hour speed. Another train starts from B at 8 a.m. and travels towards A at a speed of 25 km per hour. At what time will they meet ?

20. A train overtakes two persons who are walking in the same direction in which the train is going, at the rate of 2 kmph and 4 kmph respectively and passes them completely in 9 and 10 seconds respectively. The length of the train is _______ metres.

a) 72 metres b) 54 metres c) 50 metres d) 45 metres

ANSWERS

1. Time $= \dfrac{\text{Total distance}}{\text{Speed}} = \dfrac{0.130}{40}$ hr $= \dfrac{0.130 \times 60 \times 60}{40} = 11.7$ sec

2. Speed = 40 km/hr $= 40 \times \dfrac{5}{18}$ m/s

$\therefore$ Time $= \dfrac{(60+90)}{40 \times 5} \times 18 = \dfrac{150 \times 18}{40 \times 5} = 13.5$ seconds

3. 30 km/hr $= 30 \times \dfrac{5}{18} = \dfrac{25}{3}$ m/s

Length of train $= \dfrac{25}{3} \times \dfrac{27}{2} = \dfrac{225}{2} = 112.5$ m

4. 45 km/hr $= 45 \times \dfrac{5}{18} = \dfrac{25}{2} = 12.5$ m/s

Distance covered by the train in 30 seconds
$= 12.5 \times 30 = 375$ m

$\therefore$ length of bridge $= 375 - 130 = 245$ m

5. 36 km/hr $= 36 \times \dfrac{5}{18} = 10$ m/s

Distance covered by train in 8 seconds = length of train
$= 8 \times 10 = 80$m

6. Speed of train $= \dfrac{100+50}{10} = 15$ m/s $= \dfrac{15 \times 18}{5} = 54$ km/hr

7. Relative speed = 42 - 30 = 12 km/hr

$= 12 \times \dfrac{5}{18} = \dfrac{10}{3}$ m/s

Time $= \dfrac{\text{Total length of both the train}}{\text{Relative speed}} = \dfrac{84+60}{\frac{10}{3}}$

$= \dfrac{144 \times 3}{10} = 43.2$ seconds

8. Relative speed of train and first person

$= \dfrac{75}{\frac{15}{2}} = 10$ m/s $= 10 \times \dfrac{18}{5} = 36$ km/hr

$\therefore$ speed of train = 36 + 6 = 42 km/hr

Now, relative speed of train and 2nd person

$= \dfrac{75}{27} \times 4$ m/s $= \dfrac{300}{27} \times \dfrac{18}{5} = 40$ km/hr

$\therefore$ speed of 2nd person = 42 - 40 = 2 km/hr.

Quicker Maths (Direct formula) : Speed of 2nd person = Relative speed of train with respect to 1st person + Speed of first person - Relative speed of train with respect to 2nd person

$= \left(\dfrac{75}{\frac{15}{2}} \times \dfrac{18}{5}\right) + 6 - \left(\dfrac{75}{27} \times 4 \times \dfrac{18}{5}\right) = 36 + 6 - 40 = 2$ km/hr

9. Relative speed of two trains = 45 + 36 = 81 km/hr (when two trains are moving in opposite directions)

$= 81 \times \dfrac{5}{18} = \dfrac{45}{2} = 22\dfrac{1}{2}$ m/s

$\therefore$ length of both the trains $= \dfrac{45}{2} \times 8 = 180$ m.

Now, when two trains are moving in the same direction, the relative speed $= 45 - 36 = 9$ km/hr $= \dfrac{9 \times 5}{18} = \dfrac{5}{2}$ m/s

The man sitting in the faster train passes the length of the slower train in 30 seconds.

$\therefore$ length of the slower train $= \frac{5}{2} \times 30 = 75$ m.

$\therefore$ length of the faster train $= 180 - 75 = 105$ m.

Quicker Maths (Direct formula) :

Length of slower train $= 30 \times$ (Relative speed of two trains)

$= 30\,(45 - 36)\,\frac{5}{18} = 75$ m.

Length of faster train

= Total length of both trains - length of slower train

$= 8\,(45 + 36)\,\frac{5}{18} - 75$

$= 8 \times \frac{81 \times 5}{18} - 75 = 180 - 75 = 105$ m

10. **Quicker Maths (Direct formula) :**

R_1 = Relative speed, when they are moving in the same direction

$$= \frac{100 + 80}{18} = 10 \text{ m/s}$$

R_2 = Relative speed, when they are moving in opposite directions

$$= \frac{100 + 80}{9} = 20 \text{ m/s}$$

Speed of faster train $= \frac{R_1 + R_2}{2} = \frac{10 + 20}{2} = 15$ m/s

$= 15 \times \frac{18}{5} = 54$ km/hr

Speed of slower train $= \frac{R_2 - R_1}{2} = \frac{20 - 10}{2} = 5$ m/s

$= 5 \times \frac{18}{5} = 18$ km/hr.

11. Same as in Q. 10.

$R_1 = \frac{80 + 80}{60} = \frac{160}{60} = \frac{8}{3}$ m/s $\quad R_2 = \frac{80 + 80}{3} = \frac{160}{3}$ m/s

Speed of faster train $= \frac{\frac{8}{3} + \frac{160}{3}}{2} = \frac{168}{6} = \frac{84}{3}$ m/s

$= \frac{84}{3} \times \frac{18}{5} = 100.8$ km/hr

Speed of slower train $= \frac{\frac{160}{3} - \frac{8}{3}}{2} = \frac{152}{6}$ m/s

$= \frac{152}{6} \times \frac{18}{5} = 91.2$ km/hr

12. Speed of train $= \frac{120}{5} = 24$ m/s

$\therefore$ time taken by the train to pass the platform

$$= \frac{120 + 180}{24} = 12.5 \text{ seconds}$$

13. Length of train = Relative speed $\times$ time $= (40 - 25)\left(\frac{5}{18}\right) \times 48$

$$= \frac{15 \times 5 \times 48}{18} = 200\text{m}$$

14. Let the length of train = x m

Then speed of train $= \frac{x}{15} = \frac{x + 100}{25}$

or, $25x = 15x + 1500$ or, $10x = 1500$ $\therefore x = 150$ m

Quicker Maths (Direct formula) : (See Ex 21)

Length of train $= \frac{\text{Time to pass a pole} \times \text{Length of platform}}{\text{Time to pass platform} - \text{Time to pass a pole}}$

$= \frac{15 \times 100}{25 - 10} = 150$ m.

15. Speed of the first train $= \frac{100}{10} = 10$ m/s

Relative speed with respect to other train $= \frac{100 + 100}{8} = 25$ m/s

$\therefore$ Speed of second train $= 25 - 10 = 15$ m/s $= 15 \times \frac{18}{5} = 54$ km/hr

16. **Quicker Maths (Direct formula) :**

Length of other train = Relative speed $\times$ time to cross each other - length of first train

$= \left(102 \times \frac{5}{18}\right) \times 18 - 250 = 260$ m.

17. Same as Q. 16.

Length of the second train

= Relative speed $\times$ time taken to cross each other - length of first train

$$= \left\{(40-22)\frac{5}{18}\right\} \times 60 - 125 = 300 - 125 = 175 \text{ m.}$$

Note : In Q. 16, both the trains are moving in opposite directions; hence relative speed = 62 + 40 = 102 km/hr. In Q. 17, both the trains are moving in the same directions; hence relative speed = 40 - 22 = 18 km/hr.

18. When trains are moving in opposite directions,
Speed of second train = Relative speed - speed of first train

$$= \left\{\frac{120+100}{6} \times \frac{18}{5}\right\} - 50 = 132 - 50 = 82 \text{ km/hr.}$$

19. Till 8 a.m., the train from A covers a distance of 20 km.
Now, the remaining distance 110 - 20 = 90 km is covered by the trains with relative speed = 20 + 25 = 45 km/hr

$\therefore$ they meet after $= \frac{90}{45} = 2$ hrs.

That is, at 8 + 2 = 10 a.m.

Quicker Maths (Direct formula) :

They will meet at 8 a.m. + $\frac{110-(8\text{a.m.}-7\text{ a.m.})20}{20+25}$

= 8 a.m. + 2 hr = 10 a.m.

20. c; Let the length of train = x m.
We know that, when train & man are moving in the same direction relative speed = Speed of Train - Speed of Man

$\therefore$ Speed of train = Relative speed + Speed of man

Now,

Speed of train in two cases $= \frac{x}{9} + 2\left(\frac{5}{18}\right) = \frac{x}{10} + 4\left(\frac{5}{18}\right)$

or, $\frac{x}{9} - \frac{x}{10} = \frac{10}{9} - \frac{5}{9}$ or, $\frac{x}{90} = \frac{5}{9}$ $\therefore x = \frac{5}{9} \times 90 = 50$ m.

Quicker Maths (Direct formula) : (See Ex 19)

When all are moving in the same direction,
Length of train

$$= \frac{\text{Relative speed of two men} \times \text{Product of times to pass them}}{\text{Difference of times to pass them}}$$

$$= \frac{(4-2)\frac{5}{18} \times 9 \times 10}{10-9} = 50 \text{ m.}$$

Streams

Introduction: Normally by speed of the boat or swimmer we mean the speed of the boat (or swimmer) in still water. If the boat (or the swimmer) moves against the stream then it is called **upstream** and if it moves with the stream, it is called **downstream.**

If the speed of the boat (or the swimmer) is x and if the speed of the stream is y then, while upstream the effective speed of the boat = x - y and while downstream the effective speed of the boat = x + y.

Theorem: *If x km per hour be the man's rate in still water, and y km per hour the rate of the current. Then*

x+y = man's rate with current

x-y = man's rate against current.

Adding and subtracting and then dividing by 2.

$x = \frac{1}{2}$ *(man's rate with current + his rate against current)*

$y = \frac{1}{2}$ *(man's rate with current - his rate against current)*

Hence we have the following two facts :

(i) A man's rate in still water is half the sum of his rates with and against the current.

(ii) The rate of the current is half the difference between the rates of the man with and against the current.

Ex 1: A man can row upstream at 10km/hr and downstream at 16km/hr. Find the man's rate in still water and the rate of the current.

Soln : Rate in still water $= \frac{1}{2}(10+16) = 13$ km/hr

Rate of current $= \frac{1}{2}(16-10) = 3$ km/hr.

Ex 2: A man swims downstream 30km and upstream 18km, taking 3 hrs each time. What is the velocity of current?

Soln: Man's rate downstream $= \frac{30}{3}$ km/hr = 10 km/hr

Man's rate upstream $= \frac{18}{3}$ km/hr = 6 km/hr

$\therefore$ Velocity of stream $= \frac{(10-6)}{2} = 2$ km/hr

Ex 3: A man can row 6 km/hr in still water. It takes him twice as long to row up as to row down the river. Find the rate of the stream.

Soln . Method I :

Let man's rate upstream = x km/hr

Then, man's rate downstream = 2x km/hr

$\therefore$ Man's rate in still water $= \frac{1}{2}(x + 2x)$ km/hr

$\therefore \frac{3x}{2} = 6$ or x = 4 km/hr

Thus, man's rate upstream = 4 km/hr

Man's rate downstream = 8 km/hr

$\therefore$ rate of stream $= \frac{1}{2}(8 - 4) = 2$ km/hr

Method II :

We have,

up rate + down rate = 2 × rate in still water

$= 2 \times 6 = 12$ km/hr

Also, up rate : down rate = 1 : 2

So, dividing 12 in the ratio of 1 : 2, we get

up rate = 4 km/hr

down rate = 8 km/hr

$\therefore$ rate of stream $= \frac{8-4}{2} = 2$ km/hr

Method III (Shortest Method):

Let rate of stream = x km/h

Then, 6 + x = 2 (6 - x)

or, 3x = 6

$\therefore x = \frac{6}{3} = 2$ km/h

Theorem: *A man can row x km/hr in still waters. If in a stream which is flowing at y km/hr, it takes him z hrs to row to a place and back, the distance between the two places is* $\frac{z(x^2 - y^2)}{2x}$

Proof: Man's speed upstream = (x - y) km/hr

Man's speed downstream = (x + y) km/hr

Let the required distance be 'A' km then

$$\frac{A}{(x-y)} + \frac{A}{(x+y)} = z$$

$$\text{or, } \frac{A[x+y+x-y]}{(x-y)(x+y)} = z$$

$$\text{or, } \frac{2Ax}{x^2 - y^2} = z$$

$$\text{or, } A = \frac{z(x^2 - y^2)}{2x}$$

$\therefore$ The required distance $= \frac{z(x^2 - y^2)}{2x}$

Ex 4 : A man can row 6 km/hr in still water. When the river is running at 1.2 km/hr, it takes him 1 hour to row to a place and back. How far is the place ?

Soln : Man's rate downstream = (6 + 1.2) km/hr = 7.2 km/hr

Man's rate upstream = (6 - 1.2) km/hr = 4.8 km/hr

Let the required distance be x km. Then

$\frac{x}{7.2} + \frac{x}{4.8} = 1$ or $4.8x + 7.2x = 7.2 \times 4.8$

$\Rightarrow x = \frac{7.2 \times 4.8}{12} = 2.88$ km.

By Direct Formula : Required distance $= \frac{1 \times [6^2 - (1.2)^2]}{2 \times 6}$

$= \frac{36 - 1.44}{12} = 3 - 0.12 = 2.88$ km.

Ex 5 : A man can row 7 km/hr in still water. In a stream which is flowing at 3 km/hr, it takes him 7 hrs to row to a place and back. How far is the place ?

Soln : By the formula, distance $= 7 \times \frac{(7)^2 - (3)^2}{2 \times 7} = 20$ km

Ex 6: In a stream running at 2 km/hr, a motorboat goes 10 km upstream and back again to the starting point in 55 minutes. Find the speed of the motorboat in still water.

Soln : Using the above formula, we have,

$$10 = \frac{55}{60} \times \frac{(x)^2 - (2)^2}{2x}$$

or, $1200x = 55(x^2 - 4)$

or, $11x^2 - 240x - 44 = 0 \quad \therefore (x-22)(11x+2) = 0$

So, x = 22 km/hr (neglecting the -ve value)

Ex 7 : A man can row 30 km upstream and 44 km downstream in 10 hrs. Also, he can row 40 km upstream and 55 km downstream in 13 hrs. Find the rate of the current and the speed of the man in still water.

Soln : Let, upstream rate = x km/hr and downstream rate = y km/hr

Then, $\frac{30}{x} + \frac{44}{y} = 10$ and $\frac{40}{x} + \frac{55}{y} = 13$

or, $30u + 44v = 10$

$40u + 55v = 13$

Where $u = \frac{1}{x}$ and $v = \frac{1}{y}$

Solving, we get $u = \frac{1}{5}$ and $v = \frac{1}{11}$

$\therefore x = 5$ and $y = 11$

$\therefore$ rate in still water $= \frac{5+11}{2} = 8$ km/hr

Rate of current $= \frac{11-5}{2} = 3$ km/hr

Quicker Method : (By use of multiple cross-multiplication)

Arrange the given figures in the following form :

Upstream	Downstream	Time
30	44	10
40	55	13

Upstream speed of man $= \frac{30 \times 55 - 40 \times 44}{55 \times 10 - 44 \times 13} = \frac{-110}{-22} = 5$ km/hr

Downstream speed of man

$= \frac{30 \times 55 - 40 \times 44}{30 \times 13 - 40 \times 10} = \frac{-110}{-10} = 11$ km/hr

$\therefore$ Speed of man $= \frac{5+11}{2} = 8$ km/hr.

and speed of stream $= \frac{11-5}{2} = 3$ km/hr.

Note : How do the denominators of the above two formulae differ? For upstream speed we use the figures of downstream speed and time and for downstream speed we use the figures of upstream speed and time.

Numerators remain the same in both formulae.

Ex. 8 : A boat covers 24 km upstream and 36 km downstream in 6 hours, while it covers 36 km upstream and 24 km downstream in 6.5 hrs. Find the velocity of the current.

Soln : By Quicker Method used in Ex. 7;

US	DS	Time
24	36	6
36	24	6.5

Upstream speed of boat $= \frac{24 \times 24 - 36 \times 36}{24 \times 6 - 36 \times 6.5} = \frac{-720}{-90} = 8$ km/hr

Downstream speed of boat

$= \frac{24 \times 24 - 36 \times 36}{24 \times 6.5 - 36 \times 6} = \frac{-720}{-60} = 12$ km/hr.

$\therefore$ Speed of current $= \frac{12-8}{2} = 2$ km/hr.

Theorem : *A man rows a certain distance downstream in x hours and returns the same distance in y hrs. If the stream flows at the rate of z km/hr then the speed of the man in still water is given by* $\frac{z(x+y)}{y-x}$ *km/hr.*

Proof : Let the speed of the man in still water be 'm' km/hr

then his upstream speed = (m - z) km/hr.

and downstream speed = (m + z) km/hr.

Now, we are given that up and down journey are equal, therefore

$x(m + z) = y(m - z)$

or, $m(y - x) = z(x + y)$

$\therefore m = \frac{z(x+y)}{y-x}$ km/hr.

Ex. 9 : Ramesh can row a certain distance downstream in 6 hours and return the same distance in 9 hours. If the stream flows at the rate of 3 km per hour find the speed of Ramesh in still water.

Soln : By the above formula :

Ramesh's speed in still water $= \frac{3(9+6)}{9-6} = 15$ km/hr.

Note : Cor. (to above theorem) : If in the above case speed of man in still water is z km/hr and we are asked to find the speed of stream, then our formula is: $\frac{z(y-x)}{x+y}$ km/hr.

Ex. 10 : If in the Ex 9 given above the speed of Ramesh in still water 12 km/hr, find the speed of the stream.

Soln : By the formula given in **(Note)** :

$$\text{Speed of stream} = \frac{12(9-6)}{9+6} = 2.4 \text{ km/hr.}$$

EXERCISES

1. If a man's rate with the current is 12 km/hr and the rate of the curr is 1.5 km/hr, then the man's rate against the current is ______ km/hr.
2. A boat goes 40 km upstream in 8 hours and 36 km downstream in hours. The speed of the boat in still water is ______ km/hr.
3. A boat travels upstream from B to A and downstream from A to B 3 hours. If the speed of the boat in still water is 9 km/hr and the speed o the current is 3 km/hr, the distance between A and B is ______ km.
4. A man can row at 5 km/hr in still water and the velocity of the curr is 1 km/hr. It takes him 1 hour to row to a place and back. How far is the place?
5. The speed of a boat in still water is 6 km/hr and the speed of the strea is 1.5 km/hr. A man rows to a place at a distance of 22.5 km and comes back to the starting point. Find the total time taken by him.
6. A man rows upstream 16 km and downstream 28 km, taking 5 hour each time. The velocity of the current is ______ km/hr.
7. A boat moves upstream at the rate of 1 km in 10 minutes and downstream at the rate of 1 km in 6 minutes. The speed of the current is ______ km/hr.
8. A can row a certain distance down a stream in 6 hours and return the same distance in 9 hours. If the stream flows at the rate of $2\frac{1}{4}$ km per hour, find how far he can row in an hour in still water.
9. The current of a stream runs at the rate of 4 km an hour. A boat goes 6 km and back to the starting point in 2 hours. The speed of the boat in still water is ______ km/hr.
10. A boat covers 24 km upstream and 36 km downstream in 6 hours, while it covers 36 km upstream and 24 km downstream in $6\frac{1}{2}$ hours. The velocity of the current is ______.

    a) 1.5 km/hr　　b) 1 km/hr

    c) 2 km/hr　　d) 2.5 km/hr

11. The current of a stream runs at 1 km/hr. A motorboat goes 35 km upstream and back again to the starting point in 12 hours. The speed of the motorboat in still water is ______ km/hr.
12. A man can row $9\frac{1}{3}$ km/hr in still water and he finds that it takes him thrice as much time to row up than as to row down the same distance in river. The speed of the current is ______ km/hr.
13. A man can row three quarters of a kilometre against the stream in $11\frac{1}{4}$ minutes and return in $7\frac{1}{2}$ minutes. The speed of the man in still water is ______ km/hr.

ANSWERS

1. Man's rate with the current = 12 km/hr.

   Man's rate in still water = 12 - 1.5 = 10.5 km/hr

   Man's rate against current = 10.5 - 1.5 = 9 km/hr

Quicker Method (Direct Formula) :

Man's rate against current = Man's rate with current - 2 × rate of current = 12 - 2 × 1.5 = 9 km/hr

2. Boat's upstream speed $= \frac{40}{8} = 5$ km/hr

   Boat's downstream speed $= \frac{36}{6} = 6$ km/hr.

   $\therefore$ Speed of boat in still water $= \frac{5+6}{2} = 5.5$ km/hr.

3. Let the distance be x km.

   Now, upstream speed = 9 - 3 = 6 km/hr.

   and downstream speed = 9 + 3 = 12 km/hr.

   Total time taken in upstream and downstream journey

$$= \frac{x}{6} + \frac{x}{12} = 3 \quad \text{or, } \frac{18x}{72} = 3 \qquad \therefore x = \frac{3 \times 72}{18} = 12 \text{ km.}$$

Quicker Maths (Direct formula) :

Distance

$$= \frac{\text{Total time} \times \{(\text{speed in still water})^2 - (\text{speed of current})^2\}}{2 \times \text{speed in still water}}$$

$$= \frac{3 \times \{(9)^2 - (3)^2\}}{2 \times 9} = \frac{3 \times 72}{18} = 12 \text{ km.}$$

4. Same as Q. 3.

$$\text{Distance} = \frac{1 \times \{(5)^2 - (1)^2\}}{2 \times 5} = \frac{24}{10} = 2.4 \text{ km.}$$

Note : Try to solve by detail method.

5. The Quicker formula given in Q.3 can be written in the form:

$$\text{Total time} = \frac{2 \times \text{Distance} \times \text{Speed in still water}}{(\text{Speed in still water})^2 - (\text{Speed in current})^2}$$

$$= \frac{2 \times 22.5 \times 6}{(6)^2 - (1.5)^2} = 8 \text{ hrs.}$$

Note: For detail method:

Boat's upstream speed = 6 - 1.5 = 4.5 km/hr

Boat's downstream speed = 6 + 1.5 = 7.5 km/hr

$$\therefore \text{ Total time} = \frac{22.5}{4.5} + \frac{22.5}{7.5}$$

$$= 5 + 3 = 8 \text{ hrs}$$

6. Upstream speed $= \frac{16}{5}$ km/hr

Downstream speed $= \frac{28}{5}$ km/hr

$$\therefore \text{ Velocity of current} = \frac{\frac{28}{5} - \frac{16}{5}}{2} = \frac{12}{10} = 1.2 \text{ km/hr}$$

7. Same as Q. 6.

8. Let the speed of A in still water = x km/hr

Then, downstream speed $= \left(x + \frac{9}{4}\right)$ km/hr

and upstream speed $= \left(x - \frac{9}{4}\right)$ km/hr

Now, distance $= 6\left(x + \frac{9}{4}\right) = 9\left(x - \frac{9}{4}\right)$

or, $6x + \frac{27}{2} = 9x - \frac{81}{4}$

or, $3x = \frac{135}{4}$

$\therefore\ x = \frac{135}{4 \times 3} = \frac{45}{4} = 11\frac{1}{4}$ km/hr

Quicker Math (Direct formula):

Speed in still water

$$= \frac{\text{Rate of Stream (Sum of upstream and downstream time)}}{\text{Difference of upstream \& downstream time}}$$

$$\frac{\frac{9}{4}(6+9)}{9-6} = \frac{9 \times 15}{4 \times 3} = \frac{45}{4} = 11\frac{1}{4} \text{km/hr}$$

9. Let the speed of boat in still water = x km/hr

Speed of current = 4 km/hr.

Speed of upstream = (x - 4) km/hr

Speed downstream = (x + 4) km/hr

Now, $\frac{6}{x-4} + \frac{6}{x+4} = 2$

or, $\frac{6(x+4) + 6(x-4)}{x^2 - 16} = 2$

or, $2x^2 - 32 = 6x + 6x$

or, $x^2 - 6x - 16 = 0$

or, $(x - 8)(x + 2) = 0$

$\therefore\ x = 8$ or -2

We reject the negative value.

$\therefore$ speed of boat in still water = 8 km/hr

Direct formula: If we put the values in the formula used in Q. 3. we have,

$6 = \frac{2\{x^2 - 4^2\}}{2x}$

or, $6x = x^2 - 16$

$\therefore\ x^2 - 6x - 16 = 0$

$\therefore\ x = 8$, or -2

That is, x = 8 km/hr

10. See Ex 8.

11. Same as Q. 9. By Quicker formula (used in Q. 3)

$35 = \frac{12(x^2 - 1^2)}{2x}$

or, $35x = 6x^2 - 6$

or, $6x^2 - 35x - 6 = 0$

or, $6x^2 - 36x + x - 6 = 0$

or, $6x(x-6)+(x-6)=0$

or, $(x-6)(6x+1)=0$

$\therefore\ x=6$ or $-\frac{1}{6}$

We neglect the -ve value. Therefore, speed of boat in still wa
$=6$ km/hr.

12. Let the speed of current = x km/hr

Then, $\frac{28}{3}+x = 3\left(\frac{28}{3}-x\right)$

or, $4x=\frac{2\times 28}{3}$

$\therefore\ x=\frac{14}{3} = 4\frac{2}{3}$ km/hr.

13. Upstream speed $=\frac{3}{4}\div\frac{45}{4\times 60}$

$=\frac{3}{4}\times\frac{4\times 60}{45}=4$ km/hr.

Downstream speed $=\frac{3}{4}\div\frac{15}{2\times 60} = \frac{3\times 2\times 60}{4\times 15}=6$ km/hr

$\therefore$ Speed in still water $=\frac{4+6}{2}=5$ km/hr.

Elementary Mensuration - I

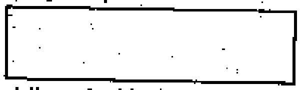
(MEASUREMENT OF AREAS)

In mensuration we often have to deal with the problem of finding the areas of plane figures. In this chapter we shall look at some of such problems and study some short-cut methods to solve such problems.

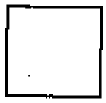
But before that, let us begin by having a look at some elementary definitions.

Some elementary definitions

1. **Rectangle :-** A quadrilateral with opposite sides equal and all the four angles equal to 90°.

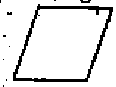


2. **Square :-** A quadrilateral with all sides equal and all the four angles equal to 90°.

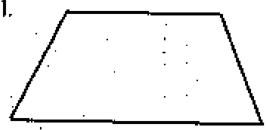


3. **Parallelogram** : A quadrilateral with opposite sides parallel and equal.

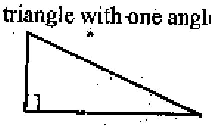
4. **Rhombus :-** A parallelogram with all four sides equal.



5. **Trapezium :-** A quadrilateral with any one pair of opposite sides parallel.

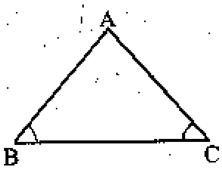


6. **Right-angled triangle :-** A triangle with one angle equal to 90°.



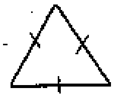
7. Isoceles triangle :- A triangle with any two sides equal.

Cor :- In an isoccles triangle, opposite angles are equal.

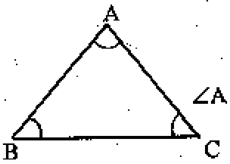


$\angle B = \angle C$

8. Equilateral triangle :- A triangle with all sides equal.

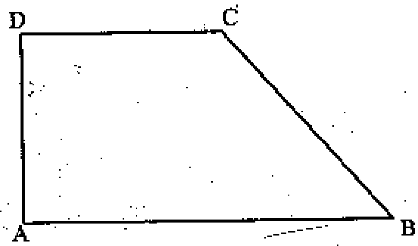


Cor :- In an equilateral triangle, all angles are equal, each being equal to 60°.



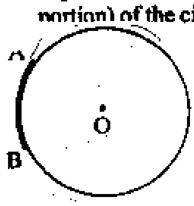
$\angle A = \angle B = \angle C = 60°$

9. Perimeter (of a geometrical figure) :- The length of the outer boundary of the geometrical figure.



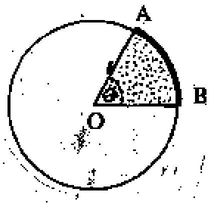
Perimeter of ABCD = AB + BC + CD + DA

10. Arc of a circle :- A portion of the perimeter (or a part of the curved portion) of the circle.



arc AB = length of AB

11. Sector of a circle :- The area covered between an arc, the centre and two radii of the circle.



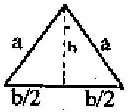
Shaded Portion = Sector AOB

List of important formulae

1. (i) Area of a rectangle = Length × breadth
   (ii) Length $= \frac{\text{Area}}{\text{Breadth}}$; Breadth $= \frac{\text{Area}}{\text{Length}}$
   (iii) $(\text{Diagonal})^2 = (\text{Length})^2 + (\text{Breadth})^2$
   (iv) Perimeter = 2(Length + Breadth)
2. (i) Area of a square $= (\text{side})^2 = \frac{1}{2}(\text{Diagonal})^2$.
   (ii) Perimeter of a square = 4× Side
3. Area of 4 walls of a room = 2 × (Length + Breadth) × Height.
4. Area of a parallelogram = (Base × Height).
5. Area of a rhombus $= \frac{1}{2} \times$ (product of diagonals).
   When d_1 and d_2 are the two diagonals then side of rhombus $= \frac{1}{2}\sqrt{d_1^2 + d_2^2}$
6. (i) Area of an equilateral triangle $= \frac{\sqrt{3}}{4} \times (\text{side})^2$.

(ii) Perimeter of an equilateral triangle = 3 × side

7. Area of an isoceles triangle $= \frac{b}{4}\sqrt{4a^2 - b^2}$



$h = \sqrt{a^2 - (b/2)^2} = \frac{1}{2}\sqrt{4a^2 - b^2}$

8. If a, b, c are the lengths of the sides of a triangle and

$s = \frac{1}{2}(a + b + c)$, then :

Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$.

9. Area of a triangle $= \frac{1}{2} \times$ Base × Height.

10. Area of a trapezium

$= \frac{1}{2}$(sum of parallel sides × perpendicular distance between them)

$= \frac{1}{2}(a + b)h$

(Where a and b are the parallel sides of the trapezium and h is the perpendicular distance between the sides a and b.

$h = \frac{2}{k}\sqrt{s(s-k)(s-c)(s-d)}$

(where k = (a - b) i.e. the difference between the parallel sides and c and d are the two non-parallel sides of the trapezium. Also,

$s = \frac{k + c + d}{2}$

$\therefore$ Area $= \frac{1}{2}(a + b)h = \frac{a + b}{k}\sqrt{s(s-k)(s-c)(s-d)}$

11. (i) Circumference of a circle = $2\pi r$.

(ii) Area of a circle = πr^2.

(iii) arc AB $= \frac{2\pi r\theta}{360°}$, where $\angle AOB = \theta$ and O is the centre.

(iv) Area of sector AOB $= \frac{\pi r^2 \theta}{360°}$ (See figure in previous page).

(v) Area of sector AOB $= \frac{1}{2} \times$ arc AB × r.

12. In a Parallelogram

Area $= 2\sqrt{s(s-a)(s-b)(s-d)}$

Where a and b are the two adjacent sides and d is the diagonal connecting the ends of the two sides.

Problems on Parallelogram

Ex. 1: The two adjacent sides of a parallelogram are 5 cm and 4 cm respectively, and if the respective diagonal is 7 cm then find the area of the parallelogram?

Sol: Required area $= 2\sqrt{s(s-a)(s-b)(s-D)}$

$\because$ Where $S = \frac{a + b + D}{2} = \frac{5 + 4 + 7}{2} = 8$

$= 2\sqrt{8(8-5)(8-4)(8-7)}$

$= 2\sqrt{8 \times 3 \times 4} = 8\sqrt{6} = 19.6$ sqcm.

Ex. 2: In a parallelogram, the lengths of adjacent sides are 12 cm and 14 cm respectively. If the length of one diagonal is 16 cm, find the length of the other diagonal.

Sol: In a parallelogram, the sum of the squares of the diagonals = 2 × (the sum of the squares of the two adjacent sides)

or, $D_1^2 + D_2^2 = 2(a^2 + b^2)$

or, $16^2 + x^2 = 2(12^2 + 14^2)$

or, $256 + x^2 = 2(144 + 196)$

or, $x^2 = 680 - 256 = 424$

$\therefore x = \sqrt{424} = 20.6$ cm

Problems on Trapezium (Trapezoid)

Ex. 1: In a trapezium, parallel sides are 60 and 90 cms respectively and non-parallel sides are 40 and 50 cms respectively. Find its area.

Sol: k = difference between the parallel sides = 90 - 60 = 30 cm

Let c be 40 cm then d = 50 cm

Now, $s = \frac{k + c + d}{2} = \frac{30 + 40 + 50}{2} = \frac{120}{2} = 60$ cm

$\therefore$ Area $= \frac{a + b}{k}\sqrt{s(s-k)(s-c)(s-d)}$

$= \frac{60 + 90}{30}\sqrt{60(60-30)(60-40)(60-50)}$

$= 5\sqrt{60 \times 30 \times 20 \times 10} = 5 \times 600 = 3000$ sq. cm.

Ex. 2: In the above question find the perpendicular distance between the two parallel sides of the trapezium.

Sol: $h = \frac{2}{k}\sqrt{s(s-k)(s-c)(s-d)}$

$= \frac{2}{30} \times \sqrt{60 \times 30 \times 20 \times 10} = \frac{1}{15} \times 600 = 40$ cm

Note: We can verify that Area $= \frac{1}{2}(a+b)h = \frac{1}{2} \times (60+90) \times 40$

$= 150 \times 20 = 3000$ sq. cm.

Ex. 3: A 5100 sq cm trapezium has the perpendicular distance between the two parallel sides 60 m. If one of the parallel sides be 40 m then find the length of the other parallel side.

Sol: $A = \frac{1}{2}(a+b)h$

or, $5100 = \frac{1}{2}(40+x) \times 60$

or, $170 = 40 + x$

$\therefore$ required other parallel side = 170 - 40 = 130 m.

Problems on Rectangles and Squares

Type I : Simple questions requiring direct application of formulae

Ex. 1. Calculate the area of a rectangle 23 metres 7 decimetres long and 14 metres 4 decimetres 8 centimetres wide.

Sol : Length = 23.70 metres.

Breadth = 14.48 metres.

$\therefore$ area = (23.70 × 14.48) square metres

= 343.18 square metres. Ans.

Ex. 2. Find the diagonal of a rectangle whose sides are 12 metres and 5 metres.

Sol : The length of the diagonal

$= \sqrt{12^2 + 5^2}$ metres $= \sqrt{169}$ metres

= 13 metres. Ans.

Type II : Carpeting a floor

Ex. 3. How many metres of a carpet 75 cm wide will be required to cover the floor of a room which is 20 metres long and 12 metres broad ?

Soln: Length required $= \frac{\text{Length of room} \times \text{breadth of room}}{\text{Width of carpet}}$

$\therefore$ length required $= \frac{20 \times 12}{0.75} = 320$m.

Cor: What amount needs to be spent in carpeting the floor if the carpet is available at Rs 20/- per metre?

Quicker method : Amount required =

Rate per metre $\times \frac{\text{length of room} \times \text{breadth of room}}{\text{width of carpet}}$

$= 20 \times \frac{20 \times 12}{0.75} =$ Rs. 6400

Type III. Paving a courtyard with tiles

Ex 4: How many paving stones each measuring 2.5m × 2 m are required to pave a rectangular courtyard 30 m long and 16.5 m wide ?

Soln : Number of tiles required $= \frac{\text{length} \times \text{breadth of courtyard}}{\text{length} \times \text{breadth of each tile}}$

$= \frac{30 \times 16.5}{2.5 \times 2} = 99$

Cor : What amount needs to be spent if the tiles of the aforesaid dimension are available at Re. 1 per piece ?

Quick method :

Amount required =

price per tile $\times \frac{\text{length} \times \text{breadth of courtyard}}{\text{length} \times \text{breadth of each tile}}$

$= 1 \times \frac{30 \times 16.5}{2.5 \times 2} =$ Rs. 99

Type IV. Paving with square tiles : largest tile

Ex 5: A hall-room 39 m 10 cm long and 35 m 70 cm broad is to be paved with equal square tiles. Find the largest tile so that the tiles exactly fit and also find the number of tiles required.

Soln : Quicker Method :

Side of largest possible tile = H.C.F. of length and breadth of the room **(Remember)**

= H.C.F. of 39.10 and 35.70m

= 1.70m.

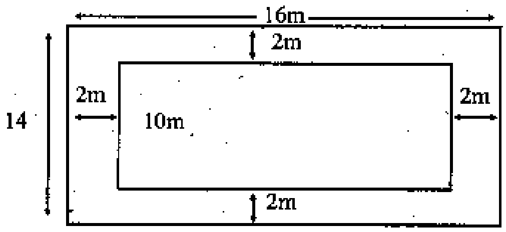
Also number of tiles required

$= \frac{\text{Length} \times \text{breadth of room}}{(\text{H.C.F. of length and breadth of the room})^2}$

$= \frac{39.10 \times 35.70}{1.70 \times 1.70} = 483.$

Type V : Path round a garden, verandah round a room

Ex 6. A rectangular hall 12 m long and 10 m broad, is surrounded by a verandah 2 metres wide. Find the area of the verandah.



Soln : Quicker Method :

In such cases,

(I) *When the verandah is outside the room, surrounding it*

Area of verandah = 2 (width of verandah)× [length + breadth of room + 2 (width of verandah)] **(Remember)**

(II) *When the path is within the garden, surrounded by it*

Area of path = 2 (width of path)× [length + breadth of garden - 2 (width of path)]

(Remember)

Now in the given question, by formula I, (since the verandah is outside the room, formula I will be applied)

area of verandah = $2 \times 2 \times (10 + 12 + 2 \times 2)$

$= 4 \times 26 = 104\ m^2$

Ex 7: A rectangular grassy plot is 112 m by 78 m. It has a gravel path 2.5 m wide all round it on the inside. Find the area of the path and the cost of constructing it at Rs. 2 per square metre ?

Soln : By quicker math formula II (since the path is inside the plot, formula II will be applied), area of the path

$= 2 \times 2.5 \times (112 + 78 - 2 \times 2.5)$

$= 5 \times 185 = 925$ sq. m.

$\therefore$ cost of construction = rate × area

$= 2 \times 925 =$ Rs. 1850

Some more cases on paths

A. When area of the path is given, to find the area of the garden enclosed (the garden is square in shape)

Ex 8: A path 2 m wide running all round a square garden has an area of 9680 sq. m. Find the area of the part of the garden enclosed by the path.

oln (Quicker Method) :

Area of the square garden =

$$\left[\frac{\text{Area of path} - 4 \times (\text{width of path})^2}{4 \times \text{width of path}}\right]^2$$

(Remember)

$\therefore$ Here in the given question,

$$\text{area of garden} = \left[\frac{9680 - 4 \times (2)^2}{4 \times 2}\right]^2$$

$$= \left[\frac{9664}{8}\right]^2 = (1208)^2$$

$= 1459264$ sq. m.

B. When area of the path be given, to find the width of the path

Ex 9: A path all around the inside of a rectangular park 37 m by 30 m occupies 570 sq. m. Find the width of the path.

oln : By quicker math (see formula II, Ex 6)

Area of path = 2 × width of path × [length + breadth of park - 2 × (width of path)]

$\Rightarrow 570 = 2 \times x \times [37 + 30 - 2x]$ (x is the width of path)

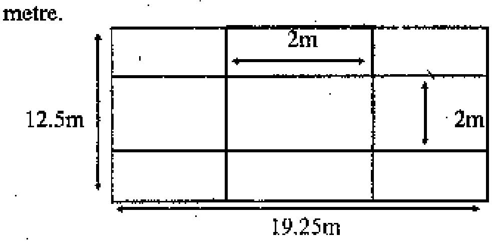
$\Rightarrow 570 = 134x - 4x^2$

$\Rightarrow 4x^2 - 134x + 570 = 0$

On solving this equation we get, x = 5m.

C. Paths crossing each other (important)

Ex 10: An oblong piece of ground measures 19 m 2.5 dm by 12 metres 5 dm. From the centre of each side a path 2 m wide goes across to the centre of the opposite side. What is the area of the path ? Find the cost of paving these paths at the rate of Rs. 1.32 per sq. metre.



Soln : Quicker method :

In such problems, use the formula given below :

I. **Area of the path** = (Width of path) (length + breadth of pa
- width of path) **(Remembe**

II. **Area of the park minus the path** =
(length of park - width of path) × (breadth of park- width of pa
(Remembe

Now, for the given question,

area of path = 2 × (19.25 + 12.5 - 2)

= 2 × 29.75 = 59.5 sq. m.

∴ cost = rate × area = Rs (59.5 × 1.32) = Rs. 78.54.

Type VI: Area and ratio

Ex 11: The sides of a rectangular field of 726 sq. m. are in the ratio o
3:2. Find the sides.

Soln : Quicker method :

Side = $\sqrt{\text{Area} \times \text{Ratio}}$

2nd side = $\sqrt{\text{Area} \times \text{Inverse Ratio}}$ **(Remembe**

∴ In the given question,

first side = $\sqrt{726 \times \frac{3}{2}} = \sqrt{1089} = 33$ m.

and second side = $\sqrt{726 \times \frac{2}{3}} = \sqrt{484} = 22$ m.

Type VII : Some Miscellaneous Cases

Turkey carpet and oilcloth

Ex 12 : In the centre of a room 10 metres square, there is a square of
turkey carpet, and the rest of the floor is covered with oilclot
The carpet and the oilcloth cost respectively Rs 15 and Rs 6.5
per square metre, and the total cost of the carpet and the oilclo
is Rs 1338.50 . Find the width of the oilcloth border.

Sol : The area of the square room = 100 sq. metres.

The mean cost per sq. metre

$= \frac{\text{Rs}1338.50}{100}$ = Rs 13.385

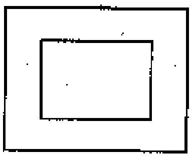
carpet oilcloth

15 6.50

13.385

6.885 1.615

= 81 : 19



y the Alligation Rule, the area of the square carpet is 81 sq. metres.
Therefore, the carpet is 9 metres in length and breadth.

But the room is 10 metres in length and breadth.

Hence double the width of the border is (10 - 9) or 1 metre.

∴ the width of the border = $\frac{1}{2}$ metre = 5 dm. Ans.

Diagonal

Ex 13: A square field of 2 sq. kilometres is to be divided into two equal parts by a fence which coincides with a diagonal. Find the length of the fence.

Soln : Area of square = 2 km^2

∴ Diagonal = $\sqrt{2 \times 2}$ km = 2 kilometres. Ans.

Ex 14: A square field of area 31684 square metres is to be enclosed with wire placed at heights 1, 2, 3, 4 metres above the ground. What length of the wire will be required, if its length required for each circuit is 5% greater than the perimeter of the field ?

Soln : Area of the field = 31684 sq. metres.

∴ perimeter = $\sqrt{31684} \times 4$metres

= 178 × 4 metres.

∴ length of each circuit = $178 \times 4 \times \frac{105}{100}$ metres.

Since the wire goes round 4 times,

∴ total length of wire required = $178 \times 4 \times \frac{105}{100} \times 4$ metres

= 2990.4 metres. Ans.

Problems on Triangles

Type I : Simple Application of Formulae

Ex 15: The base of a triangular field is 880 metres and its height 550 metres. Find the area of the field. Also calculate the charges for supplying water to the field at the rate of Rs 24.25 per sq. hectometre.

Soln : Area of the field = $\frac{\text{Base} \times \text{Height}}{2}$

$= \frac{880 \times 550}{2}$ sq. metres = $\frac{440 \times 550}{100 \times 100}$ sq. hectometres

= 24.20 sq. hectometres. Ans.

Cost of supplying water to 1 sq. hectometre

= Rs 24.25

$\therefore$ cost of supplying water to the whole field

$$= \text{Rs } 24.20 \times 24.25$$
$$= \text{Rs } 586.85 \text{ Ans.}$$

Ex 16 : The base of a triangular field is three times its height. If the c of cultivating the field at Rs 36.72 per hectare is Rs 495.72, fi its base and height.

Soln : Area of the field $= \dfrac{\text{Rs } 495.72}{\text{Rs}36.72}$ hectares

$$= \frac{27}{2} \text{ hectares.}$$

Also, area of the field $= \dfrac{1}{2} \times 3 \times \text{Height} \times \text{Height}$

$$= \frac{3}{2} (\text{Height})^2$$

$\therefore \dfrac{3}{2} (\text{Height})^2 = \dfrac{27}{2}$ hectares.

$\therefore (\text{Height})^2 = \dfrac{27}{2} \times \dfrac{2}{3}$ hectares = 9 hectares

= 90000 sq. metres.

Height $= \sqrt{90000}$ m = 300 m. Ans.

Also Base = 3 × Height = 900 m. Ans.

Ex 17 : Find the area of a triangle whose sides are 50 metres, 78 metres 112 metres respectively and also find the perpendicular from the opposite angle on the side 112 metres.

Soln : Here a = 50 metres, b = 78 metres, c = 112 metres.

$\therefore s = \dfrac{1}{2}$ (50 + 78 + 112) metres

$= \dfrac{1}{2} \times 240$ metres = 120 metres.

$\therefore$ s - a = (120 - 50) metres = 70 metres

s - b = (120 - 78) metres = 42 metres

s - c = (120 - 112) metres = 8 metres

$\therefore$ area $= \sqrt{120 \times 70 \times 42 \times 8}$ sq. metres

= 1680 sq. metres. Ans.

Perpendicular $= \dfrac{2 \text{ Area}}{\text{Base}} = \dfrac{1680 \times 2}{112}$ metres = 30 metres. Ans.

Type II: Quicker Methods for triangle problems

Ex (18-20) : Solve the previous three examples Ex (15-17) by quicker methods.

Soln:

Ex 18 : Since (Ex. 15) involves direct application of formula, a method quicker than the one employed can not be used.

Ex 19 : The ratio between base and height in Ex. 16 is 3 :1.

In such questions use the rule :

Base $= \sqrt{2 \times \text{Area} \times \text{Ratio}}$

Height $= \sqrt{2 \times \text{Area} \times \text{Inverse Ratio}}$

(Remember)

Now, ratio of base and height is 3:1. Hence, the ratio attached with base is 3, the ratio attached with height is 1.

$\therefore$ Base $= \sqrt{2 \times \dfrac{27}{2} \times \dfrac{3}{1}} = 900$ m.

Height $= \sqrt{2 \times \dfrac{27}{2} \times \dfrac{1}{3}} = 300$ m.

Ex 20 : Since Ex. 17 involves direct application of formula, no quicker method can be employed.

Problems on Parallelogram, Rhombus and Trapezium

Type I: Questions Requiring Direct Application of Formulae

Ex 21 : Find the surface of a piece of metal which is in the form of a parallelogram whose base is 10 cm and height is 6.4 cm.

Soln : Surface area = height × base = 6.4 × 10 = 64 sq. cm.

Ex 22 : Find the area of a rhombus one of whose diagonals measures 8 cm and the other 10 cm.

Soln : Area $= \dfrac{1}{2}$ product of diagonals

$$= \frac{8 \times 10}{2} = 40 \text{ sq. cm.}$$

Ex 23 : Find the distance between the two parallel sides of a trapezium if the area of the trapezium is 250 sq. m. and the two parallel sides are equal to 15 m and 10 m respectively.

Soln : We have

Area $= \dfrac{1}{2} \times$ height × (sum of parallel sides)

or, $250 = \frac{1}{2} \times \text{height} \times (15 + 10)$

or, $\text{height} = \frac{250 \times 2}{25}$

$\therefore$ height = 20 m.

Type II: Some Quicker Methods

A : To find the area of a rhombus with one side and one diagonal given

Ex 24 : Find the area of a rhombus one side of which measures 20 cm. and one diagonal 24 cm.

Soln : Quicker Method

$$\text{Area of a rhombus} = \text{diagonal} \times \sqrt{(\text{side})^2 - \left(\frac{\text{diagonal}}{2}\right)^2}$$ **(Remember)**

$\therefore$ In the given question,

$$\text{Area} = 24 \times \sqrt{(20)^2 - \left(\frac{24}{2}\right)^2} = 24 \times \sqrt{400 - 144}$$

$$= 24 \times 16 = 384 \text{ cm}^2$$

Ex 25: The perimeter of a rhombus is 146 cm and one of its diagonals is 55 cm. Find the other diagonal and the area of the rhombus.

Soln : (Quicker Method) :

$$\text{Other diagonal} = 2 \times \sqrt{(\text{side})^2 - \left(\frac{\text{diagonal}}{2}\right)^2}$$ **(Remember)**

Now, one side of a rhombus $= \frac{146}{4} = 36.5$ cm.

$\therefore$ other diagonal $= 2 \times \sqrt{(36.5)^2 - \left(\frac{55}{2}\right)^2} = 48$ cm.

Now, area $= \frac{1}{2}$ (product of diagonals)

$= \frac{1}{2} \times 48 \times 55 = 1320$ sq. cm.

Problems on Regular Polygons

A regular polygon is a polygon (triangle, quadrilateral, pentagon, hexagon, heptagon, octagon etc.) which has all sides equal.

The following formulae may prove useful :

A. *Area of a regular polygon* $= \frac{1}{2} \times$ *number of sides* $\times$ *radius of the inscribed circle.*

B. *Area of a hexagon* $= \frac{3\sqrt{3}}{2} \times (\text{side})^2$

C. *Area of an octagon* $= 2(\sqrt{2} + 1)(\text{side})^2$

Ex 26 : Find the area of a regular hexagon whose side measures 9 cm.

Soln : Area of a regular hexagon $= \frac{3\sqrt{3}a^2}{2}$.

Here $a = 9$ cm.

$\therefore$ area $= \frac{3\sqrt{3} \times 9^2}{2}$ aq. cm = 210.4 sq cm approx. Ans.

Ex 27 : Find to the nearest metre the side of a regular octagonal enclosure whose area is 1 hectare.

Soln : Area of a regular octagon $= 2(1+\sqrt{2})a^2$

Now, $2(1+\sqrt{2})a^2 = 1$ hectare.

$a^2 = \frac{10000}{2(1+\sqrt{2})}$ sq. m.

or, $a^2 = 2071$ sq m approx.

$\therefore$ $a = 46$ metres approx. Ans.

Problems on Rooms And Walls

Papering the walls and allowing for doors etc.

Ex 28 : A room 8 metres long, 6 metres broad and 3 metres high has two windows $1\frac{1}{2}$ m $\times$ 1 m and a door 2 m $\times$ $1\frac{1}{2}$ m. Find the cost of papering the walls with paper 50 cm wide at 25 P per metre.

Soln : Area of walls = 2(8+6)3 = 84 sq. m.

Area of two windows and door $= 2 \times 1\frac{1}{2} \times 1 + 2 \times 1\frac{1}{2} = 6$ sq. m.

Area to be covered = 84 - 6 = 78 sq. m.

$\therefore$ length of paper $= \frac{78 \times 100}{50}$ m. = 156 m.

$\therefore$ cost = Rs $\frac{156 \times 25}{100}$ = Rs 39. Ans.

Height of the room

Ex 29 : A room is 7 metres long and 5 metres broad; the doors and windows occupy 5 sq. metres, and the cost of papering the remaining part of the surface of the walls with paper 75 cm wide at Rs 4.20 per piece of 13 m is Rs 39.20. Find the height of the room.

Soln : Length of paper $= \frac{\text{Rs } 39.20}{\text{Rs } 4.20} \times 13 \text{ m} = \frac{364}{3}$ m.

Area of paper $= \frac{364}{3} \times \frac{75}{100} = 91$ sq. m.

Area of walls $= 91 + 5 = 96$ sq. m.

Now area of walls $= 2(7+5) \times$ height $= (24 \times \text{height})$ sq. m.

$\therefore 24 \times$ height $= 96$

$\therefore$ height $= \frac{96}{24} = 4$ metres. Ans.

Ex 30: A hall, whose length is 16 metres and breadth twice its height, takes 168 metres of paper 2 metres wide for its four walls. Find the area of the floor.

Soln : Let the breadth = 2h metres, then height = h metres.

Area of walls $= 2(16+2h)h$ sq. metres.

Area of paper $= 168 \times 2$ sq. metres.

$\therefore 2(16+2h)h = 168 \times 2 \quad \therefore (8+h)h = 84.$

On solving, h = 6, -14; -14 is not acceptable

$\therefore$ h = 6, and breadth = 12

$\therefore$ area of the floor $= 16 \times 12$ sq. metres $= 192$ sq. metres. Ans.

Lining a box with metal

Ex 31 : A closed box measures externally 9 dm long, 6 dm broad, $4\frac{1}{2}$ dm high, and is made of wood $2\frac{1}{2}$ cm thick. Find the cost of lining it on the inside with metal at 6 P per sq. m.

Soln : The internal dimensions are $8\frac{1}{2}$ dm, $5\frac{1}{2}$ dm, 4 dm.

Area of the 4 sides $= 2\left(8\frac{1}{2} + 5\frac{1}{2}\right) \times 4$ sq. dm. $= 112$ sq. dm.

Area of bottom and top $= 2 \times 8\frac{1}{2} \times 5\frac{1}{2}$ sq. dm. $= \frac{187}{2}$ sq. dm.

Total area to be lined $= \left(112 + \frac{187}{2}\right)$ sq. dm. $= \frac{411}{2}$ sq. dm.

$\therefore$ cost $= \frac{411}{2} \times 6$ P = Rs 12.33 . Ans.

Problems on Circles

The following formulae may be used for *quick* solutions :

(i) Area $= \pi\,(\text{radius})^2$ (ii) Radius $= \sqrt{\left(\frac{\text{Area}}{\pi}\right)}$

(iii) Diameter $= 2\sqrt{\left(\frac{\text{Area}}{\pi}\right)}$ (iv) Area $= \pi\left(\frac{\text{diameter}}{2}\right)^2$

(v) Perimeter $= 2\pi\,(\text{radius})$ (vi) Radius $= \left(\frac{\text{Perimeter}}{2\pi}\right)$

(vii) Perimeter $= \pi\,(\text{diameter})$ (viii) Diameter $= \left(\frac{\text{Perimeter}}{\pi}\right)$

(ix) Arc of a sector $= \left(\frac{\theta^o}{360^o}\right) \times$ circumference

(x) Area of a sector $= \left(\frac{\theta^o}{360^o}\right) \times \pi \times (\text{radius})^2$

Let us look at some examples now :

I. Simple Application of Formulae

Ex 32 : (a) Find the circumference of a circle whose radius is 42 metres.

(b) Find the radius of a circular field whose circumference measures $5\frac{1}{2}$ km. $\left(\text{Take } \pi = \frac{22}{7}\right)$

Soln : (a) $C = 2\pi r$

$\therefore$ required circumference $= 2 \times \frac{22}{7} \times 42$ metres

$= 264$ metres. Ans.

(b) $r = \frac{C}{2\pi}$

$\therefore$ reqd. radius $= \frac{\frac{11}{2} \times 1000}{2\pi}$ m

$= \frac{\frac{11}{2} \times 1000 \times 7}{2 \times 22}$ m = 875 metres. Ans.

Ex 33. The radius of a circular wheel is $1\frac{3}{4}$ m. How many revolutions will it make in travelling 11 km?

Soln : Distance to be travelled = 11 km = 11000 m.

Radius of the wheel = $1\frac{3}{4}$ m.

$\therefore$ circumference of the wheel $= 2 \times \frac{22}{7} \times 1\frac{3}{4}$ m $= 11$ m.

$\therefore$ in travelling 11 m the wheel makes 1 revolution.

$\therefore$ in travelling 11000 m the wheel makes $\frac{1}{11} \times 11000$ revolutions, i.e. 1000 revolutions. Ans.

Direct Formula :

$$\text{No. of revolutions} = \frac{\text{Distance}}{2\pi r} = \frac{11000}{2 \times \frac{22}{7} \times \frac{7}{4}} = 1000$$

II. Some Quicker Methods

A. Area of a ring :

Ex 34 : The circumference of a circular garden is 1012 m. Find the area. Outside the garden, a road of 3.5 m width runs round it. Calculate the area of this road and find the cost of gravelling it at the rate of 32 paise per sq. m.

Soln (Quicker Method) :

$$\text{Area} = \frac{(\text{circumference})^2}{4\pi} \qquad \textbf{(Remember)}$$

$$= \frac{(1012)^2}{4 \times \frac{22}{7}} = 81466 \text{ sq. m.}$$

Area of ring $= \pi$ [(width of ring) (2 × inner radius + width of ring)] **(Remember)**

$$\text{Now, inner radius} = \sqrt{\frac{\text{Area}}{\pi}} = \sqrt{\frac{81466 \times 7}{22}} = 161 \text{ m.}$$

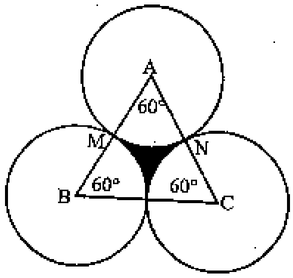
$$\therefore \text{area of ring-shaped road} = \frac{22}{7} \times 3.5 \times (3.5 + 2 \times 161)$$

$$= \frac{22}{7} \times 3.5 \times (3.5 + 322) = 3580.5 \text{ sq.m.}$$

$\therefore$ cost of gravelling = 3580.5 × 0.32 = 1145.76 rupees.

B. Identical Circles Placed Together

Ex 35 : There is an equilateral triangle of which each side is 2 m. With all the three corners as centres circles are described each of radius 1 m. (i) Calculate the area common to all the circles and the triangle. (ii) Find the area of the remaining portion of the triangle. (Take $\pi = 3.1416$)



Soln (Quicker Method) :

When the side of the equilateral triangle is double the radius of the circles, all circles touch each other and in such cases the following formula may be used :

$$\text{Area of each sector} = \frac{1}{6} \times \pi \times (\text{radius})^2$$

$$\text{Area of remaining (shaded) portion} = \left(\sqrt{3} - \frac{\pi}{2}\right)(\text{radius})^2$$

$$= (0.162)\,(\text{radius})^2$$

(Remember)

$\therefore$ In this given question, the area common to all circles and triangle

= sum of the areas of three sectors AMN, BML and CLN

$$= \frac{1}{6}\pi r^2 + \frac{1}{6}\pi r^2 + \frac{1}{6}\pi r^2 = \frac{1}{2}\pi r^2$$

$$= \frac{1}{2} \times \frac{22}{7} \times (1)^2 = 1.57 \text{ sq.m.}$$

(ii) The area of the remaining portion of the triangle

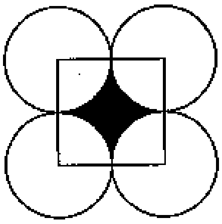
= The area of the shaded portion

$= 0.162 \times (1)^2 = 0.162$ sq. m.

Ex 36: The diameter of a coin is 1 cm. If four of these coins be placed

on a table so that the rim of each touches that of the other,
find the area of the unoccupied space between them.

(Take $\pi = 3.14$



Soln (Quicker Method):

Again, if the circles be placed in such a way that they to each other, then the square's side is double the radius. In such cases following formulae may be used :

Area of each sector $= \frac{1}{4} \times \pi \times (\text{radius})^2$

(Remember)

Area of remaining portion (shaded part) $= (4 - \pi)(\text{radius})^2$

$= (0.86)(\text{radius})^2$

(Remember)

Now, in the given question,

area of the unoccupied space $= (0.86)(\text{radius})^2$

$= (0.86)\left(\frac{1}{2}\right)^2 = 0.215$ sq. cm.

Miscellaneous Example :

Ex. 37 : The length of a rectangle is increased by 60%. By what per ce should the width be decreased to maintain the same area?

Soln : Let the length and breadth of the rectangle be x and y.

Then, its area = xy

New length $= x\left(\frac{160}{100}\right) = \frac{8x}{5}$

As the area remains the same, the new breadth of the rectangle

$= \frac{xy}{\frac{8x}{5}} = \frac{5y}{8}$

$\therefore$ decrease in breadth $= y - \frac{5y}{8} = \frac{3y}{8}$

$\therefore$ % decrease in breadth $= \frac{3y \times 100}{8 \times y} = \frac{75}{2} = 37\frac{1}{2}\%$

Quicker Method (Direct Formula) : You must have gone through similar examples in the chapter 'Percentage'. If you recall, you find the formula as

Required percentage decrease in breadth

$$= 60\left(\frac{100}{100 + 60}\right) = \frac{75}{2} = 37\frac{1}{2}\%$$

Ex. 38 : If the length of a rectangle is decreased by 20%, by what per cent should the width be increased to maintain the same area?

Soln : Quicker Method (Direct Formula) :

Required percentage increase in breadth $= 20\left(\frac{100}{100 - 20}\right) = 25\%$

Note : To find the above formulae, we have used the rule of fraction.

Theorem : *If length and breadth of a rectangle is increased by x and y per cent respectively, then area is increased by* $\left(x + y + \frac{xy}{100}\right)\%$.

Proof : See chapter "Percentage". You will find the proof for a similar case.

Note : If any of the two measuring sides of rectangle is decreased then put negative value for that in the given formula.

Ex. 39 : If the length and the breadth of a rectangle is increased by 5% and 4% respectively, then by what per cent does the area of that rectangle increase?

Soln : By direct formula :

% increase in area $= 5 + 4 + \frac{5 \times 4}{100} = 9 + 0.2 = 9.2\%$

Ex. 40 : If the length of a rectangle increases by 10% and the breadth of that rectangle decreases by 12%, then find the % change in area.

Soln : Since breadth decreases by y = -12, then

% change in area $= 10 - 12 + \frac{10 \times (-12)}{100}$

$= -2 - 1.2 = -3.2\%$

Since there is -ve sign, the area decreases by 3.2%.

Ex. 41 : If the length of a rectangle decreases by 4% and breadth is increased by 6% find the percentage change in area.

(Ans. 1.76% increase)

Soln : Try yourself.

Ex. 42 : If sides of a square are increased by 10%, then its area is increased by ____.

Soln : We can apply the above theorem here also by putting x = y = 10%

$\therefore$ % increase in area $= 10 + 10 + \frac{10 \times 10}{100} = 21\%$

Theorem : *If all the measuring sides of any two dimensional figure is changed by x%, then its area changes by* $\left(2x + \frac{x^2}{100}\right)\%$.

Proof : The above theorem is true for any two-dimensional figure such as heptagon, hexagon, pentagon, quadrilateral, triangle, circle, rhombus, parallelogram etc.

We will prove the theorem for triangle, rectangle and circle. Thereafter we generalise the theorem for all 2-dimensional figures.

For Triangle : Suppose the three sides of a triangle be a, b and c.

Then area of the triangle $= A = \sqrt{s(s-a)(s-b)(s-c)}$

where $s = \frac{a+b+c}{2}$

Now, when all the sides are increased by x%, the sides become

$\frac{a(100+x)}{100}, \frac{b(100+x)}{100}$ and $\frac{c(100+x)}{100}$

Now, $s_1 = \frac{100+x}{100}\left[\frac{a+b+c}{2}\right] = \frac{100+x}{100}s$

$\therefore$ New area, A_1

$$= \sqrt{s_1\left(s_1 - \frac{a(100+x)}{100}\right)\left(s_1 - \frac{b(100+x)}{100}\right)\left(s_1 - \frac{c(100+x)}{100}\right)}$$

$$= \sqrt{\left(\frac{100+x}{100}\right)^4 s(s-a)(s-b)(s-c)}$$

$$\therefore A_1 = \left(\frac{100+x}{100}\right)^2 A$$

Now, % increase in area $= \frac{A_1 - A}{A} \times 100$

$$= \frac{\left[\left(\frac{100+x}{100}\right)^2 - 1\right]A}{A} \times 100$$

$$= \left[\left(\frac{100+x}{100}\right)^2 - 1\right] \times 100$$

$$= \left[1 + \frac{x^2}{(100)^2} + \frac{2x}{(100)} - 1\right] \times 100 = \left[2x + \frac{x^2}{100}\right]$$

Thus, the theorem is true for triangle.

For Rectangle : Let the sides of a rectangle be a and b. Now when its sides are changed by x%, they become $\frac{a(100+x)}{100}$ and $\frac{b(100+x)}{100}$ respectively.

Now, new area $= A_1 = ab\left[\frac{100+x}{100}\right]^2 = \left[\frac{100+x}{100}\right]^2 A$

$\therefore$ % increase in area $= \frac{A_1 - A}{A} \times 100$

$$= \left[\left(\frac{100+x}{100}\right)^2 - 1\right] \times 100$$

$$= \left[2x + \frac{x^2}{100}\right]$$

Thus, the theorem is also true for rectangle.

For Circle : The circle has two measuring sides which are the same and the side is known as its radius (since, area $= \pi r^2$, r is used twice).

Let the radius of the circle be r.

$\therefore$ Area $= A = \pi r^2$

When its radius is changed (increased say) by x%, it becomes $\frac{r(100+x)}{100}$

$\therefore$ its new area $= A_1 = \pi\left[\frac{r(100+x)}{100}\right]^2$

$$= \pi r^2\left[\frac{100+x}{100}\right]^2 = \left[\frac{100+x}{100}\right]^2 A$$

$\therefore$ % increase in area $= \frac{A_1 - A}{A} \times 100$

$$= \left[\left(\frac{100 + x}{100}\right)^2 - 1\right] \times 100 = \left[2x + \frac{x^2}{100}\right]$$

Thus, the theorem is also true for circle.

Final conclusion : We now conclude that the above theorem is true f any two-dimensional figure.

Note: (1) We can use this theorem for Ex. 42.

(2) Whenever there is decrease, use -ve value for x. Whenever you get the -ve value, don't hesitate to say that there is decrease in th area. In Ex. 42, if there is decrease, we put x = -10 in the formula. Th is, $2x + \frac{x^2}{100} = -20 + \frac{(-10)^2}{100} = -19$ which implies that there is decreas of 19% oin area.

In that case, % increase in area $= 2 \times 10 + \frac{(10)^2}{100} = 21\%$

Ex. 43 : If radius of a circle is increased by 5%, find the percentage increase in its area.

Soln : By the theorem :

% increase in its area $= 2 \times 5 + \frac{5^2}{100} = 10 + 0.25 = 10.25\%$

Ex. 44 : If all the sides of a hexagon (six-sided figure) is increased by 2%, find the % increase in its area.

Soln : Required % increase $= 2 \times 2 + \frac{2^2}{100} = 4 + 0.04 = 4.04\%$

Note : If there is decrease in the above cases, find the percentage decrease in area. (Ans Ex. 43 = 9.75% and Ex. 44 = 3.96%)

Theorem : *If all the measuring sides of any two-dimensional figure are changed (increased or decreased) by x% then its perimeter also changes by the same, ie, x%.*

Proof : It is easy to prove it. Try yourself.

Ex. 45 : If diameter of a circle is increased by 12%, find the % increase in its circumference.

Soln : Although diameter is rarely used as the measuring side of a circle, the above theorem holds good for it. Thus, by the theorem, % increase in circumference = 12%.

Theorem : *If all sides of a quadrilateral are increased by x% then its corresponding diagonals also increase by x%.*

Proof : Try yourself.

Ex. 46 : If the sides of a rectangle are increased each by 10%, find the percent increase in its diagonals.

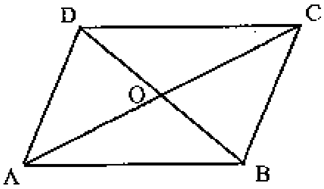
Soln : Required % increase in diagonals = 10%

Ex. 47 : If the length and the two diagonals of a rectangle are each increased by 9%, then find the % increase in its breadth.

Soln : From the above theorem it can be concluded that its breadth also increases by the same value, i.e. 9%.

Ex. 48 : A parallelogram, the length of whose sides are 12 cm and 8 cm, has one diagonal 10 cm long. Find the length of the other diagonal.

Soln :



Diagonals of a parallelogram bisect each other.

Let BD = 10 cm.

$\therefore$ OB = 5 cm

In triangle ABC, O is the mid-point of AC.

By a very important theorem in plane geometry, we have in triangle ABC

$AB^2 + BC^2 = 2(OB^2 + AO^2)$

$\Rightarrow 12^2 + 8^2 = 2(5^2 + AO^2)$

$\Rightarrow 144 + 64 = 50 + 2\,AO^2$

$\Rightarrow AO^2 = 79$

$\therefore$ AO = 8.9 (Approximately)

$\therefore$ the other diagonal = AC = 2AO = 2 × 8.9 = 17.8 cm.

Quicker Method (Direct Formula) :

By the above-mentioned theorem, we have

$AB^2 + BC^2 = 2(OB^2 + AO)^2$

or, $2AO^2 = AB^2 + BC^2 - 2(OB)^2$

$\therefore AO = \sqrt{\frac{1}{2}\left\{AB^2 + BC^2 - 2(OB)^2\right\}}$

$\therefore$ Other diagonal $= 2AO = 2\sqrt{\frac{1}{2}\{AB^2 + BC^2 - 2(OB)^2\}}$

or, Other diagonal $= \sqrt{2\{AB^2 + BC^2 - 2(OB)^2\}}$

Thus, in this case, other diagonal $= \sqrt{2\{144 + 64 - 2 \times 25\}}$

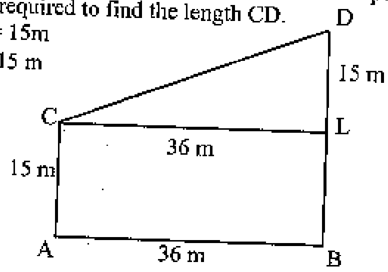
$= \sqrt{2 \times 158} = \sqrt{316} = 17.8$ (Approx.)

Ex. 49 : Two poles 15 m and 30 m high stand upright in a playground. If their feet be 36 m apart, find the distance between their tops.

Soln : From the figure it is required to find the length CD.

We have CA = LB = 15m

$\therefore$ LD = BD - LB = 15 m



$\therefore CD = \sqrt{CL^2 + DL^2} = \sqrt{36^2 + 15^2} = \sqrt{1521} = 39$ cm.

Ex. 50 : A semi-circle is constructed on each side of a square of length 2 m. Find the area of the whole figure.

Soln : Total area = Area of square + 4 (Area of a semi-circle)

$= 2^2 + 4\left(\frac{1}{2}\pi r^2\right)$ (radius $= \frac{2}{2} = 1$)

$= (4 + 2\pi)\ m^2$

Ex. 51 : The area of a circle is halved when its radius is decreased by n. Find its radius.

Soln : By the question we have, $\frac{\pi(r-n)^2}{\pi r^2} = \frac{1}{2}$

or, $r^2 = 2(r-n)^2$

or, $r^2 - \{\sqrt{2}(r-n)\}^2 = 0$

or, $\{r - \sqrt{2}(r-n)\}\{r + \sqrt{2}(r-n)\} = 0$

Since $r + \sqrt{2}(r-n) \neq 0$, we have

$r - \sqrt{2}(r-n) = 0$

or, $r(\sqrt{2} - 1) = \sqrt{2}n$

$\therefore r = \frac{\sqrt{2}n}{\sqrt{2} - 1}$

Quicker Method : We have,

$\frac{\pi(r-n)^2}{\pi r^2} = \frac{1}{2}$

or, $\left\{\frac{\sqrt{2}(r-n)}{r}\right\}^2 = 1$

or, $\frac{\sqrt{2}(r-n)}{r} = 1$

or, $r(\sqrt{2} - 1) = \sqrt{2}n$

$\therefore r = \frac{\sqrt{2}n}{\sqrt{2} - 1}$

Ex. 52 : A cord is in the form of a square enclosing an area of 22 cm^2. If the same cord is bent into a circle, then find the area of that circle.

Soln : Area of square = 22 cm^2

$\therefore$ Perimeter of the square $= 4\sqrt{22}$ cm.

Now, this perimeter is the circumference of the circle.

$\therefore$ circumference of the circle $= 2\pi r = 4\sqrt{22}$

$\therefore r = \frac{2\sqrt{22}}{\pi}$

$\therefore$ area of the circle $= \pi r^2$

$= \pi\left(\frac{2\sqrt{22}}{\pi}\right)^2$

$= \frac{\pi \times 4 \times 22}{\pi^2}$

$= \frac{4 \times 22}{\pi} = \frac{4 \times 22 \times 7}{22} = 28\ cm^2$

Quicker Method (Direct Formula) : *If the area of a square is x sq cm, then area of the circle formed by the same perimeter is given by* $\frac{4x}{\pi}$ *cm*^{2}. **(Remember)**

Thus, in this case, area of circle $= \frac{4 \times 22 \times 7}{22} = 28\ cm^2$

Note : Proof for the above statement can be seen in the detailed method for the above example.

Theorem : *Area of a square inscribed in a circle of radius r is $2r^2$.* **(Remember)**

Proof : When radius is r, diameter is 2r.

Now, we know that the diagonal of the square inscribed in a circle is equal to its diameter.

Thus, diagonal of the inscribed square = 2r.

$\therefore$ Area of square $= \frac{(\text{diagonal})^2}{2} = \frac{(2r)^2}{2} = 2r^2$.

Cor : *Side of a square inscribed in a circle of radius r is $\sqrt{2}\ r$.*

Proof : The theorem gives area of square $= 2r^2$

$\therefore$ side of the square $= \sqrt{2r^2} = \sqrt{2}\ r$.

Note : Such a square is the largest quadrilateral inscribed in a circle.

Ex. 53 : The circumference of a circle is 100 cm. Find the side of the square inscribed in the circle.

Soln : Circumference of the circle $= 2\pi r = 100$

$\therefore\ r = \frac{50}{\pi}$

$\therefore$ side of the inscribed square $= \sqrt{2}\ r = \sqrt{2} \times \frac{50}{\pi}$

Theorem : *The area of the largest triangle inscribed in a semi-circle of radius r is r^2.*

(Remember)

Proof : The largest such triangle is an isosceles triangle (the triangle whose two sides are equal) with diameter as its base and radius as its height.

$\therefore$ area $= \frac{1}{2} \times$ base $\times$ height $= \frac{1}{2} \times 2r \times r = r^2$.

Ex. 54 : The largest triangle is inscribed in a semi-circle of radius 14 cm. Find the area inside the semi-circle which is not occupied by the triangle.

Soln : Such area = Area of semicircle - Area of such largest triangle

$$= \frac{\pi}{2} r^2 - r^2$$

$$= r^2\left(\frac{\pi}{2} - 1\right) = 14^2 \times \frac{(22-14)}{14} = 112 \text{ cm}^2$$

Theorem : *The area of the largest circle that can be drawn in a square of side x is $\pi\left(\frac{x}{2}\right)^2$.*

Proof : The diameter of such a circle is equal to the side of square. Then radius of the largest such circle $= \frac{x}{2}$

$\therefore$ Area $= \pi\left(\frac{x}{2}\right)^2$

Ex. 55 : Find the area of the largest circle that can be drawn in a square of side 14 cm.

Soln : By the formula:

Required area $= \pi\left(\frac{14}{2}\right)^2 = \frac{22}{7} \times 7^2 = 154 \text{ cm}^2$

Ex. 56: In a quadrilateral, the length of one of its diagonal is 23 cm and the perpendiculars drawn on this diagonal from other two vertices measure 17 cm and 7 cm respectively. Find the area of the quadrilateral.

Soln: In any quadrilateral,

Area of the quadrilateral $= \frac{1}{2} \times$ any diagonal $\times$ (sum of perpendiculars drawn on diagonal from two vertices)

$= \frac{1}{2} \times D \times (P_1 + P_2)$

$= \frac{1}{2} \times 23 \times (17 + 7) = 12 \times 23 = 276$ sq cm.

Ex. 57: (a) What is the relation between a circle and an equilateral triangle which is inscribed in the circle?

(b) What is the relation between an equilateral triangle and a circle inscribed in that triangle?

(c) An equilateral triangle is circumscribed by a circle and another circle is inscribed in that triangle. Find the ratio of the areas of the two circles.

Soln: Remember the theorems:

(a) "The area of a circle circumscribing an equilateraltriangle of side x is $\frac{\pi}{3}\ x^2$".

(b) The area of a circle inscribed in an equilateral triangle of
x is $\frac{\pi}{12}x^2$.

(c) From the abvoe two theorems, we can say that the requi
ratio $= \frac{\pi}{3}x^2 : \frac{\pi}{12}x^2 = \frac{1}{3} : \frac{1}{12} = 4 : 1$

Ex. 58: Is there any relation between the number of sides and the num of diagonals in a polygon?

Soln : YES. There exists such a relationship.

Number of diagonals $= \frac{n(n-3)}{2}$. Where n = no. of sides in t polygon.

e.g., for a hexagon, there are $\frac{6(6-3)}{2} = 9$ diagonals.

Note : The proof for the above statement is very easy. Try to prove it

If you can't prove, see the following hints.

There are n corners in a polygon of n sides. Each corner ca make diagonals with other corners except the two adjuscent corners, means that one corner can make (n - 3) diagonals. Thus total diagona by n corners are n(n - 3). If you look carefully you will find that eac diagonal is repeated when we take the other corner for drawing diagonal

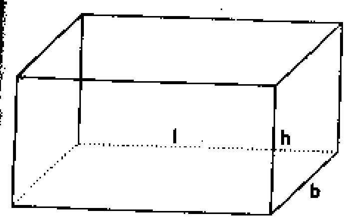
so, the exact no. of diagonals $= \frac{n(n-3)}{2}$.

Elementary Mensuration - II

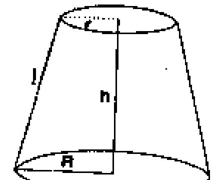
(Measurement of Volume and Surface Areas)

*An object which occupies space has usually three dimensions: length, breadth and depth. Such an object is usually called a **solid**.*

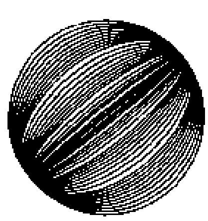
Given below are some commonly known solids :



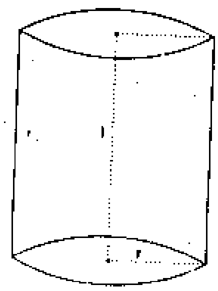
CUBOID



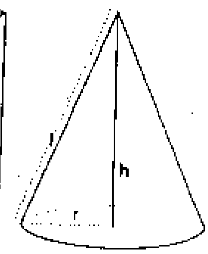
FRUSTUM OF A CONE



SPHERE



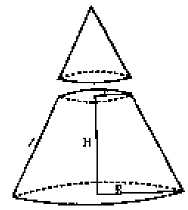
CYLINDER



CONE

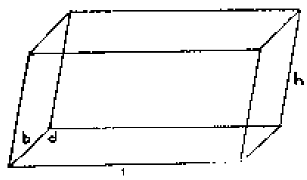
List of important formulas

1. **Cuboid :** Let length= l, breadth = b and height = h units.
   (i) volume of cuboid = $(l \times b \times h)$ cubic units = $\sqrt{A_1 \times A_2 \times A}$ cu. units where A_1 = area of base or top, A_2 = area of side fa
   A_3 = area of other side face.
   (ii) whole surface of cuboid = $2(lb + bh + lh)$ sq. units.
   (iii) diagonal of cuboid = $\sqrt{l^2 + b^2 + h^2}$ units.
2. **Cube :** Let each edge (or side) of a cube be a units. Then :
   (i) Volume of the cube = a^3 cubic units.
   (ii) Whole surface of the cube = $(6a^2)$ sq. units.
   (iii) diagonal of the cube = $(\sqrt{3}\ a)$ units.
3. **Cylinder :** Let the radius of the base of a cylinder be r units and its height (or length) be h units. Then :
   (i) volume of the cylinder = $(\pi r^2 h)$ cu. units.
   (ii) curved surface area of the cylinder = $(2\pi rh)$ sq. units.
   (iii) total surface area of the cylinder = $(2\pi rh + 2\pi r^2)$ sq. units.
4. **Sphere:** Let the radius of a sphere be r units. Then :
   (i) volume of the sphere = $\left(\frac{4}{3}\pi r^3\right)$ cu. units.
   (ii) surface area of the sphere = $(4\pi r^2)$ sq. units.
   (iii) volume of a hemisphere = $\left(\frac{2}{3}\pi r^3\right)$ cu. units.
   (iv) curved surface area of the hemisphere = $(2\pi r^2)$ sq. units.
   (v) whole surface area of the hemisphere = $(3\pi r^2)$ sq. units.
5. **Right Circular Cone:** Let r be the radius of the base, h the height and l the slant height of a cone. Then:
   (i) slant height $l = \sqrt{h^2 + r^2}$
   (ii) volume of the cone = $\left(\frac{1}{3}\pi r^2 h\right)$ cu. units.
   (iii) curved suface area of the cone:
   $= (\pi rl)$ sq. units = $(\pi r\sqrt{r^2 + h^2})$ sq. units.
   (iv) total surface area of the cone = $(\pi rl + \pi r^2) = \pi r(l + r)$
6. **Fructum of a right circular cone:** If a cone is cut by a plane parallel to the base so as to divide the cone into two parts as shown in the figure, the lower part is called the fructum of the cone.



Let the radius of the base of the fructum = R, the radius of the top = r, height = h and slant height = l units
Slant height, $l = \sqrt{h^2 + (R - r)^2}$ units
Curved surface area = $\pi(r + R)l$ sq. units
Total surface area = $\pi\{(r + R)l + r^2 + R^2\}$ sq. units
Volume = $\frac{\pi h}{3}(r^2 + R^2 + rR)$ cu. units

7. **Right Parallelopiped:** It is such type of a cuboid in which the shape of a side face is rectangular whereas the shape of the base or the top face is a parallelogram (neither a rectangle nor a square).



Surface area (of the side faces)
Surface area (of the base or the top face)
$= 2\sqrt{s(s-a)(s-b)(s-d)}$ sq. units
Total surface area = $2h(b + l) + 4\sqrt{s(s-a)(s-b)(s-d)}$ sq. units
Volume = Base area × height

Solving Problems

Usually, the formulae given above are sufficient for solving problems on mensuration. However, depending upon some typical cases, we may have some quicker methods for some indirect problems. Let us see the application of both these approaches by way of a few solved examples:

PROBLEMS ON CUBES AND CUBOIDS

Type I : Direct Application of Formulas

Ex.1. Find the volume and the surface of a slab of stone measuring 4 metres in length, 2 metres in width and $\frac{1}{4}$ metre in thickness.

Sol. Volume $= 4 \times 2 \times \frac{1}{4} = 2$ cu. metres. Ans.

Surface $= 2(lb + lh + bh)$

$= 2(4 \times 2 + 4 \times \frac{1}{4} + 2 \times \frac{1}{4}) = 19$ sq. metres. Ans.

Ex. 2. A rectangular tank, measuring internally $37\frac{1}{3}$ metres in length, 1[illegible] metres in breadth and 8 metres in depth, is full of water. Find th[e] weight of water in metric tons, given that one cubic metre of wate[r] weighs 1000 kilograms.

Soln. Volume of water $= 37\frac{1}{3} \times 12 \times 8$ cu. metres.

Weight of water $= \frac{112}{3} \times 12 \times 8 \times 1000$ kg

$= 3584000$ kg $= 3584$ metric tons. Ans.

Ex. 3(a). A brick measures 20 cm by 10 cm by $7\frac{1}{2}$ cm. How many bricks will be required for a wall 25 m long, 2 m high and $\frac{3}{4}$ m thick ?

Soln. Volume of wall $= 25 \times 2 \times \frac{3}{4}$ cu. m.

Volume of one brick $= \frac{20}{100} \times \frac{10}{100} \times \frac{15}{200}$

$= \frac{3}{2000}$ cu. m.

Reqd. number of bricks $= \left(25 \times 2 \times \frac{3}{4}\right) \div \frac{3}{2000} = 25000$. Ans.

Ex. 3(b). Find the volume of a cuboid whose areas of base and two adjacent faces are 180 sq. cm, 96 sq. cm and 120 sq. cm respectively.

Soln. We have,

Volume of a cuboid

$= \sqrt{\text{area of base} \times \text{area of one face} \times \text{area of the other face}}$

$= \sqrt{180 \times 96 \times 120} = 1440$ cu. cm.

Type II : Some Quicker Methods

Ex.4. A closed wooden box measures externally 9 cm long, 7 cm broad, 6 cm high. If the thickness of the wood is half a cm, find (i) the capacity of the box and (ii) the weight supposing that one cubic cm. of wood weighs 0.9 gm.

Soln: Quicker Method

In such cases,

Capacity = (external length - 2 × thickness) × (external breadth - 2 × thickness) × (external height - 2 × thickness) **(Remember)**

Volume of material = External volume - Capacity **(Remember)**

$\therefore$ in the given question,

Capacity $= (9 - 2 \times 0.5)(7 - 2 \times 0.5)(6 - 2 \times 0.5)$

$= 8 \times 6 \times 5 = 240 \text{ cm}^3$.

$\therefore$ Volume of wood = external volume - capacity

$= 9 \times 7 \times 6 - 240 = 138$ cu. cm.

$\therefore$ **Weight of wood = Volume of wood × density of wood** **(Note)**

$= 138 \times 0.9 = 124.2$ g.

Ex.5: The surface of a cube is $30\frac{3}{8}$ sq. metres. Find its volume.

Soln: Quicker Method

Volume of cube $= \left(\sqrt{\frac{\text{Surface area}}{6}}\right)^3$ **(Remember)**

$\therefore$ In the given question,

Volume $= \left(\sqrt{\frac{\frac{243}{8}}{6}}\right)^3 = 11\frac{25}{64}$ cu. m.

SOME SPECIAL CASES

Rainfall in a given area and similar problems

Ex.6: The annual rainfall at a place is 43 cm. Find the weight in metric tonnes of the annual rainfall there on a hectare of land, taking the weight of water to be 1 metric tonne for 1 cubic metre.

Soln:- Quicker Method :-

Volume of water = height (level) of water × base area

(Remember

In the given question, level of rainfall is 43 cm.

$\therefore$ volume of water $= \frac{43}{100}$ m × 10000 sq. m = 4300 cu. m.

(As 1 hectare = 10,000 sq.m.)

$\therefore$ weight of water = 4300 × 1 = 4300 metric tonnes.

Ex.7: A rectangular tank is 50 metres long and 29 m deep. If 1000 cub metres of water be drawn off the tank, the level of the water in the ta goes down by 2 metres. How many cubic metres of water can the ta hold ?

Soln:- Quicker Method:-

By the formula given in the previous example and the second li of this question, we have :

Volume = 1000 = [level (= 2m) × base area]

$\therefore$ base area $= \frac{1000}{2} = 500.$

$\therefore$ Total volume = depth × base area = 29 × 500 = 14500 cub m

An Exact Cube Cut Off From A Square Bar

Ex.8: A cubic metre of copper weighing 9000 kilograms is rolled into bar 9 metres long. An exact cube is cut off from the bar. Ho much does it weigh?

Soln:- General Method

In this case a given volume of copper is rolled into a square ba (basically a cuboid with square base) of given length. Then a exact cube is cut off from this square bar. Obviously, the exac cube should have the same dimensions as that of the square bas of the square bar.

Now, given volume = 1 cu. m.

= Area of square base × length

$\Rightarrow$ Area of square base × length = 1

$\Rightarrow$ Area of square base $= \frac{1}{\text{length}} = \frac{1}{9}.$

$\therefore$ side of square base $= \sqrt{\frac{1}{9}} = \frac{1}{3}.$

$\therefore$ Vol. of the cut off cube = (side of the square base)^3

$= \left(\frac{1}{3}\right)^3 = \frac{1}{27}.$

$\therefore$ weight of cube $= \frac{1}{27} \times 9000 = 333.3$ kg.

Quicker Method

In this type of question, use the formula:

$$\textbf{Volume of cube cut off} = \left(\sqrt{\frac{\text{Volume of original solid}}{\text{length of the solid}}}\right)^3$$

(Remember)

$\therefore$ Volume $= \left(\sqrt{\frac{1}{9}}\right)^3 = \frac{1}{27}$

$\therefore$ weight $= \frac{9000}{27} = 333.3$ kg.

Case of Object Changing Its Shape

Ex.9 : A cubic metre of gold is extended by hammering so as to cover an area of 6 hectares. Find the thickness of the gold.

Soln:- The underlying concept for these type of questions is that the **total. volume of a solid does not change even when its shape changes.**

$\therefore$ old volume = new volume

$\Rightarrow$ 1 cu m. = 60000 × thickness

$\Rightarrow$ thickness $= \frac{1}{60,000}$ m = 0.0017 cm.

Integration of cubes

Ex.10 : Three cubes of metals whose edges are 3, 4 and 5 cm respectively are melted and formed into a single cube. If there be no loss of metal in the process find the side of the new cube.

Soln: Quicker Method

When many cubes integrate into one cube, the side of the new cube is given by

side $= \sqrt[3]{\text{Sum of cubes of sides of all the cubes}}$

(remember)

$\therefore$ Here, side $= \sqrt[3]{3^3 + 4^3 + 5^3} = \sqrt[3]{27 + 64 + 125}$

$= \sqrt[3]{216} = 6$ cm.

(Note: Can you solve this question by the 'volume remains unchanged' principle of the previous examples?)

Disintegration of a Cube into Identical Cubes

Ex.11 A cube of sides 3 cm is melted and smaller cubes of sides 1 cm each are formed. How many such cubes are possible ?

Soln: Quicker Method

In such questions use the rule:

$$\textbf{number possible} = \left(\frac{\text{original length of side}}{\text{new length of side}}\right)^3$$

(Remember)

$\therefore$ In this question, possible number of cubes $= \left(\frac{3}{1}\right)^3 = 27.$

Rate Of Water Issuing From A Jet

Ex.12: A stream which flows at a uniform rate of 2.5 km. per hour, is 20 metres wide, the depth of a certain ferry being 1.2 m. How many litres pass the ferry in a minute ?

Soln: Solve such problems using the rule :

volume = time x speed x area of cross section

(Remember)

Now in this question, time = 1 minute, speed = 2.5 km per hour $= \frac{125}{3}$ m/min., area of cross section $= 20 \times 1.2 = 24$

$\therefore$ volume $= 1 \times \frac{125}{3} \times 24 = 1000$ cu m $= 1{,}000{,}000$ litre.

PROBLEMS ON CYLINDERS

Type I. Direct Application of Formulas

Ex. 13: Find the volume of a cylinder which has a height of 14 metres and a base of radius 3 metres. Also find the curved surface of the cylinder.

Sol. Volume $= 3 \times 3 \times \frac{22}{7} \times 14$ cu. metres

$= 396$ cu. metres. Ans.

Curved surface = circumference × height

$= 2 \times 3 \times \frac{22}{7} \times 14$ sq. metres $= 264$ sq. metres.

Type II : Some Quicker Methods

Case Of Hollow Tube With Some Thickness

Ex.14: A hollow cylindrical tube open at both ends is made of iron 2 cm thick. If the external diameter be 50 cm and the length of the tube be 140 cm, find the volume of iron in it.

Soln: Quicker Method

In such cases, use the rule:

Volume of metal

$= \pi$ height $\times \left[(\text{outer radius})^2 - (\text{inner radius})^2\right]$

(Remember)

Now in the given question, external radius $= 50 \div 2 = 25$ cm.

inner radius = outer radius - thickness $= 25 - 2 = 23$.

$\therefore$ Volume $= \frac{22}{7} \times 140 \times (25^2 - 23^2) = 42240$ cu. cm.

[**Note:** Still quicker formulas can be used in the following from:

Volume = π × height × (2 × outer radius - thickness) (thickness)

(when outer radius is given)

Volume = π × height × (2 × inner radius + thickness) (thickness)

(when inner radius is given)

For example, in the above question, outer radius is given and hence we use the first relation:

Volume $= \frac{22}{7} \times 140 \times (25 \times 2 - 2)(2) = \frac{22}{7} \times 140 \times 48 \times 2$

$= 42240$ cu. cm.]

Ex.15: Find the weight of a lead pipe 3.5 cm long if the external diameter is 2.4 cm, the thickness of the lead is 2 mm and 1 c.c. of lead weighs 11.4 gms.

Soln: Try yourself.

Case of Rolling A Square Into A Cylinder

Ex.16: A rectangular sheet with dimensions 22m x 10 m is rolled into a cylinder so that the smaller side becomes the height of the cylinder. What is the volume of the cylinder so formed ?

Soln: Quicker Method

In such cases, use the rule:

$$\textbf{Volume} = \frac{\text{height} \times (\text{other side of the sheet})^2}{4\pi}$$

(Remember)

$\therefore$ In the given question,

$$\text{Volume} = \frac{10 \times (22)^2}{4 \times \frac{22}{7}} = 385 \text{ cu. m.}$$

[**Note:** The height is 10 m since it is the smaller side. The other side is obviously 22 m.]

PROBLEMS ON CONES

Ex.17: Find what length of canvas 2 metres in width is required to make a conical tent 8 metres in diameter and 5.6 metres in slant height; also find the cost of the canvas at the rate of Rs 3.20 per metre.

Sol. Curved surface area = πrl

$$= \frac{22}{7} \times \frac{1}{2} \times 8 \times 5.6 \text{ sq. metres}$$
$$= 22 \times 4 \times 0.8 \text{ sq. metres}$$
$$= 70.4 \text{ sq. metres.}$$

$\therefore$ length of the canvas = 70.4 ÷ 2 metres
= 35.2 metres. Ans.

Cost of the canvas = Rs 35.2 × 3.20
= Rs 112.64 Ans.

Ex.18: The diameter of a right circular cone is 14 metres and its slant height is 12 metres. Find its
(i) curved surface area, (ii) total surface area,
(iii) volume, and
(iv) the cost of colouring its total surface at the rate of 14 P per sq. metre.

Soln. (i) Curved surface area = $\pi rl = \frac{22}{7} \times \frac{14}{2} \times 12$ sq. metres
= 264 sq. metres. Ans.

(ii) Total surface area = $\pi r(r+l)$
$$= \frac{22}{7} \times \frac{14}{2}\left(\frac{14}{2} + 12\right) \text{ sq. metres}$$
$$= \frac{22}{7} \times \frac{14}{2} \times 19 \text{ sq. metres}$$
= 418 sq. metres. Ans.

(iii) Volume = $\frac{1}{3}\pi r^2 h$.

Now let us find h.
$$h = \sqrt{l^2 - r^2}.$$
$$= \sqrt{144 - 49} \text{ metres} = 9.75 \text{ metres.}$$

Volume = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 9.75$ cu. metres
= 500.5 cu. metres. Ans.

(iv) Reqd. cost = Rs 418 × $\frac{14}{100}$

= Rs 58.52 Ans.

Ex.19: A frustum of a right circular cone has a diameter of base 10 cm, of top 6 cm and a height of 5 cm. Find the area of its whole surface and volume.

Soln. R = 5 cm, r = 3 cm, and h = 5 cm.
$$\therefore s = \sqrt{h^2 + (R - r)^2} \text{ cm.} = \sqrt{5^2 + (5-3)^2} \text{ cm}$$
$$= \sqrt{29} \text{ cm} = 5.385 \text{ cm.}$$

$\therefore$ whole surface of the frustum = $\pi(R^2 + r^2 + Rl + rl)$
$$= \frac{22}{7}(5^2 + 3^2 + 5 \times 5.385 + 3 \times 5.385) = 242.25 \text{ sq. cm.}$$

Volume = $\frac{\pi h}{3}(R^2 + r^2 + Rr)$
$$= \frac{22 \times 5}{7 \times 3}[5^2 + 3^2 + 5 \times 3] \text{ cu. cm.}$$
= 256.67 cu. cm. Ans.

PROBLEMS ON SPHERES

Type I: Direct Application of Formulas

Ex.20: Find the volume and the surface area of a sphere of diameter 42 cm.

Sol. Radius = 21cm.

$\therefore$ Volume
$$= \left(\frac{4}{3}\pi r^3\right) = \left(\frac{4}{3} \times \frac{22}{7} \times 21 \times 21 \times 21\right) \text{cm}^3 = 38808 \text{ cm}^3.$$

Surface area
$$= 4\pi r^2 = \left(4 \times \frac{22}{7} \times 21 \times 21\right) \text{cm}^2 = 5544 \text{ cm}^2.$$

Ex.21: Find the volume, curved surface area and the total surface area of a hemisphere of radius 21 cm.

Sol. Volume = $\frac{2}{3}\pi r^3$
$$= \left(\frac{2}{3} \times \frac{22}{7} \times 21 \times 21 \times 21\right) \text{cm}^3 = 19404 \text{ cm}^3.$$

Curved surface area = $2\pi r^2$
$$= \left(2 \times \frac{22}{7} \times 21 \times 21\right) \text{cm}^2 = 2772 \text{ cm}^2.$$

Total surface area = $3\pi r^2$

$$= \left(3 \times \frac{22}{7} \times 21 \times 21\right) cm^2 = 4158\ cm^2.$$

Type II: Some Quicker Methods

Cases of spheres changing shapes

The basic principle for solving such questions where the solid in the shape of any particular object (sphere, cylinder, cone etc.) changes into some other shape, is : *the volume remains unchanged.* But we can develop some quicker methods for finding the answers. Let us see some examples:

Ex.22: A copper sphere of diameter 18 cm. is drawn into a wire of diameter 4 mm. Find the length of the wire.

Soln: Quicker Method

When a sphere is converted into a cylinder **(Note that wire is basically a cylinder)** the length of the wire is given by the rule:

(i) **length of cylinder** $= \frac{4 \times (\text{radius of sphere})^3}{3 \times (\text{radius of cylinder})^2}$

(Remember)

$\therefore$ In the given question, length $= \frac{4 \times (90)^3}{3 \times (2)^2} = 243000$ mm $= 24300$ cm.

Note : When a sphere is converted into a cylinder we may have three types of question. One, when the radii of the cylinder and the sphere are given and the length of the cylinder is to be found. Second, when the radius and the length of the cylinder are given and the radius of the sphere is to be found. And third, when the length of the cylinder and the radius of the sphere is given and the radius of the cylinder is to be found. In the first case the formula given above may be used for a quick solution. In the second and third cases the following formulas should be used:

(ii) **radius of sphere**

$$= \sqrt[3]{\frac{3}{4}(\text{length of cylinder})(\text{radius of cylinder})^2}$$

(iii) **radius of cylinder** $= \sqrt{\frac{4 \times (\text{radius of sphere})^3}{3 \times (\text{length of cylinder})}}$

Ex.23: A copper sphere of 36 m diameter is drawn into a cylindrical wire of length 7.29 km. What is the radius of wire ?

Soln: Try yourself.

Ex.24: A cylinder of radius 2 cm and height 15 cm is melted and the same mass is used to create a sphere. What will be the radius of the sphere ?

Soln: Try yourself. [**Hint:** Sphere converted into a cylinder and cylinder converted into a sphere are one and the same thing. Use formula (iii) above.]

Ex.25: How many bullets can be made out of a lead cylinder 28 cm high and 6 cm radius, each bullet being 1.5 cm in diameter ?

Soln: In this case, one cylinder is not converted into just one sphere but many spheres are being made. Here we will use the following formula:

Number of bullets $= \frac{\text{volume of cylinder}}{\text{volume of 1 bullet}}$

(Remember)

$$= \frac{\pi \times 6 \times 6 \times 28}{\frac{4}{3} \times \pi \times 0.75 \times 0.75 \times 0.75} = 1792.$$

Ex.26: Find the number of lead balls of diameter 1 cm each that can be made from a sphere of diameter 16 cm.

Soln: Number of balls $= \frac{\text{Volume of big sphere}}{\text{Volume of 1 small sphere}}$

$$= \frac{\frac{4}{3}\pi \times 8 \times 8 \times 8}{\frac{4}{3}\pi \times 0.5 \times 0.5 \times 0.5} = 4096.$$

Quicker Method:

When a sphere disintegrates into many identical spheres use the formula: number $= \left(\frac{\text{bigger radius}}{\text{smaller radius}}\right)^3$ **(Remember)**

$\therefore$ number $= \left(\frac{8}{0.5}\right)^3 = 16^3 = 4096.$

SOME MISCELLANEOUS CASES

Problems Involving Ratios

It will be helpful if you remember the following results:

I.Two Spheres

(i) $(\text{ratio of radii})^2 =$ ratio of surface areas

(ii) $(\text{ratio of radii})^3 =$ ratio of volume

(iii) $(\text{ratio of surface areas})^3 = (\text{ratio of volumes})^2$

II. Two Cylinders

A. When volumes are equal

(i) ratio of radii = $\sqrt{\text{inverse ratio of heights}}$

(ii) ratio of curved surface areas = inverse ratio of radii

(iii) ratio of curved surface areas = $\sqrt{\text{ratio of heights}}$

B. When radii are equal

(i) ratio of volumes = ratio of heights

(ii) ratio of curved surface areas = ratio of heights

(iii) ratio of volume = ratio of curved surface areas

C. When heights are equal

(i) ratio of volumes = $(\text{ratio of radii})^2$

(ii) ratio of curved surface areas = ratio of radii

(iii) ratio of volumes = $(\text{ratio of curved surface areas})^2$

D. When curved surface areas are equal

(i) ratio of volumes = ratio of radii

(ii) ratio of volumes = inverse ratio of heights

(iii) ratio of radii = inverse ratio of heights

III. Two Cubes

(i) ratio of volumes = $(\text{ratio of sides})^3$

(ii) ratio of surface areas = $(\text{ratio of sides})^2$

(iii) $(\text{ratio of surface areas})^3 = (\text{ratio of volumes})^2$

[Note : See the similarity of the formulas of spheres and of cubes.]

IV. Two Cones

A. When volumes are equal

Formula (i) of cylinders holds.

B. When radii are equal

Formula (i) of cylinders holds.

C. When heights are equal

Formula (i) of cylinders holds.

D. When curved surface areas are equal

Formulas (iii) of cylinders holds in a changed form :

ratio of radii = inverse ratio of **slant** heights.

Some solved examples

Ex.27: The curved surface areas of two spheres are in the ratio 1: 4. Find the ratio of their volumes.

Soln: By formula I (iii), we have $(\text{ratio of surface areas})^3$

$= (\text{ratio of volumes})^2$

$\therefore \quad (1:4)^3 = (\text{ratio of volumes})^2$

$\therefore \quad (1:64) = (\text{ratio of volumes})^2$

$\therefore \quad \sqrt{1:64} = 1:8 = \text{ratio of volumes}$

Other method: Ratio of sides $= \sqrt{\frac{1}{4}} = \frac{1}{2} = 1:2$

$\therefore$ Ratio of volumes $= \left(\frac{1}{2}\right)^3 = \frac{1}{8} = 1:8$

Ex.28: The radii of two spheres are in the ratio of 1 : 2. What is the ratio of their surface areas ?

Soln: By formula, I, (i), we have :

$(\text{ratio of surface areas})^2 = (1:2)^2 = 1:4$.

Ex.29: Two circular cylinders of equal volume have their heights in the ratio of 1 : 2. Ratio of their radii is?

Soln : By formula II, A, (i), we have; ratio of radii

$= \sqrt{\text{inverse ratio of heights}} = \sqrt{2:1} = \sqrt{2}:1$

Ex.30: If the heights of two cones are in the ratio 1 : 4 and their diameters are in the ratio 4 : 5, what is the ratio of their volumes ?

Soln: Since there is no defined short-cut method for this type of question, we will solve it by the general method. We know that,

Volume $= \frac{1}{3}\pi\ (\text{radius})^2(\text{height})$

$$\therefore \frac{\text{Volume of first cone}}{\text{Volume of second cone}} = \frac{\frac{1}{3}\pi\ (\text{radius of first cone})^2\ (\text{height})}{\frac{1}{3}\pi\ (\text{radius of second cone})^2\ (\text{height})}$$

$$= \left(\frac{\text{radius of first cone}}{\text{radius of second cone}}\right)^2 \left(\frac{\text{height of first cone}}{\text{height of second cone}}\right)$$

Thus, ratio of volumes = $(\text{ratio of radii})^2$ (ratio of heights)

$= (4:5)^2\ (1:4) = \frac{16}{25} \times \frac{1}{4} = 4:25$

Note: If you don't want to go into the detail of the above derived method:

Since volume $= \frac{\pi}{3}\ (\text{radius})^2\ (\text{height})$.

$\therefore$ ratio of volumes = $(\text{ratio of radii or diameters})^2$ (ratio of height)

($\frac{\pi}{3}$ is a constant value, which is cancelled out)

Remember :

V. **For a cone and also for a cylinder :**

(i) ratio of volumes = (ratio of radii)^2 (ratio of heights)

(ii) ratio of heights = (inverse ratio of radii)^2 (ratio of volumes)

(iii) ratio of radii

= $\sqrt{\text{(ratio of volumes) (inverse ratio of heights)}}$

VI. **For a cylinder**

(i) ratio of curved surface areas = (ratio of radii) (ratio of heights)

(ii) ratio of heights = (ratio of curved surface areas) (inverse ratio radii)

(iii) ratio of radii = (ratio of curved surface areas) (inverse ratio of heights)

VII. **For a cone**

All the three formulas in VI hold; just change to **slant heights** inste of **heights.**

Ex. 31 : Two cm of rain has fallen on a square km of land. Assuming that 50% of the raindrops could have been collected and contain in a pool having a 100m × 10m base, by what level would th water level in the pool have increased?

Soln : Volume of rain water = Area × height

$= (1\text{km})^2 \times 2 \text{ cm}$

$= (1000\text{m})^2 \times 0.02 \text{ m} = 20{,}000 \text{ m}^3$

Quantity of collected water = 50% of 20,000 m^3

$= \frac{1}{2} \times 20{,}000 = 10{,}000 \text{m}^3$.

$\therefore$ Increased level in pool $= \frac{\text{Volume collected}}{\text{Base area of pool}}$

$= \frac{10{,}000}{10 \times 100} = 10 \text{ m}$

$\therefore$ the water level would be increased by 10 m.

Ex. 32 : If the radius of a cylinder is doubled and the height is halved, what is the ratio between the new volume and the previous volume?

Soln : Let the initial radius and height of the cylinder be r cm and h cm respectively.

Then $V_1 = \pi r^2 h$

and $V_2 = \pi (2r)^2 \frac{h}{2} = 2 \pi r^2 h$

$\frac{\text{New volume}}{\text{Previous volume}} = \frac{2\pi r^2 h}{\pi r^2 h} = \frac{2}{1} = 2 : 1$

. 33 : A well of 11.2 m diameter is dug 8 m deep. The earth taken out has been spread all round it to a width of 7 m to form a circular embankment. Find the height of this embankment.

ln : Volume of earth dug out

$= \pi r^2 h = \frac{22}{7} \times \left(\frac{11.2}{2}\right)^2 \times 8 = \frac{22}{7} \times 5.6 \times 5.6 \times 8 = 788.48 \text{ m}^3$

rea of embankment $= \pi (5.6 + 7)^2 - \pi (5.6)^2$

$= \pi\left[(5.6 + 7)^2 - (5.6)^2\right]$

$= \pi\left[(5.6 + 7 - 5.6)(5.6 + 5.6 + 7)\right]$

$= \frac{22}{7} \times 7 \times 18.2 = 400.4 \text{ m}^2$

$\therefore$ height of embankment $= \frac{\text{Volume of earth}}{\text{Area of embankment}}$

$= \frac{788.48}{400.4} = 1.97$ m.

Ex. 34 : A right-angle triangle having base 6.3 m and height equal to 10 cm. is turned around the height. Find the volume of the cone thus formed. Also find the surface area.

Soln : Hint : The cone thus formed has height = the height of triangle radius = base of the triangle

slant height = hypotenuse of the triangle.

Now, solve yourself.

Theorem : *If length, breadth and height of a cuboid is increased by x%, y% and z% respectively, then its volume is increased by*

$$\left[x + y + z + \frac{xy + xz + yz}{100} + \frac{xyz}{(100)^2}\right]\%$$

Proof : For the ease of calculations let us suppose that each side of the cuboid be 100 units.

Then its volume = 100^3 units.

Increased sides of the cuboid are: (100 + x), (100 + y) & (100 + z)

Then its new volume = (100 + x) (100 + y) (100 + z)

$= 100^3 + 100^2(x + y + z) + 100(xy + xz + yz) + xyz$

Change in volume $= 100^2(x + y + z) + 100(xy + xz + yz) + x$

% Change in volume

$$= \frac{100^2(x + y + z) + 100(xy + xz + yz) + xyz}{100^3} \times 10$$

$$= x + y + z + \frac{xy + xz + yz}{100} + \frac{xyz}{100^2}$$

Note : This theorem is considered as a basic for all the three-dimensio figures. See how the following theorems are derived from th one.

Theorem : *(For Cube) : If side of a cube is increased by x%, then it volume increases by*

$$\left[3x + \frac{3x^2}{100} + \frac{x^3}{100^2}\right]\% \text{ or } \left[\left(1 + \frac{x}{100}\right)^3 - 1\right] \times 100\%$$

Proof : In a cube, all the sides are equal; hence all the sides are increase by equal %. Put $x = y = z$ in the basic theorem.

$\therefore$ We get $3x + \dfrac{3x^2}{100} + \dfrac{x^3}{100^2}$

This can also be written in the form $\left[\left(1 + \dfrac{x}{100}\right)^3 - 1\right] \times 100\%$

Theorem : *(For Sphere) : If the radius (or diameter) of a sphere is changed by x% then its volume changes by* $\left[3x + \dfrac{3x^2}{100} + \dfrac{x^3}{100^2}\right]$ %.

Proof : A sphere has all the three measuring sides equal which is its radius. Thus, here also we put all the three values equal **in basic theorem** and hence get the result.

Note : All the three-dimensional figures have three measuring sides. In this case also, the three equal measuring sides are r because in r^3, three r's are used

Theorems : (For cylinder)

I. *If height is changed by x% and radius remains the same then its volume changes by x%*

oof : Volume $= \pi r^2 h$

Since only height changes and there is no change in radius, so consider that radius changes by 0%. And also, since r^2 has two asuring sides we put the two other values equal to zero in the basic orem.

Thus, it becomes (Put $y = z = 0$)

$$x + 0 + 0 + \frac{0 + 0 + 0}{100} + \frac{0 \times 0 \times 0}{100^2} = x$$

II. *If radius is changed by x% and height remains the same the volume changes by* $\left[2x + \dfrac{x^2}{100}\right]$ *% or,* $\left[\left(1 + \dfrac{x}{100}\right)^2 - 1\right] \times 100\%$.

roof : Only two measuring sides (r^2) change; so put two of the three radius equal and the third as zero. Following so, we have, (put $y = x$ & $z = 0$)

$$x + x + 0 + \frac{x^2 + 0 + 0}{100} + \frac{x \times x \times 0}{100^2} = 2x + \frac{x^2}{100}$$

III. *If radius is changed by x% and height is changed by y% then volume changes by* $\left[2x + y + \dfrac{x^2 + 2xy}{100} + \dfrac{x^2 y}{100^2}\right]$%.

Proof : Two equal measuring sides (r^2) change by x% while the third measuring side changes by y%, therefore put two values equal and third different. (Put x and z as x and y as it is.) We have

$$x + y + z + \frac{xy + x^2 + xy}{100} + \frac{x^2 y}{100^2}$$

$$= 2x + y + \frac{x^2 + 2xy}{100} + \frac{x^2 y}{100^2}$$

IV. *If height and radius both change by x% then volume changes by* $\left[3x + \dfrac{3x}{100} + \dfrac{x^3}{100^2}\right]$%.

Proof : As in cube and sphere, here also all the three measuring sides (2 radius and one height) change, so put x, y and z as x.

Note : (1) We suggest you to remember only the basic theorem and learn how it changes according to change in measuring sides of any three-dimensional figure.

(2) We have used the word "change" in place of increase decrease in some cases. By this we conclude that if there increase use the +ve value and if there is decrease then use -ve value. If we get the answers +ve or -ve then there is respectively increase or decrease in the volume. 'Change' mentioned the above theorems is always one way i.e. if one value is increas then other also increases.

(3) Establish the theorem for cone, considering all the ca separately.

Ex. 35 : Each edge of a cube is increased by 50%. What is the percent increase in its volume? Also find the % increase in its surf area.

Soln : From the theorem :

$$\% \text{ increase in volume} = 3 \times 50 + \frac{3(50)^2}{100} + \frac{(50)^3}{100^2}$$

$$= 150 + 75 + 12.5 = 237.5\%$$

For the area, we see that only two measuring sides are involv (as area has 2 dimensions). So we use the formula (see previous chapt :

$$\% \text{ increase in area} = 2x + \frac{x^2}{100} = 2 \times 50 + \frac{50 \times 50}{100} = 125\ \%$$

Ex. 36 : Each of the radius and the height of a right circular cylinder both increased by 10%. Find the % by which the volume i creases.

Soln: Since all the three (two radius + one height) measuring sides increase by the same value, we use the formula

$$\% \text{ increase in volume} = 3 \times 10 + \frac{3(10)^2}{100} + \frac{(10)^3}{100^2}$$

$$= 30 + 3 + 0.1 = 33.1\%$$

Ex. 37: Each of the radius and the height of a cone is increased by 20% Then find the % increase in volume.

Soln : Since all the three measuring sides (two radius + one height) increase by the same percent value, we use the same formula as i previous examples.

$$\% \text{ increase in volume} = 3 \times 20 + \frac{3(20)^2}{100} + \frac{(20)^3}{(100)^2}$$

$$= 60 + 12 + 0.8 = 72.8\ \%$$

x. 38: The radius of a sphere is increased by 5%. Find the % increase in its surface area.

oln : We are asked for the % increase in area, so we use the formula for two-dimensional figures (given in previous chapter) :

$$\text{Required percentage value} = 2 \times 5 + \frac{5 \times 5}{100} = 10 + 0.25 = 10.25$$

x. 39 : Each edge of a cube is decreased by 50%. Find the percentage decrease in its surface area and volume.

oln : For surface area (2-dimensional figure) we use the formula (used in previous chapter) : $\left[2x + \frac{x^2}{100}\right]\%$

As there is decrease, put the -ve value of x.

Therefore, required % decrease in surface area

$$= 2(-50) + \frac{(-50)^2}{100}$$

$$= -100 + 25 = -75\%$$

$\therefore$ area decreases by 75%.

Now, percentage decrease in volume

$$= 3(-50) + \frac{3(-50)^2}{100} + \frac{(-50)^3}{100^2}$$

$$= -150 + 75 - 12.5 = -87.5\%$$

$\therefore$ volume decreases by 87.5%.

x. 40: A cylinder, a hemisphere and a cone stand on the same base and have the same heights. Then find the ratio of their volumes and also the ratio of the areas of their curved surface.

oln : Let the diameters of the bases for all the three be x cm and height be y cm.

or hemisphere :

Radius $= \frac{x}{2}$ cm & height $= y = \frac{x}{2}$ cm ---- (*)

or cone :

Radius $= \frac{x}{2}$ cm & height $= y = \frac{x}{2}$ cm(As height is the same for all)

or cylinder :

Radius $= \frac{x}{2}$ cm & height $= y = \frac{x}{2}$ cm

Cylinder's volume : Hemisphere's volume : Cone's volume

Now, perimeter of the rectangle = 264 cm.

Since, perimeter includes double the length and breadth, wh finding the sides we divide by double the sum of ratio.

Therefore, length $= \frac{264}{2(6+5)} \times 6 = 72$ cm

and breadth $= \frac{264}{2(6+5)} \times 5 = 60$ cm. Ans.

Ex. 47: A right circular cone is exactly fitted inside a cube in such a wa that the edges of the base of the cone are touching the edges of one of the faces of the cube and the vertex is on the opposite fac of the cube. If the volume of the cube is 343 cc, what **approxi mately** is the volume of the cone?

1) 80 cc 2) 90 cc 3) 110 cc 4) 105 cc 5) 100 cc

Soln: Edge of the cube $= \sqrt[3]{343} = 7$ cm

$\therefore$ Radius of cone = 3.5 cm

height = 7 cm

volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 7 = \frac{1}{3} \times 22 \times 12.25 \approx 90$ cc

Series

A **Series** is a sequence of numbers obtained by some particula predefined rule and applying that predefined rule it is possible to finc out the next term of the series.

A series can be created in many ways. Some of these ar discussed below :

(i) Arithmetic Series. An arithmetic series is one in whicl successive numbers are obtained by adding (or subtracting) a fixe number to the previous number. For example,

(i) 3,5,7,9,11,......
(ii) 10,8,6,4,2,......
(iii) 13,22,31,40,49,.....
(iv) 31,27,23,19,15,.....etc.

are arithmetic series because in each of them the next number can b obtained by adding or subtracting a fixed number. (For example, i 3,5,7,9,11, ... every successive number is obtained by adding 2 to th previous number).

(ii) Geometric Series. A geometrical series is one in whic each successive number is obtained by multiplying (or dividing) a fixe number by the previous number.

For example,

(i) 4,8,16,32,64,....
(ii) 15,-30,60,-120,240,.....
(iii) 1024,512,256,128,64,.....
(iv) 3125,-625,125,-25,5,.....

are geometric series because, in each of them, the next number can b obtained by multiplying (or dividing) the previous number by a fixe number. (For example, in : 3125,-625, 125,-25,5,... every successi number is obtained by dividing the previous number by -5.)

(iii) **Series of squares, cubes etc**. These series can be forme by squaring or cubing every successive number.

For example,

(i) 2, 4, 16, 256, ...
(ii) 3, 9, 81, 6561, ...
(iii) 2, 8, 512, ... etc.

are such series. (In the first and second, every number is squared to g the next number while in the third it is cubed).

(iv) **Mixed Series.** A mixed series is basically the one we need to have a sound practice of because it is generally the mixed series which is asked in the examination. By a mixed series, we mean a series which is created according to any non-conventional (but logical) rule. Because there is no limitation to people's imagination, there are infinite ways in which a series can be created and naturally it is not possible to club together all of them. Still we are giving examples of some more popular ways of creating these mixed series. (We shall be giving them names, which are NOT generalised and probably not found in any other book, but which are given with the purpose of clarifying their logic without difficulty).

I) Two-tier Arithmetic Series. We have seen that in an arithmetic series the difference of any two successive numbers is fixed. A Two-tier Arithmetic Series shall be the one in which the differences of successive numbers themselves form an arithmetic series.

Examples

(a) 1, 2, 5, 10, 17, 26, 37, ...

(b) 3, 5, 9, 15, 23, 33, ... etc.

are examples of such series. [In 1, 2, 5, 10, 17, 26, 37, ...; for example, the differences of successive numbers are 1, 3, 5, 7, 9, 11,... which is an arithmetic series.

Note : Two-tier arithmetic series can be denoted as a quadratic function.

For example, the above series

(a) is $0^2 + 1$, $1^2 + 1$, $2^2 + 1$, $3^2 + 1$, ... which can be denoted as

$f(x) = x^2 + 1$, where x = 0, 1, 2, ...

Similarly example (b) can be denoted as

$f(x) = x^2 + x + 3$, x = 0, 1, 2, 3, ...

II) Three-tier Arithmetic Series. This, as the name suggests, is a series in which the differences of successive numbers form a two-tier arithmetic series; whose successive term's differences, in turn, form an arithmetic series.

For example

a) 336, 210, 120, 60, 24, 6, 0, ...

is an example of three-tier arithmetic series.

[The differences of successive terms are

126, 90, 60, 36, 18, 6, ...

The differences of successive terms of this new series are

36, 30, 24, 18, 12, ...

which is an arithmetic series.]

Note : Three-tier arithmetic series can be denoted as a cubic function.

For example, the above series is (from right end) $1^3 - 1$, $2^3 - 2$, $3^3 - 3$, $4^3 - 4$, ... which can also be denoted as

$f(x) = x^3 - x$; x = 1, 2, ...

III) We know that,

i) In an arithmetic series we add (or deduct) a fixed number to find the next number, and

ii) In a geometric series we multiply (or divide) a fixed number to find the next number.

We can combine these two ideas into one to form

a) **Arithmetico-Geometric Series.** As the name suggests, in this series each successive term should be found by first adding a fixed number to the previous term and then multiplying it by another fixed number.

For example

1,6, 21, 66, 201......

is an arithmetico-geometric series. (Each successive term is obtained by first adding 1 to the previous term and then multiplying it by 3).

Note : The differences of successive numbers should be in Geometric Progression.

In this case, the successive differences are 5, 15, 45, 135, ... which are in GP.

b) **Geometrico-Arithmetic Series.** As the name suggests, a geometrico-arithmetic series should be the one in which each successive term is found by first multiplying (or dividing) the previous term by a fixed number and and then adding (or deducting) another fixed number.

For example

3, 4, 7, 16, 43, 124, ...

is a geometrico-arithmetic series. (Each successive term is obtained by first multiplying the previous number by 3 and then subtracting 5 from it.)

Note : The differences of successive numbers should be in geometric progression. In this case, the successive differences are 1, 3, 9, 27, 81, ... which are in GP.

IV) Twin Series. We shall call these twin series, because they are two series packed in one .

1, 3, 5, 1, 9, -3, 13, -11, 17, ...

is an example of twin series. (The first, third, fifth etc. terms are 1, 5, 9, 13, 17 which is an arithmetic series. The second, fourth, sixth etc. are 3, 1, -3, -11 which is a geometrico-arithmetic series in which successive terms are obtained by multiplying the previous term by 2 and then subtracting 5.)

V) Other Series. Besides, numerous other series are possible and it is impossible to even think of (let alone write them down) all of them. It is only through a lot of practice and by keeping abreast with the latest trends that one can expect to master the series.

SUGGESTED STEPS FOR SOLVING SERIES QUESTIONS

Despite the fact that it is extremely difficult to lay down all possible combinations of series, still, if you follow the following step-by-step approach, you may solve a series question easily and quickly.

Step I : Preliminary Screening

First check the series by having a look at it. It may be that the series is very simple and just a first look may be enough and you may know the next term. Some examples are given below, where preliminary screening is sufficient to tell you the next term.

Ex. i) 4, -8, 16, -32, 64, ?
ii) 1, 4, 9, 16, 25, 36, 49, ?
iii) 1, 3, 6, 10, 15, 21, ?
iv) 2,6, 18, 54, 162, ?

Answer i) Each term is multiplied by -2.
Next term : -132.
ii) The series is +3, +5, +7, +9,+11, +13, +15.
Next term: 49 + 15 = 64.
Another approach: The series is, $1^2, 2^2, 3^2$ etc.
Next term: $8^2 = 64$.
iii) The series is +2, +3, +4, +5, +6, +7.
Next term: 21 + 7 = 28.
iv) Each term is multiplied by 3.
Next term: 162 x 3 = 486.

Step II : Check Trend: Increasing / Decreasing / Alternating

If you fail to see the rule of the series by just preliminary screening you should see the trend of the series. By this we mean that you should check whether the series increases continuously or decreases continuously or whether it alternates, ie, increases and decreases alternately. For example, the series i) and ii) in the following examples are increasing, the series iii) is decreasing and the series iv) is alternating.

Ex. i) 3, 10, 21, 36, 55, 78.
ii) 5,10,13,26,29, 58.
iii) 125, 123, 120, 115, 108, 97.
iv) 253, 136, 352, 460, 324, 631, 244.

Step III (A) (to be employed if the series is increasing or decreasing) Feel the rate of increase or decrease

For an increasing (or decreasing) series, start with the first term and move onwards. You will notice that the series proceeds either arithmetically or geometrically or alternately. By an arithmetic increase, we mean that there is an increase (or decrease) of terms by virtue of addition (or subtraction). In such cases you will 'feel' that the series rises (or falls) rather slowly. By a geometric increase (or decrease) we mean that there is an increase (or decrease) of terms by virtue of multiplication (or division) or if there is addition it is of squares or of cubes. In such cases, you will 'feel' that the series rises (or falls) very sharply. By an alternative increase (or decrease) we mean that the series may be irregularly increasing or decreasing. In such cases, the rise (or fall) may be sharp then slow and then again sharp and so on.

For example, consider the series: 4, 5, 7, 10, 14, 19, 25. Here the series increases and the increase is slow. A gradual, slow increase. So you should try to test for an arithmetic type of increase. Indeed, it turns out to be a two-tier arithmetic series, the differences 1, 2, 3, 4, 5, forming a simple series.

Again, consider the series: 1, 2, 6, 15, 31, 56. Here you may immediately 'feel' that the series rises very sharply. So, you should try to test for a geometric type of increase. On trial you may see that the series is not formed by successive multiplications. So, you should check for addition of squared numbers, cubed numbers etc. Indeed the series turns out to be 1, $1+1^2$, $1+1^2+2^2$, $1+1^2+2^2+3^2$ etc. Another similar example could be of the series 1, 5, 14, 30, 55, 91. This is 1^2, 1^2+2^2, $1^2+2^2+3^2$, $1^2+2^2+3^2+4^2$ etc. Another example could be : 2, 9, 28, 65, 126, 217. This is : 1^3+1, 2^3+1, 3^3+1 etc.

[Note: We have seen that there may be two ways in which a geometric increase (or decrease) may take place. In one case it is because of multiplications (or divisions) by terms and in other case it is because of addition (or substraction) of squared or cubed terms. How do we differentiate between the two? We can differentiate between the two by looking at the trend of the increase. If the increase is because of addition of squared or cubed terms, the increase will be not very sharp in the later terms (fourth, fifth,

sixth terms etc.) For example, watch the series: 1, 2, 6, 15, 31, 56. Here the series appears to rise very steeply: 1, 1 × 2=2, 2 × 3=6, 6 × 2.5=15, 15 × 2 ≈ 31, 31 × (1.something) = 56. Thus we see multiplications are by 2, 3, 2.5, 2, respectively. That is, the rise is very sharp initially but later it slows down. The same can be said to be true of the series: 1, 5, 14, 30, 55, 91. Here, 1 × 5=5, 5 × 3 ≈ 14, 14 × 2 ≈ 30, 30 × 1.8 ≈ 55, 55 × 1.6 ≈ 91. Here too, the rise is very sharp initially, but later it slows down. **In such cases, therefore, where the rise is very sharp initially but slows down later on, you should check for addition of squared or cubed numbers.]**

As our next example, consider the series : 3, 5, 11, 25, 55, 117. We see that this series, too, rises very sharply. Hence, there must be a geometric type of increase. Further, the rate of increase does not die down in later terms. In fact, it picks up as the series progresses. Hence, this time the geometric increase should be of the first kind, i.e., through multiplication. The series must be formed by multiplications by 2 and some further operation. Now it is easy. A little more exercise will tell us that the series is : × 2-1, × 2+1, × 2+3, × 2+5, × 2+7 etc. Another and similar example could be : 7, 8, 18, 57, 232, 1165. Here the series is × 1+1, × 2+2, × 3+ 3, × 4+ 4, × 5+ 5.

As our last example we will take up a series which shows an alternating increase. In such cases there are two possibilities: one, that two different series may be intermixed or the other, that two different kinds of operations may be being performed on successive terms. To understand this, let us see the following examples. Consider the series: 1, 3, 5, 10, 14, 29, 30, 84. You can see that this series increases gradually and hence it is an increasing series but the increase, in itself, is irregular, haphazard. In fact, it is a mix of two series: 1, 5, 14, 30 which is a series: 1, $1+2^2$, $1+2^2+3^2$, $1+2^2+3^2+4^2$; and the other series: 3, 10, 29, 84 which is another series: × 3+ 1, × 3-1, × 3-3 etc. Again, consider the series: 3, 13, 20, 84, 91, 459. This is also an increasing series with a haphazard increase (alternating increase) with sharp and then slow rises coming alternately. Here, two different kinds of operations are being performed alternately: the first operation is that of multiplication by 3, 4, 5 successively and adding a constant number 4 and the second operation is that of adding 7. Hence the series is : × 3+4, + 7, × 4+4, +7, × 5+4.

Step III (B) (to be employed if the series is neither increasing nor decreasing but alternating) Check two possibilities

For an alternating series, where the terms increase and decrease alternately, the rules remain, more or less, the same as those for a series showing alternating increase.

(Note: Please note the *difference* between *an alternating increase* and *a series having alternating increase* carefully. In an alternating increase terms increase, decrease alternately. But a series having alternating increase increases continiously [and on having alternating decrease decreases continuously]. The increase may be haphazard and irregular - alternately, sharp and slow - but the increase is continuous. For example, 15, 22, 20, 27, 25 is an alternating series because there is increase and decrease in terms, alternately. On the other hand, 1, 3, 5, 10, 14, 29, 30 is an increasing series having alternating increase.)

For an alternating series you should check for two possibilities: One, that the series may be a mix of two series (twin series) and two, that two different kinds of operations may be going on. For example, consider the series: 4, 8, 6, 12, 9, 16, 13. This is an alternating series. It is a mix of two simple series: 4, 6, 9, 13 and 8, 12, 16 etc. Again, consider the series: 800, 1200, 600, 1000, 500, 900. Here, two different kinds of operations are going on. One, addition of 400 and two, division by 2.

A SUMMARY OF THE THREE STEPS [Very Important]

Step I : Do a preliminary screening of the series. If it is a simple series you will be able to solve it easily.

Step II: If you fail in preliminary screening then determine the trend of the series. Determine whether it is increasing, decreasing or alternating.

Step III (A): Perform this step only if a series is increasing or decreasing. Use the following rules:

i) If the rise of a series is slow or gradual, the series is likely to have an addition-based increase; successive numbers are obtained by adding some numbers.

ii) if the rise of a series is very sharp initially but slows down later on, the series is likely to be formed by adding squared or cubed numbers.

iii) if the rise of a series is throughout equally sharp, the series is likely to be multiplication-based; successive terms are obtained by multiply-

ing by some terms (and, maybe, some addition or subtraction could be there, too.)

iv) if the rise of a series is irregular and haphazard there may be two possibilities. Either there may be a mix of two series or two different kinds of operations may be going on alternately. (The first is more likely when the increase is very irregular: the second is more likely when there is a pattern, even in the irregularity of the series).

Step III (B): (to be performed when the series is alternating)
[Same as (iv) of step (iii). Check two possibilities]

Some solved examples -

Ex: Find the next number of the series

i) 8,14,26,50,98,194	ii) 8,8,9,9,11,10,14,11
iii) 325,259,204,160,127,105	iv) 54,43,34,27,22,19
v) 824,408,200,96,44,18	vi) 16,17,21,30,46,71
vii) 3,3,6,18,72,360	viii) 3,4,8,17,33,58
ix) 6,16,36,76,156,316	x) -2,4,22,58,118,208

Solutions:

i) Sharp increase and terms roughly doubling every time. On checking with 2 as multiple the series is:
next term = previous term × 2 -2. Next term = 382.

ii) Irregular. Very irregular. Likely to be, therefore, mixed. On checking it is a mix of two series:
8, 9, 11, 14, (+1,+2,+3 etc.) and 8, 9, 10, 11.
Next term =14+4=18.

iii) Gradual slow decrease. Likely to be arithmetical decrease. Check the differences of successive terms. They are: 66, 55, 44, 33, 22. Hence, next decrease will be : 11.
Next term= 105-11= 94.

iv) Gradual slow decrease. Likely to be arithmetical decrease. Check differences. They are 11, 9, 7, 5, 3. Hence, next decrease will be 1.
Next term =19-1= 18.

v) Sharp decrease and terms roughly being halved everytime. Checking with 2 as divisor the series is:
Next term = (previous term - 8) ÷ 2. Next term = 5.

vi) Preliminary screening tells us that each term is obtained by adding $1^2, 2^2, 3^2, 4^2, 5^2$ ...., respectively.
Next term = 71 + 6^2= 107

vii) Sharp increase. The series is: × 1, × 2, × 3, × 4, × 5, ... Next term =360 × 6 = 2160

viii) Sharp increase that slows down later on. (Ratios of successive terms rise sharply from 4 ÷3=1.3 to 8 ÷ 4 = 2 to 17 ÷ 8 =2.125 and then start falling to 33÷ 17≈ 1.9 and then to 58÷ 33 ≈ 1.8). Hence likely to be addition of squared or cubed numbers. On checking the series is : $+1^2, +2^2, +3^2, +4^2, +5^2$, ... Next term = 58 + 6^2 = 94.

ix) Sharp increase with terms roughly doubling each time. Likely to have geometrical nature with 2 as multiple. On checking the series is: ×2 + 4. Next term = 316 × 2 + 4 = 636

x) Series increases sharply but then its speed of rise slows down. Likely to be addition of squared or cubed numbers. On checking, the series is: $1^3 - 3, 2^3 - 4, 3^3 - 5, 4^3 - 6$ ..... Next term =$7^3 - 9$ = 334

FINDING WRONG NUMBERS IN A SERIES

In today's examinations, a series is more likely to be given in the format of a complete series in which an incorrect number is included. The candidate is required to find out the wrong number.

Obviously, finding the wrong number in a series is very easy once you have mastered the art of understanding how the series is likely to be formed. On studying a given series and applying the concepts employed so far you should be able to understand and thus "decode" the formation of the series. This should not prove very difficult because usually six terms are given and it means that at least five correct terms are given. This should be sufficient to follow the series.

We are giving below some solved examples on this particular type where you are required to find out the wrong numbers in a series:

SELECTED NUMBER SERIES (ASKED IN PREVIOUS EXAMS)

Which Of The Following Does Not Fit In The Series ?

1) 2, 6, 12, 27, 58, 121, 248	2) 3,9,18,54,110,324,648
3) 1,1.5,3,6,22.5,78.75,315	4) 190,166,145,128,112,100,91
5) 895,870,821,740,619,445,225	6) 1,2,6,21,86, 445,2676
7) 864,420,200,96,40,16,6	8) 4,12,30,68,146,302,622
9) 7,10,12,14,17,19,22,22	10) 196,168,143,120,99,80,63
11) 258,130,66,34,18,8,6	12) 2,6,24,96,285,568,567
13) 6072,1008,200,48,14,5,3	14) 2,1,10,19,14,7,16
15) 318,368,345,395,372,422,400,449	16) 2807,1400,697,347,171,84,41,20
17) 824,408,396,96,44,18,5	18) 5,7,13,25,45,87,117
19) 2185,727,241,79,30,7,1	20) 2,3,10,15,25,35,50,63

21) 2,7,28,60,126,215,344

22) 0,4,19,48,100,180,294

23) 1,2,7,34,202,1420

24) 823, 734, 645, 556, 476, 37
289

25) 1,4,11,34,102,304,911

26) 5, 8, 20, 42, 124, 246, 736

27) 13700, 1957, 326, 65, 16, 6, 2

28) 1, 1.5, 3, 20.25, 121.5, 911.25, 8201.25

29) 3, 6, 10, 20, 33, 62, 94

30) 0, 6, 23, 56, 108, 184, 279

31) 1, 2, 6, 12, 66, 197, 786

32) 1, 2, 6, 144, 2880, 86400, 3628800

33) -1, 5, 20, 59, 119, 209, 335

34) 1, 2, 4, 8, 15, 60, 64

35) 49, 56, 64, 71, 81, 90, 100, 110

36) 1, 3, 10, 29, 74, 172, 382

37) 25, 26, 24, 29, 27, 36, 33

38) 36, 54, 18, 27, 9, 18.5, 4.5

39) 144, 132, 125, 113, 105, 93, 84, 72, 61, 50

40) 3, 9, 36, 72, 216, 864, 172
3468

41) 1, 1, 1, 4, 2, 1, 9, 5, 1, 16

ANSWERS

1) 6;($2\times2+1=5$; $5\times2+2=12$; $12\times2+3=27$; $27\times2+4=58$; and so on)
2) 110;(Multiply by 3 and 2 alternately)
3) 6;($1\times1.5=1.5$; $1.5\times2=3$; $3\times2.5=7.5$; $7.5\times3=22.5$;-------------)
4) 128;(190-24=166; 166-21=145; 145-18=127; 127-15=112;----)
5) 445;(reduce the successive numbers by $5^2, 7^2, 9^2, 11^2$,------)
6) 86;($1\times1+1=2$; $2\times2+2=6$; $6\times3+3=21$; $21\times4+4=88$;---------)
7)96;(Start from right end; 2(6+2)=16; 2(16+4)=40; 2(40+6)=92; 2(92+8)=200 ----------)
8) 302;(Add 8,18, 38, 78,158 and 318 to the successive numbers)
9) 19;(There are two series; S_1=7,12,17,22; S_2=10,14,18,22)
10) 196;(Add 17,19,21,23,---------to the successive numbers from RE)
11) 8;(Add 4,8,16,32,64,128 to the successive numbers from RE)
12) 24;($2\times6-6=6$; $6\times5-5=25$; $25\times4-4=96$; $96\times3-3=285$;----------)
13) 1008;(From RHS ;$3\times1+2=5$; $5\times2+4=14$; $14\times3+6=48$; $48\times4+8=200$; $200\times5+10=1010$)
14) 19;($2\div2=1$; 1+9=10; $10\div2=5$; 5+9=14; $14\div2=7$; 7+9=16)
15) 400;(There are two series; S_1 =318+27=345; 345+27 =372; 372+27 =399;
S_2=368+27=395; 395+27=422;----)
16)347;($20\times2+1=41$; $41\times2+2=84$; $84\times2+3=171$;------)

17) 396;[(824-8) ÷2 =408; (408-8) ÷ 2 =200; (200-8) ÷2=96;--------]
18) 87;(Add 2,6,12,20,30 and 42 to the successive numbers)
19) 30;[(2185-4)÷ 3=727; (727-4)÷ 3=241; (241-4)÷ 3=79;----------]
20) 25;[$1^2+1=2$; $2^2-1=3$; $3^2+1=10$; $4^2-1=15$; $5^2+1=26$;-------------]
21) 60;[$1^3+1=2$; $2^3-1=7$; $3^3+1=28$; $4^3-1=63$;--------------]
22) 19;[$1^3-1^2=0$; $2^3-2^2=4$; $3^3-3^2=18$; $4^3-4^2=48$;$5^3-5^2=100$;----------]
23) 202;[$1\times2-1=1$; $1\times3-1=2$; $2\times4-1=7$; $7\times5-1=34$; $34\times6-1=203$;----]
24) 476;[Hundred-digit of each number is decreasing by one and unit- and ten-digits are increasing by one.]
25) 102;[$1\times3+1=4$; $4\times3-1=11$; $11\times3+1=34$--------]
26) 20; [Series is $\times2-2$, $\times3-2$, $\times2-2$, $\times3-2$, ...]
27) 6; [Series is $-1\div7$, $-1\div6$, $-1\div5$, $-1\div4$, $-1\div3$, ...]
28) 3; [Series is $\times1.5$, $\times3$, $\times4.5$, $\times6$, $\times7.5$, $\times9$]
29) 33; [Series is $\times2$, $\times1.5+1$, $\times2$, $\times1.5+1$, $\times2$, $\times1.5+1$]
30) 108; [Series is 1^3-2^0, 2^3-2^1, 3^3-2^2, 4^3-2^3, 5^3-2^4 ......]
31) 12; [Series is $\times3-1$, $\times4-2$, $\times3-1$, $\times4-2$, ...]
32) 6; [Series is $\times1\times2$, $\times2\times3$, $\times3\times4$, $\times4\times5$, $\times5\times6$, ...]
33) 20; [Series is 1^3-2, 2^3-3, 3^3-4, 4^3-5, 5^3-6, ...]
34) 8; [Series is $\times2$, +2, $\times3$, +3, $\times4$, +4, ...]
35) 71; [Series is 7^2, 7^2+7, 8^2, 8^2+8, 9^2, 9^2+9,......]
36) 172; [Series is $\times2+1$, $\times2+4$, $\times2+9$, $\times2+16$, $\times2+25$, ...]
37) 24; [Series is $+1^2$, -1, $+2^2$, -2, $+3^2$, -3,........]
38) 18.5; [Series is $\times1.5$, $\div3$, $\times1.5$, $\div3$, $\times1.5$, $\div3$]
39) 61; [Series is -12, -7, -12, -8, -12, -9, -12, -10, -12,....]
40) 3468; [Series is $\times3$, $\times4$, $\times2$, $\times3$, $\times4$, $\times2$, $\times3$,.....]
41) 5; [Series is $1^2, 1^1, 1^0, 2^2, 2^1, 2^0, 3^2, 3^1, 3^0, 4^2$,......]

Some Unique Series: These series may be asked in examinations, so you must be aware of them.

I. Series of Date or Time:

1) Which of the following doesn't fit into the series?
5-1-96, 27-1-96, 18-2-96, 12-3-96, 2-4-96

Soln: Each successive date differs by 22 days. If you recall that 96 is a leap year, you will find that 12-3-96 should be replaced by 11-3-96.

2) Which of the following doesn't fit into the series?
5.40, 8.00, 10.20, 12.30, 3.00, 5.20

Soln: Each successive time differs by 2 hrs 20 minutes. So 12.30 should

be replaced by 12.40.

Note: Keep in mind that the problem of series may be based on dates times. Sometimes it doesn't strike our mind and the question is solv wrongly.

II. Fractional series:

Which of the following doesn't fit into the series?

1) $\frac{4}{5}, \frac{7}{15}, \frac{1}{15}, -\frac{1}{5}, -\frac{8}{15}$

Soln: Whenever you find that most of the fractions have the same denominators, change all the denominators to the same value. F example, in this question, the series becomes:

$\frac{12}{15}, \frac{7}{15}, \frac{1}{15}, -\frac{3}{15}, -\frac{8}{15}$

Now, it is clear that numerators must decrease successively b 5. Therefore, $\frac{1}{15}$ should be replaced by $\frac{2}{15}$.

Note: The above method is useful when the fractional values are decreased by a constant value (a constant fraction). In this cas the values are decreased by $\frac{5}{15}$ or $\frac{1}{3}$.

2) $\frac{4}{5}, \frac{23}{35}, \frac{18}{35}, \frac{12}{35}, \frac{8}{35}, \frac{3}{35}$

Soln: By the above rule if we change all the fractions with the same denominators, the series is $\frac{28}{35}, \frac{23}{35}, \frac{18}{35}, \frac{12}{35}, \frac{8}{35}, \frac{3}{35}$.

We see that numerators decrease by 5, thus $\frac{12}{35}$ should be replace by $\frac{13}{35}$.

Now, we conclude that the above fractions decrease successivel by $\frac{5}{35}$ or $\frac{1}{7}$.

3) $\frac{118}{225}, \frac{100}{199}, \frac{82}{173}, \frac{66}{147}, \frac{46}{121}, \frac{28}{95}$

Soln: We see that all the denominators differ, so we can't use the above rule. In this case usually, the numerators and denominators change in a definite pattern. Here, numerators decrease successively by 18 whereas denominators decrease successively by 26. Thus $\frac{66}{147}$ should be replaced by $\frac{64}{147}$.

4) $\frac{12}{89}, \frac{15}{86}, \frac{18}{82}, \frac{21}{80}, \frac{24}{77}, \frac{27}{74}$

Soln: Numerators increase successively by 3 whereas denominators decrease successively by 3. Thus $\frac{18}{82}$ should be replaced by $\frac{18}{83}$.

Note: More complicated questions based on fractions are not expected in the exams because it is not easy to find the solution in complicated cases.

III. Some numbers followed by their LCM or HCF

1) 1, 2, 3, 6, 4, 5, 6, 60, 5, 6, 7, ... (Fill up the blank)

Soln : The series can be separated in three parts. 1, 2, 3, 6/ 4, 5, 6, 60/ 5, 6, 7 .... In each part fourth number is LCM of first three numbers. Thus the answer should be 210.

2) 8, 6, 24, 7, 3, 21, 5, 4, 20, ..., 9, 18

1) 1 2) 3 3) 4 4) 5 5) 6

Soln : 8, 6, 24/ 7, 3, 21/ 5, 4, 20/_ , 9, 18

Third number in each part is LCM of first two numbers. Thus, the answer should be 6.

3) 8, 4, 4, 7, 8, 1, 3, 9, 3, 2, 1, ...

1) 1 2) 2 3) 3 4) 5 5) None of these

Soln : 8, 4, 4/ 7, 8, 1/ 3, 9, 3/ 2, 1 ...

In each part, third number is HCF of first to numbers. Thus our answer should be 1.

IV. Some numbers followed by their product

1) 2, 3, 6, 18, 108, 1844

Which of the above numbers does not fit into the series?

Soln : $2 \times 3 = 6$

$3 \times 6 = 18$

$6 \times 18 = 108$

$18 \times 108 = 1944$

Thus, 1844 is wrong.

2) 5, 7, 35, 8, 9, 72 , 11, 12, 132, _, 3, 6. Fill up the blank.

Soln : 5, 7, 35/ 8, 9, 72/ 11, 12, 132/ 2, 3, 6

In each group third number is the multiplication of first and second. Thus our answer is 2.

V. By use of digit-sum

1) 14, 19, 29, 40, 44, 51, 59, 73

Which of the above numbers doesn't fit into the series?

Soln : Next number = Previous number + Digit-sum of previous num

Like, 19 = 14 + (4 + 1)

29 = 19 + (1 + 9)

40 = 29 + (2 + 9)

Thus, we see that 51 should be replaced by 52.

2) 14, 5, 18, 9, 22, 4, 26, 8, 30, 3, __, __. Fill up the blanks.

Soln : 1st, 3rd, 5th, 7th, ... numbers follow the pattern of +4 (14 +
18, 18 + 4 = 22, ...). Whereas 2nd, 4th, 6th are the digit-sum
their respective previous number (5 = 1 + 4, 9 = 1 + 8), ...) Th
our answer is 34 and 7.

VI. Odd-number out: Sometimes a group of numbers is written ou
which one is different from others.

1) 22, 44, 88, 132, 165, 191, 242. Find the number which doesn't fit
the above series (or group).

Soln : 191; Others are divisible by 11 or 191 is the single prime numb

2) Which one of the following series doesn't fit into the series?

29, 31, 37, 43, 47, 51, 53

Soln : 51; All other are prime numbers.

A note on Arithmetic Progressions. Arithmetic progressi
is basically the arithmetic series.

A succession of numbers is said to be in Arithmetic Progressi
(A.P.) if the difference between any term and the term preceding it
constant throughout. This constant is called the common difference (c.
of the **A.P.**

To find the nth term of an A.P. Let the first term of an A.P.
a and the common difference be d.

Then the A.P. will be a, a+d, a+2d, a+3d, ...

Now first term $t_1 = a = a + (1-1)d$

second term $t_2 = a + d = a + (2-1)d$

third term $t_3 = a + 2d = a + (3-1)d$

fourth term $t_4 = a + 3d = a + (4-1)d$

fifth term $t_5 = a + 4d = a + (5-1)d$

Proceeding in this way, we get nth term $t_n = a + (n-1)d$

Thus nth term of an A.P. whose first term is a and commo
difference is d is given by $t_n = a + (n-1)d$

SOME SOLVED EXAMPLES

Example 1. Find the first five terms of the sequence for which $t_1=1, t_2=2$ and $t_{n+2}=t_n+t_{n+1}$.

Solution : Given, $t_1 = 1$, $t_2 = 2$, $t_{n+2} = t_n+t_{n+1}$

$\therefore$ Putting n = 1, we get $t_3 = t_1+t_2 = 1+2 = 3$

n = 2, we get $t_4 = t_2+t_3 = 2+3 = 5$

n = 3, we get $t_5 = t_3+t_4 = 3+5 = 8$

Thus the first five terms of the given sequence are 1,2,3,5 and 8.

Example 2. How many terms are there in the A.P. 20,25,30,...100?

Solution: Let the number of terms be n.

Given $t_n = 100$, a = 20, d = 5, we have to find n.

Now $t_n = a+(n-1)d$ $\therefore$ 100 = 20+ (n-1)5

or 80 = (n-1)5 or, n-1 = 16 $\therefore$ n = 17.

Example 3. A person was appointed in the pay scale of Rs. 700-40-1500. Find in how many years he will reach maximum of the scale.

Solution: Let the required number of years be n.

Given $t_n = 1500$, a = 700, d = 40, to find n.

$\therefore t_n = a+(n-1)d$

$\therefore$ 1500 = 700+(n-1)40

or, (n-1)40 = 800 or, n-1 = 20 or, n = 21.

Two-line number series

Nowadays this type of number series is also being asked in examinations.

In this type of no. series one complete series is given while the other is incomplete. Both the series have the same definite rule. Applying the very definite rule of the complete series, you have to determine the required no. of the incomplete series. For example:

Ex. 1: 4 14 36 114 460

2 a b c d e

Find the value of e.

Soln: The first series is ×1 + 10, ×2 + 8, ×3 + 6, ×4 + 4, ...

$\therefore$ a = 2 × 1 + 10 = 12, b = 12 × 2 + 8 = 32, c = 32 × 3 + 6 = 102,
d = 102 × 4 + 4 = 412, and finally e = 412 × 5 + 2 = 2062

Ex. 2: 5 6 11 28 71 160

2 3 a b c d e

What is the value of e?

Soln: The differences of two successive terms of the first series are 1
17, 43, 89, the sequence of which is $0^3 + 1^2$, $1^3 + 2^2$, $2^3 +$
$3^3 + 4^2$, $4^3 + 5^2$.

$\therefore$ a = 3 + 5 = 8, b = 8 + 17 = 25, c = 25 + 43 = 68, d = 68 + 8
= 157, and finally e = 157 + ($5^3 + 6^2$ = 125 + 36 =) 161 = 318

Ex. 3: 1296 864 576 384 256
1080 a b c d e
What should replace c?

Soln: The first series is $\div 3 \times 2$

$\therefore$ a = 1080 ÷ 3 × 2 = 720, b = 720 ÷ 3 × 2 = 480, and finally c = 480 ÷ 3 × 2 = 320

Ex. 4: 7 13 78 83 415
3 a b c d e
Find the value of b.

Soln: The first series is +6, ×6, +5, ×5

$\therefore$ a = 3 + 6 = 9 and b = 9 × 6 = 54

Ex. 5: 3240 540 108 27 9
3720 a b c d e
What is the value of d?

Soln: The first series is ÷6, ÷5, ÷4, ÷3

$\therefore$ a = 3720 ÷ 6 = 620, b = 620 ÷ 5 = 124, c = 124 ÷ 4 = 31, and finally d = 31 ÷ 3 = 10.33

Ex. 6: 27 44 71 108 155
34 a b c d e
What value should replace e?

Soln: The differences of two successive terms of the series are 17, 27, 37, 47.

$\therefore$ a = 34 + 17 = 51, b = 51 + 27 = 78, c = 78 + 37 = 115,
d = 115 + 47 = 162, and finally e = 162 + 57 = 219

Ex. 7: 108 52 24 10 3
64 a b c d e
What is the value of c?

Soln: The series is -4 ÷ 2

$\therefore$ a = (64 - 4) ÷ 2 = 30, b = (30 - 4) ÷ 2 = 13,
c = (13 - 4) ÷ 2 = 4.5

Ex. 8: -4 -2 1 8 31
-1 a b c d e
Find the value of b.

Soln: The series is repeated as ×2 + 6 and ×3 + 7 alternately.

$\therefore$ a = -1 × 2 + 6 = 4 and b = 4 × 3 + 7 = 19

Ex. 9: 5 8 41 33 57 42 61
3 4 a b c d e
Find the value of d.

Soln: This is an alternate number series having two series:

S_1 = 5 41 57 61.
The differences between two successive terms are 36 (= 6^2), 16 (= 4^2), 4 (= 2^2); and
S_2 = 8 33 42
The differences between two successive terms are 25 (= 5^2), 9 (= 3^2)

$\therefore$ b = 4 + 25 = 29 and d = 29 + 9 = 38

Remember: In such type of series the first and the second term of the two series may and may not have the similar relationship. As here, for the first series 8 - 5 = 3 but for the second series 4 - 3 = 1 ≠ 3. However, the series 3 a c e will always follow the same property as that of the series S_1 and the series 4 b d will always follow the same property as that of the series S_2.

Ex. 10: 1 3 2 10 4 28
2 a b c d e
What is the value of e?

Soln: This series is of grouping-type. Here we consider each two terms of the series separately and each group separately. That is, for the first series: the first group g_1 = 1 and 3; g_2 = 2 and 10; g_3 = 4 and 28. Here for the two numbers of each group we have to find the relevant property. For example g_1, holds the property ×3, g_2 holds the property ×5 and g_3 holds the property ×7.

The property of multiplication by 3, 5 and 7 is a relevant property. Here, if we consider these groups in the way that the differences between the two numbers of the groups are 2, 8 and 24, It is not as relevant as the former property of multiplication by 3, 5, and 7. After determining the property between the two numbers of each group, to determine the property between the groups we consider the first numbers only of each group in the fashion 1, 2 and 4.

The property is ×2.

Now, we directly conclude

e = 7 × d

and $b = 2 \times 2 = 4$ and $d = 2 \times 4 = 8$
Thus, $e = 7 \times 8 = 56$.

Note: When the alternate no. series fails to determine the property of the given series, then the grouping type of series is applied. Here, for a moment, if we consider for alternate no. series, we get
$S_1 = 1\ \ 2\ \ 4$. The property is $\times 2$
$S_2 = 3\ \ 10\ \ 28$. From merely these three numbers it is not proper to say that S_2 holds a property of $\times 3 + 1$ and $\times 3 - 2$ (as $3 \times 3 + 1 = 10$ and $10 \times 3 - 2 = 28$) or it holds the property of 3, $3^2 + 1$ and $3^3 + 1$ (as in this very case 3 should be replaced by $3^1 + 1$ i.e. 4).
Thus we observe that the property of the given series cannot be obtained by applying the method of the alternate no. series. So we proceed for the method of the grouping no. series.

Ex. 11: 220 96 347 77 516 60 733
68 a b c d e
What is the value of d?

Soln: Clearly, this no. series is of the type of alternate no. series. So, to find out the value of d, we are only concerned about the series
$S_1 = 220\ \ 347\ \ 516\ \ 733$
We observe that $220 = 6^3 + 4$, $347 = 7^3 + 4$, $516 = 8^3 + 4$, $733 = 9^3 + 4$
Now, we get $(68 - 4)^{1/3} = (4^3)^{1/3} = 4$
So, $b = (4 + 1 =)\ 5^3 + 4 = 129$ and $d = (5 + 1 =)\ 6^3 + 4 = 220$

Ex. 12: 2 5 17.5 43.75 153.125
1 a b c d e
Find the value of c.

Soln: The series is $\times 2.5, \times 3.5, \times 2.5, \times 3.5, ...$
$\therefore a = 1 \times 2.5 = 2.5$, $b = 2.5 \times 3.5 = 8.75$ and $c = 8.75 \times 2.5 = 21.875$
Here, after finding out the property of the given series as the direct repeated multiplication by 2.5 and 3.5 (the series is not of the type $\times m \pm n$ that is, $\times 2.5 + 2$, $\times 3.5 - 6$, $\times 3 - 2$ etc.), we also observe that 1, the first no. of the second series is half of 2, the first no. of the first series. So, without finding a and b, we can directly find out c as it is equal to half of the corresponding number of the first series. i.e. $c = \frac{43.75}{2} = 21.875$

Ex. 13: 3 6 24 72 144 576
1 a b c d e
What value should replace e?

Soln: The series is $\times 2, \times 4, \times 3, \times 2, \times 4, ...$
$\therefore a = 1 \times 2 = 2$, $b = 2 \times 4 = 8$, $c = 8 \times 3 = 24$, $d = 24 \times 2 = 48$, $e = 48 \times 4 = 192$
The property of the first series is direct repeated multiplication by 2, 4 and 3.
So, we can find out e directly as e = one-third of the corresponding number of the first series, i.e. $\frac{576}{3} = 192$

Ex. 14: 575 552 533 518 507
225 a b c d e
Find the value of e.

Soln: The difference of the successive terms of the first series are 23, 19, 15, 11.
$\therefore a = 225 - 23 = 202$, $b = 202 - 19 = 183$, $c = 183 - 15 = 168$, $d = 168 - 11 = 157$, and finally $e = 157 - (11 - 4 =)\ 7 = 150$.

Note: When the series holds the property of the difference of the successive terms, you can directly proceed as follows:
Difference between the first terms of the two series
$= 575 - 225 = 350$
$\therefore$ d = corresponding number of the first series i.e.
$507 - 350 = 157$
And then we have $e = 157 - (11 - 4) = 150$.

Ex. 15: 15 31 11 23 5 11
21 43 a b c d e
What is the value of d?

Soln: As the numbers are regularly increasing and then decreasing so you can consider for the alternate no. series in the way:
$S_1 = 15\ \ 11\ \ 5$; the difference of the successive terms are 4 and 6 and $S_2 = 31\ \ 23\ \ 11$; the difference of the successive terms are 8 $(= 4 \times 2)$ and 12 $(= 6 \times 2)$
Now, in order to determine the value of d, we have to consider S_2 for the second given series as 43 b d.
$\therefore b = 43 - 8 = 35$ (As the numbers of S_1 and S_2 for the first given series are continuously decreasing, we cannot have the difference of the successive term = 8 as $b = 43 + 8 = 51$)
Finally, $d = b - 12 = 35 - 12 = 23$.

Note: Here, if we apply the process of grouping type no. series, for the first given series: $g_1 = 15, 31$, $g_2 = 11, 23$; $g_3 = 5, 11$.
The property between the numbers of each group is $\times 2 + 1$.
For the second given series: $g_1 = 21, 43$; the property where is also $\times 2 + 1$.
Now, the first numbers of the groups are 15, 11, 5; the property is -4, -6, -8, ...
$\therefore$ a = 21 - 4 = 17
and c = 17 - 6 = 11 and then d = 11 × 2 + 1 = 23.
Thus, we get the same result.

Ex. 16: 5 17 13 41 29 89 61
3 11 a b c d e
What is the value of e and d?

Soln: S_1 = 5 13 29 61, the property is ×2 + 3
S_2 = 17 41 89, the property is ×2 + 7
In order to determine the value of e, we are only concerned with the series S_1 for the second given series as 3 a c e.
$\therefore$ a = 3 × 2 + 3 = 9, c = 9 × 2 + 3 = 21 and e = 21 × 2 + 3 = 45.
Also, in order to determine the value of d, we are only concerned with the series S_2 for the second given series as 11 b d.
$\therefore$ b = 11 × 2 + 7 = 29 and d = 29 × 2 + 7 = 65
Thus e = 45 and d = 65

Note: If we solve this sum by the the process of grouping no. series:
For the first given series: $g_1 = 5, 17$; g_2 13, 41; $g_3 = 29, 89$; the property is ×3 + 2.
Also for the second given series $g_1 = 3, 11$. The property is ×3 + 2.
Now the first numbers of the groups are 5, 13, 29, 61; the property is ×2 + 3.
$\therefore$ a = 3 × 2 + 3 = 9 and c = 9 × 2 + 3 = 21 and
e = 21 × 2 + 3 = 45.
d = c × 3 + 2, i.e. 21 × 3 + 2 = 65
Thus, we get the same result. However, the grouping process fails in the previous solved questions 9 and 11.
You can check it yourself.
We finally suggest you to apply the process of alternate series first and only if it fails to serve the purpose, then proceed for grouping-type number series.

Ex. 17: 9 19 39 79 159
7 a b c d e
What is the value of e?

Soln: First method: The series is ×2 + 1, i.e. 9 × 2 + 1 = 19, 19 × 2 + 1 = 39, 39 × 2 + 1 = 79, and 79 × 2 + 1 = 159
$\therefore$ a = 7 × 2 + 1 = 15, b = 15 × 2 + 1 = 31, c = 31 × 2 + 1 = 63, d = 63 × 2 + 1 = 127, and finally e = 127 × 2 + 1 = 255

Other method: The difference between the successive terms of the first series are (19 - 9 =)10, (39 - 19 =) 20, (79 - 39 =) 40 and (159 - 79 =) 80. These numbers are in geometric progression having common ratio = 2. It is obviously a systematic sequence of numbers. Applying this very property for the second series, we get
a = 7 + 10 = 17, b = 17 + 20 = 37, c = 37 + 40 = 77, d = 77 + 80 = 157 and e = 157 + (2 × 80 =) 160 = 317
Here we see that the values of each of a, b, c, d and e is entirely different from the values obtained by the first method. Both the methods have their respective systematic properties, but which of the two has to be applied depends on the provided options.
In such a case, in exams, you have to answer according to the suitability of the given options.

Note: Whenever the chain rule is single throughout the series of the type ×m ± n(where m and n are integers, e.g. ×2 + 1, ×2 - 3, ×4 + 6, ×3 + 7, etc.) this difference of answers will come; so be cautious. In the chain rule when it is not single (e.g. × 2 + 1 and then ×2 - 1 alternately, ×3 + 2 and then ×2.5 alternately etc, or ×2 + 1, ×2 + 3, ×2 + 5, ..., ×3 -7, ×3 - 14, ×3 - 21, ..., ×3, ×2, ×4 and again ×3, ×2, ×4 etc.) this difference will not appear.

Directions (Ex. 18-22): In each of the following questions, a number series is established if the positions of two out of the five marked numbers are interchanged. The position of the first unmarked number remains the same and it is the beginning of the series. The earlier of the two marked numbers whose positions are interchanged is the answer. For example, if an interchane of number of marked '1' and the number marked '4' is required to establish the series, your answer is '1'. If it is not necessary to interchange the position of the numbers to establish the series, give 5 as your answer. Remember that when the series is established, the numbers change

from left to right (i.e. from the unmarked number to the last marked number) in a specific order.

Ex. 18: 17 16 15 13 7 -17
(1) (2) (3) (4) (5)

Soln: 5; The series is: -0!, -1!, -2!, -3! ....

Ex. 19: 2 1 195 9 40 4
(1) (2) (3) (4) (5)

Soln: 2; The series is: $\times 1 - 1$, $\times 2 + 2$, $\times 3 - 3$, $\times 4 + 4$ ....
Replace (2) with (4).

Ex. 20: 16 15 29 343 86 1714
(1) (2) (3) (4) (5)

Soln: 3; The series is: $\times 1 - 1^2$, $\times 2 - 1^2$, $\times 3 - 1^2$, $\times 4 - 1^2$, ...
Replace (3) with (4).

Ex. 21: 1728 1452 1526 1477 1607 1443
(1) (2) (3) (4) (5)

Soln: 1; The series is: -11^2, -9^2, -7^2, -5^2, ...
Replace (1) with (4).

Ex. 22: 1 1 1 2 8 4
(1) (2) (3) (4) (5)

Soln: 4; The series is: 1, 1^2, 1^3, 2, 2^2, 2^3, ...
Replace (4) with (5).

Questions asked in Previous Years Exams

BSRB Chennai PO held on July 1998

Directions (Q. 1-5): In each of the following questions a number series is given. After the series, below it, a number is given followed by (a), (b), (c), (d) and (e). You have to complete the series starting with the given number following the sequence for the given series. Then answer the questions given below it.

1. 18 22 38 74
121 (a) (b) (c) (d) (e)
Which of the following numbers will come in place of (c)?
1) 141 2) 125 3) 341 4) 177 5) 241

2. 4 7 24 93
2 (a) (b) (c) (d) (e)
Which of the following numbers will come in place of (d)?
1) 12 2) 230 3) 3 4) 51 5) 1205

3. 4 2 2 3
12 (a) (b) (c) (d) (e)
Which of the following numbers will come in place of (e)?
1) 45 2) 6 3) 9 4) 18 5) None of these

4. 264 136 72 40
488 (a) (b) (c) (d) (e)
Which of the following numbers will come in place of (a)?
1) 128 2) 248 3) 38 4) 23 5) 68

5. 2 17 121 729
5 (a) (b) (c) (d) (e)
Which of the following numbers will come in place of (b)?
1) 289 2) 41 3) 17393 4) 1448 5) 5796

Solutions:

1. 4; The series is $+2^2$, $+4^2$, $+6^2$ .....
2. 2; The series is $\times 2 - 1$, $\times 3 + 3$, $\times 4 - 3$, $\times 5 + 5$ .....
3. 1; The series is $\times 0.5$, $\times 1$, $\times 1.5$, $\times 2$ .....
4. 2; The series is $\div 2 + 4$, $\div 2 + 4$ .....
5. 1; The series is $\times 8 + 1$, $\times 7 + 2$, $\times 6 + 3$ ....

BSRB Mumbai PO held on April 1998

Directions (Q. 1-5): In each of the following questions, a number series is given. After the series, below it, a number is given followed by (a), (b), (c), (d) and (e). You have to complete the series starting with the given number following the sequence of the given series. Then answer the questions given below it.

1. 11 15 38 126
7 (a) (b) (c) (d) (e)
Which of the following will come in place of (c)?
1) 102 2) 30 3) 2140 4) 80 5) 424

2. 2 3 8 27
5 (a) (b) (c) (d) (e)
Which of the following will come in place of (e)?
1) 184 2) 6 3) 925 4) 45 5) 14

3. 2 3 9 40.5
4 (a) (b) (c) (d) (e)
Which of the following will come in place of (b)?
1) 486 2) 81 3) 3645 4) 18 5) 6

4. 12, 28 64 140
   37 (a) (b) (c) (d) (e)
   Which of the following will come in place of (e)?
   1) 1412 2) 164 3) 696 4) 78 5) 340
5. 5 12 60 340
   7 (a) (b) (c) (d) (e)
   Which of the following will come in place of (d)?
   1) 172 2) 5044 3) 1012
   4) 20164 5) 28

Solutions:

1. 1; The series is $\times 1 + 4, \times 2 + 8, \times 3 + 12$ ....
2. 3; The series is $\times 1 + 1, \times 2 + 2, \times 3 + 3$, ...
3. 4; The series is $\times 1.5, \times 3, \times 4.5$, ...
4. 1; The series is $\times 2 + 4, \times 2 + 8, \times 2 + 12$, ...
5. 2; The series is: $\times 8 - 28, \times 7 - 24, \times 6 - 20$, ...

SBI PO held on 14th February, 1999

Directions (Q. 1-5): One number is wrong in each of the number series given in each of the following questions. You have to identify that number and assuming that a new series starts with that number following the same logic as in the given series, which of the numbers given in (1), (2), (3), (4) and (5) given below each series will be the third number in the new series?

1. 3 5 12 38 154 914 4634
   1) 1636 2) 1222 3) 1834
   4) 3312 5) 1488
2. 3 4 10 34 136 685 1446
   1) 22 2) 276 3) 72 4) 1374 5) 12
3. 214 18 162 62 143 90 106
   1) -34 2) 110 3) 10 4) 91 5) 38
4. 160 80 120 180 1050 4725 25987.5
   1) 60 2) 90 3) 3564
   4) 787.5 5) 135
5. 2 3 7 13 26 47 78
   1) 11 2) 13 3) 15 4) 18 5) 20

Solutions:

1. 3; The series is $\times 1 + 2, \times 2 + 2, \times 3 + 2, \times 4 + 2, \times 5 + 2, \times 6 + 2$. 914 is incorrect. It should be 772. The new series begins with 914.

2. 3; The series is $\times 1 + 1, \times 2 + 2, \times 3 + 3, \times 4 + 4, \times 5 + 5, \times 6 + 6$. 34 should be 33 and thus the new series starts with 34.
3. 4; The series is $-(14)^2, +(12)^2, -(10)^2, +(8)^2, -(6)^2$ and so on. 143 is incorrect
4. 5; The series is $\times\frac{1}{2}, \times\frac{3}{2}, \times\frac{5}{2}, \times\frac{7}{2}, \times\frac{9}{2}, \times\frac{11}{2}$. 180 is incorrect.
5. 1; The series is $+1^2 - 0, +2^2 - 1, +3^2 - 2, +4^2 - 3, +5^2 - 4, +6^2 - 5$.
   Thus, 7 is the wrong number.

BSRB Baroda PO held on 21st March, 1999

Directions (Q. 1-5): In each of the questions given below there is a mathematical series. After the series a number is being given followed by a, b, c, d and e. You have to create another series after understanding the sequence of the given series which starts with the given number. Then answer the questions given below.

1. 1 9 65 393
   2 (a) (b) (c) (d) (e)
   Out of the following numbers which would come in the place of c?
   1) 490 2) 853 3) 731 4) 729 5) None of these
2. 8 8 12 24
   36 (a) (b) (c) (d) (e)
   Out of the following numbers which would come in the place of e?
   1) 810 2) 36 3) 54 4) 108 5) None of these
3. 424 208 100 46
   888 (a) (b) (c) (d) (e)
   What number would come in the place of b?
   1) 20 2) 440 3) 216 4) 56 5) None of these
4. 4 5 9.75 23.5
   7 (a) (b) (c) (d) (e)
   What number would come in the place of d?
   1) 32.5 2) 271.5 3) 8 4) 14.25 5) None of these
5. 5 294 69 238
   13 (a) (b) (c) (d) (e)
   Which of the following numbers would come in the place of e?
   1) 246 2) 206 3) 125 4) 302 5) None of these

Solutions:

1. 4; The series is $\times 8 + 1, \times 7 + 2, \times 6 + 3$.
   $\therefore a = 2 \times 8 + 1 = 17, b = 17 \times 7 + 2 = 121, c = 121 \times 6 + 3 = 729$

2. 1; The series is $\times 1, \times 1.5, \times 2$
$\therefore$ a = 36 × 1 = 36, b = 36 × 1.5 = 54, c = 54 × 2 = 108, d = 108 × 2.5 = 270 and e = 270 × 3 = 810
3. 3; The series is ÷ 2 - 4 $\therefore$ a = 888 ÷ 2 - 4 = 440 and b = 440 ÷ 2 - 4 = 216
4. 5; The series is × 1 + 1, × 1.5 + 2.25, × 2 + 4, ×2.5 + 6.25, × 3 + 9, ...
$\therefore$ a = 7 × 1 + 1 = 8, b = 8 × 1.5 + 2.25 = 14.25, c = 14.25 × 2 + 4 = 32.5 and d = 32.5 × 2.5 + 6.25 = 81.25 + 6.25 = 87.5
5. 2; The series is $+(17)^2, -(15)^2, +(13)^2, -(11)^2, +(9)^2$, ...
$\therefore$ c = 13 + (238 - 5 =) 233 = 246, d = 246 - $(11)^2$ = 246 - 121 = 125 and e = 125 + $(9)^2$ = 125 + 81 = 206

BSRB Bangalore PO held on 7th March, 1999

Directions (1-5): In each of the following questions, a number series is given. Only one number is wrong in this series. Find out that wrong number, and taking this wrong number as the first term of the second series formed, following the same logic, find out the fourth term of the second series.

1. 8 4 4 6 12 28 90
1) 18 2) 42 3) 21 4) 24 5) None of these
2. 17 17.25 18.25 20.75 24.5 30.75
1) 23.25 2) 24.25 3) 24.5 4) 24.75 5) None of these
3. 438 487 447 476 460 469
1) 485 2) 425 3) 475 4) 496 5) None of these
4. 2 7 18 45 99 209 431
1) 172 2) 171 3) 174 4) 175 5) None of these
5. 6 8 10 42 146 770 4578
1) 868 2) 8872 3) 858 4) 882 5) None of these

Solutions:

1. 3; The series is $\times\frac{1}{2}, \times 1, \times 1\frac{1}{2}, \times 2, \times 2\frac{1}{2}, \times 3$. So 28 is wrong. The new series begins with 28.
2. 2; The series is based on the following pattern:
17 + (0.25 × 1^2 =) 0.25 = 17.25, 17.25 + (0.25 × 2^2 =) 1 = 18.25, 18.25 + (0.25 × 3^2 =) 2.25 = 20.5, 20.5 + (0.25 × 4^2 =) 4 = 24.5 and 24.5 + (0.25 × 5^2 =) 6.25 = 30.75.
So, starting with the wrong no. 20.75, we get the reqd fourth no. = 20.75 + (0.25 + 1 + 2.25) = 24.25
3. 1; The series is $+7^2, -6^2, +5^2, -4^2, +3^2$, ...
4. 5; The series is ×2 + 3, ×2 + 5, ×2 + 7, ×2 + 9, ...
5. 4; The series is based on the following pattern:
6 × 1 + 1 × 2 = 8, 8 × 2 - 2 × 3 = 10, 10 × 3 + 3 × 4 = 42, 42 × 4 - 4 × 5 (= 37 × 4) = 148, 148 × 5 + 5 × 6 (= 154 × 5) = 770, 770 × 6 - 6 (= 763 × 6) = 4578
Starting with the wrong no. 146, we get the second no. = 146 × 1 + 1 × 2 = 148, third n. = 148 × 2 - 2 × 3 = 145 × 2 = 290 and the reqd fourth no. = 290 × 3 + 3 × 4 = 294 × 3 = 882

SBI Associates PO held on 18th July, 1999

Directions (Q. 1-5): In each of the following questions a number series is given. Only one number is wrong in each series. Find out that wrong number, and taking this wrong number as the first term of the second series formed following the same logic, find out the third term of the second series.

1. 1 2 8 21 88 445
1) 24.5 2) 25 3) 25.5 4) 25 5) None of these
2. 6 7 18 63 265 1365
1) 530 2) 534 3) 526 4) 562 5) None of these
3. 7 23 58 127 269 555
1) 263 2) 261 3) 299 4) 286 5) None of these
4. 5 4 9 18 57 168
1) 12 2) 25 3) 20
4) 18 5) None of these
5. 2 7 28 146 877 6140
1) 242 2) 246 3) 252
4) 341 5) None of these

Solutions:

1. 5; The series is ×1 + 1, ×2 + 2, ×3 + 3, ... So 8 is wrong. Beginning with 8 we get 20 as third term.
2. 5; The series is $\times 1 + 1^2, \times 2 + 2^2, \times 3 + 3^2$ .... So 265 is wrong.
3. 2; The series is ×2 + 9, ×2 + 11, ×2 + 13 .... So 58 is wrong.
4. 4; The series is ×1 - 1, ×2 + 2, ×2 - 2, ×3 + 3, ... So 9 is wrong.
5. 4; The series is ×3 + 1, ×4 + 1, ×5 + 1, ... So 28 is wrong.

BSRB Guwahati PO held on 8th August, 1999

Directions (Q. 1-5): In each of the following questions a number series is given. After the series, a number is given followed by (a), (b), (c), (d) and (e). You have to complete the series starting with the number given following the sequence of the given series. Then answer the question given below it.

1. 9 19.5 41 84.5
   12 (a) (b) (c) (d) (e)
   Which of the following numbers will come in place of (c)?
   1) 111.5 2) 118.5 3) 108.25
   4) 106.75 5) None of these
2. 4 5 22 201
   7 (a) (b) (c) (d) (e)
   Which of the following numbers will come in place of (d)?
   1) 4948 2) 4840 3) 4048
   4) 4984 5) None of these
3. 5 5.25 11.5 36.75
   3 (a) (b) (c) (d) (e)
   Which of the following numbers will come in place of (c)?
   1) 34.75 2) 24.75 3) 24.5
   4) 34.5 5) None of these
4. 38 19 28.5 71.25
   18 (a) (b) (c) (d) (e)
   Which of the following numbers will come in place of (d)?
   1) 118.75 2) 118.25 3) 108.25
   4) 118.125 5) None of these
5. 25 146 65 114
   39 (a) (b) (c) (d) (e)
   Which of the following numbers will come in place of (e)?
   1) 122 2) 119 3) 112 4) 94 5) None of these

Solutions:

1. 5; The series is $\times 2 + 1.5, \times 2 + 2, \times 2 + 2.5$ ....
   So, 108.5 should come in place of (c).
2. 1; The series is $\times 1^2 + 1, \times 2^2 + 2, \times 3^2 + 3, \times 4^2 + 4$, ... So 4948 should come in place of (d).
3. 2; The series is $\times 1 + 0.25 \times 1, \times 2 + 0.25 \times 4, \times 3 + 0.25 \times 9$ ..... So 24.75 should come in place of (c).

4. 4; The series is $\times 0.5, \times 1.5, \times 2.5$ ..... So 118.125 should come in place of (d).
5. 3; The series is $+11^2, -9^2, +7^2 -5^2$, ... So 112 should come in place of (e).

BSRB Mumbai PO held on 25th July, 1999

Directions (Q. 1-5): In each of the following questions a number series is given. A number in the series is suppressed by letter 'A'. You have to find out the number in the place of 'A' and use this number to find out the value in the place of the question mark in the equation following the series.

1. 36 216 64.8 388.8 A 699.84 209.952
   $A \div 36 = ?$
   1) 61.39 2) 0.324 3) 3.24 4) 6.139 5) 32.4
2. 42 62 92 132 A 242 312
   $A + 14 = ? \times 14$
   1) $11\frac{6}{7}$ 2) 14 3) $12\frac{5}{7}$ 4) $12\frac{1}{2}$ 5) $12\frac{1}{6}$
3. 4 7 12 19 28 A 52
   $A^2 - 4 = ?$
   1) 1365 2) 1353 3) 1505 4) 1435 5) 1517
4. 18 24 A 51 72 98 129
   $A \times \frac{3}{7} \times \frac{4}{5} = ?$
   1) 12 2) $11\frac{23}{35}$ 3) $12\frac{12}{35}$ 4) $14\frac{2}{5}$ 5) $10\frac{2}{7}$
5. $\frac{3}{8}$ $\frac{3}{4}$ $\frac{9}{16}$ $\frac{9}{8}$ $\frac{27}{32}$ $\frac{27}{16}$ A
   $\sqrt{A} = ?$
   1) $\frac{3}{2}$ 2) $\frac{6}{8}$ 3) $\frac{6}{4}$ 4) $\frac{3}{4}$ 5) $\frac{9}{8}$

Solutions:

1. 3; The series is $\times 6$ and $\times \frac{3}{10}$ alternately. So, 116.64 will come in place of A. $116.64 \div 36 = 3.24$
2. 2; The series is +20, +30, +40..... So 182 will come in place of A.
   $$? = \frac{182 + 14}{14} = 14$$

3. 5; The series is +3, +5, +7, +9 .... So 39 will come in place of A.

$? = 39^2 - 4 = 1517$

4. 1; The series is +6, +11, +16, +21 .... So 35 will come in place of A.

$? = 35 \times \frac{3}{7} \times \frac{4}{5} = 12$

5. 5; The series is $\times 2$ and $\times \frac{3}{4}$ alternately. So $\frac{81}{64}$ will come in place of A. $? = \sqrt{\frac{81}{64}} = \frac{9}{8}$

BSRB Calcutta PO held on 4th July, 1999

Directions (Q. 1-5): In each of the following questions, a number series is given. After the series, a number is given followed by (a), (b), (c), (d) and (e). You have to complete the series starting with the number given, following the sequence of the given series.

1. 15 16 25 50
189 (a) (b) (c) (d) (e)
Which of the following numbers will come in place of (e)?
1) 354 2) 273 3) 394 4) 426 5) None of these

2. 6 3.5 4.5 8.25
40 (a) (b) (c) (d) (e)
Which of the following numbers will come in place of (c)?
1) 20.5 2) 21.5 3) 33.75 4) 69.5 5) None of these

3. 9 10 22 69
5 (a) (b) (c) (d) (e)
Which of the following numbers will come in place of (b)?
1) 15 2) 28 3) 14 4) 45 5) None of these

4. 2 10 27 60
5 (a) (b) (c) (d) (e)
Which of the following numbers will come in place of (b)?
1) 39 2) 13 3) 34 4) 38 5) None of these

5. 5 149 49 113
146 (a) (b) (c) (d) (e)
Which of the following numbers will come in place of (d)?
1) 290 2) 234 3) 254 4) 218 5) None of these

Solutions:

1. 1; The series is $+1^2, +3^2, +5^2, +7^2$ .....
2. 3; The series is $\times 0.5 + 0.5, \times 1 + 1, \times 1.5 + 1.5, \times 2 + 2$ ...
3. 3; The series is $\times 1 + 1, \times 2 + 2, \times 3 + 3$ ....

4. 1; The series is $\times 2 + 6, \times 2 + 7, \times 2 + 6$ ....
5. 4; The series is $+(12)^2, -(10)^2, +(8)^2, -(6)^2$ .....

BSRB Delhi PO held on 1st August, 1999

Directions (Q. 1-5): In each of the following questions a number series is given. A number in the series is suppressed by letter 'A'. You have to find out the number in the place of 'A' and the use this number to find out the value in the place of the question mark in the equation following the series.

1. 6 7 11 20 36 61 A   12% of 2A = ?
1) 21.60 2) 23.28 3) 23.04 4) 22.56 5) 23.52

2. 168 A 167 151 166 152 165
75% of A = ?
1) 112.5 2) 120.0 3) 123.0 4) 111.75 5) 113.25

3. 18 6 A $\frac{2}{3}$ $\frac{2}{9}$ $\frac{2}{27}$ $\frac{2}{81}$
$A \div 2 - 1 = ?$
1) 1 2) 2 3) 0 4) $\frac{1}{3}$ 5) $1\frac{1}{3}$

4. 26 52 104 208 A 832 1664
$\sqrt{A - 4^2}$
1) 19 2) 20.29 3) 23.28 4) 19.62 5) 20

5. 6 8.5 7 9.5 A 10.5 9
$A^2 \times 2A + 3A = ?$
1) 1485 2) 3645 3) 2560 4) 1048 5) 4120

Solutions:

1. 2; The series is: $+1^2, +2^2, +3^2, +4^2$, ...
$\therefore A = 61 + 36 = 97 \quad \therefore 12\% \text{ of } 2A = 23.28$

2. 1; The series is -1 subtracted in 1st term gives third term and +1 added in second term gives fourth term and so on.
$\therefore A = 151 - 1 = 150 \quad \therefore 75\% \text{ of } A = 112.50$

3. 3; The series is $\div 3$ in each term.
$\therefore A = 6 \div 3 = 2 \quad \therefore A \div 2 - 1 = 2 \div 2 - 1 = 0$

4. 5; The series is $\times 2$ in each term.
$\therefore A = 416 \quad \therefore \sqrt{A - 4^2} = \sqrt{416 - 16} = \sqrt{400} = 20$

5. 4; The series is: +1 added to 1st term gives third, and +1 is added to 2nd term gives fourth, and so on.
$\therefore A = 7 + 1 = 8 \quad \therefore A^2 \times 2A + 3A = 2A^3 + 3A$
$= 2 \times 8^3 + 3 \times 8 = 1048$

BSRB Hyderabad PO held on 29th August, 1999

Directions (Q. 1-5): In each of the following questions, a number series is given in which one number is wrong. You have to find out that number and have to follow the new series which will be started by that number. By following this, which will be the third number of the new series?

1. 1 2 6 33 148 765 4626
   1) 46 2) 124 3) 18 4) 82 5) None of these
2. 2 9 5 36 125 648 3861
   1) 12 2) 11 3) 75 4) 72 5) None of these
3. 3 4 12 45 190 1005 6066
   1) 98 2) 96 3) 384 4) 386 5) None of these
4. 6 10.5 23 59.5 183 644 2580
   1) 183.5 2) 182.5 3) 183 4) 182 5) None of these
5. 2 7 19 43 99 209 431
   1) 181 2) 183 3) 87 4) 85 5) None of these

Solutions:

1. 3; The series is $\times 1 + 1^2, \times 2 + 2^2, \times 3 + 3^2, \times 4 + 4^2$ ....
2. 5; Ans = 13. The series is $\times 1 + 7, \times 2 - 11, \times 3 + 15$, ...
3. 4; The series is $\times 1 + 1^2, \times 2 + 2^2, \times 3 + 3^2, \times 4 + 4^2$ ......
4. 1; The series is $\times 1.5 + 1.5, \times 2 + 2, \times 2.5 + 2.5, \times 3 + 3$ ....
5. 2; The series is $\times 2 - 3, \times 2 + 5, \times 2 + 7, \times 2 - 9$ ....

NABARD held on 18th July, 1999

Directions (Q. 1-5): In each of the following question a number series is given. A number in the series is suppressed by 'P' mark. First you have to find out the number in the place of the 'P' mark and use this number to find out the answer of the question following the series.

1. 188 186 P 174 158 126
   $\sqrt{P - 13} = ?$
   1) 14.03 2) 14.10 3) 13.00 4) 13.67 5) None of these
2. 3.2 4.8 2.4 3.6 P 2.7
   0.06% of 54 ÷ P = ?
   1) 0.18 2) 1.62 3) 0.62 4) 18.0 5) 0.018
3. 4 $6\frac{1}{3}$ $8\frac{2}{3}$ P $13\frac{1}{3}$ $15\frac{2}{3}$
   30% of $(P^2 + 13^2) = ?$

   1) 78.73 2) 87.00 3) 98.83 4) 172.80 5) None of these
4. 220 182 146 114 84 58 P
   $P \times \frac{1}{\sqrt{256}} = ?$
   1) $2\frac{1}{8}$ 2) 2 3) $2\frac{1}{4}$ 4) $3\frac{7}{8}$ 5) None of these
5. 25 37 51 67 85 P 127
   20% of $(P \times \sqrt{625}) = ?$
   1) 625 2) 550 3) 450 4) 525 5) None of these

Solutions:

1. 3; The series is -2, -4, -8, -16, ...
   So, P = 186 - 4 = 182
   $\therefore ? = \sqrt{P - 13} = \sqrt{182 - 13} = 13$
2. 5; The series is $\times 1.5, \div 2, \times 1.5, \div 2$, ...
3. 2; The series is $+2\frac{1}{3}$ in each term
4. 1; The series is -38, -36, -32, -30, -26, -24
5. 4; The series is +12, +14, +16, +18 ...

BSRB Chennai PO held on 30th January, 2000

Directions (Q. 1-5): In each of the following questions a number series is given. A number is given after the series and then (a), (b), (c), (d) and (e) are given. According to the given series, you have to form a new series which begins with the given number, and then answer the question asked.

1. 6 3.0 4.5 2.25
   40 (a) (b) (c) (d) (e)
   Which of the following numbers will come in place of (c)?
   1) 20.5 2) 21.5 3) 33.75 4) 69.5 5) 15
2. 5 9 26 90
   13 (a) (b) (c) (d) (e)
   Which of the following numbers will come in place of (e)?
   1) 2880 2) 2292 3) 1716 4) 3432 5) None of these
3. 4 9 25 103
   3 (a) (b) (c) (d) (e)
   Which of the following numbers will come in place of (c)?
   1) 391 2) 81 3) 91 4) 79 5) None of these

4. 6 10 32 126
   2 (a) (b) (c) (d) (e)
   Which of the following numbers will come in place of (a)?
   1) 4 2) 6 3) 2 4) 3 5) None of these
5. 1260 628 312 154
   788 (a) (b) (c) (d) (e)
   Which of the following numbers will come in place of (d)?
   1) 194 2) 45.5 3) 48 4) 72.5 5) None of these

Solutions:

1. 5; The series is $\div 2, \times 1.5$ ......
2. 5; Ans = 2860. The series is $\times 1 + 4, \times 2 + 8, \times 3 + 12, ...$
3. 4; The series is $\times 2 + 1, \times 3 - 2, \times 4 + 3, \times 4 + 3, \times 5 - 4$ ...
4. 3; The series is $\times 2 - 2, \times 3 + 2, \times 4 - 2, \times 6 + 2$ .....
5. 2; The series is $\div 2 - 2$ in each steps.

Data Sufficiency

Introduction

Data sufficiency has recently become a favourite question for many of the recent examinations. In this type of questions, usually a question is given followed by two or three statements. These two or three statements contain data or some pieces of information using which the question can possibly be solved. You are required to judge whether the data given is sufficient to answer the question or not.

An analysis

Data sufficiency questions are not new topics in themselves. They may be covering any of the topics already covered; for example : percentage, time and work, algebra, time and distance etc. Hence you should treat these questions as old-type only. Only these questions are asked in a different pattern and not the conventional pattern.

Suggested steps

When you are attempting a question of data sufficiency you should follow a systematic approach as laid down below. This approach being a systematic one, will save your time. Also in case you are stuck up at any point, it will help your chances of guessing a correct answer because it narrows down the possible answers from 5 to 3 or 2.

To understand this approach let us first look at the way in which such questions are usually asked :

Two Statements Data Sufficiency

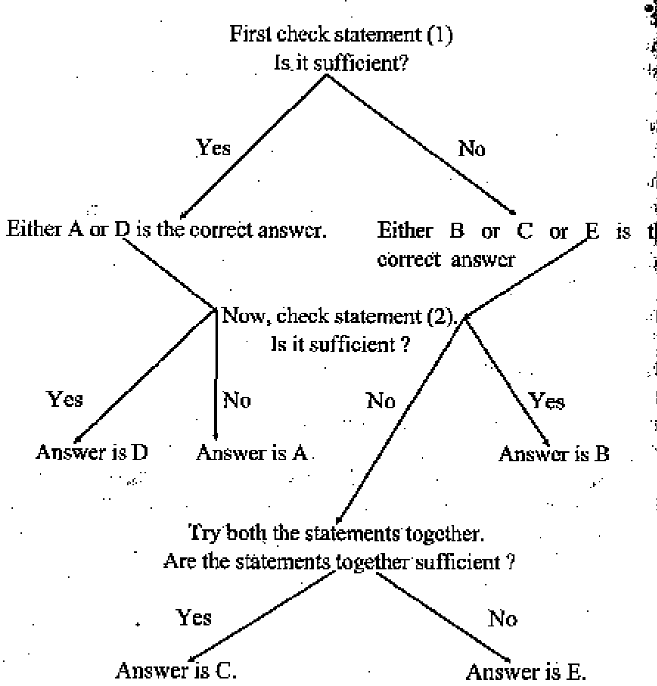
Directions : The questions below consist of a question followed by two statements labelled as (1) and (2). You have to decide if these statements are sufficient to answer the question. Give answer:

(A) if statement (1) alone is sufficient to answer the question but statement (2) alone is not sufficient to answer the question.
(B) if statement (2) alone is sufficient to answer the question but statement (1) alone is not sufficient to answer the question.
(C) if you can get the answer from (1) and (2) together although neither statement by itself suffices.
(D) if statement (1) alone is sufficient and statement (2), too, alone is sufficient.
(E) if you cannot get the answer from statements (1) and (2) together but

still more data are needed.

By looking at this format of the question we would suggest yo to try the first statement [labelled (1)] and see if this statement sufficient. There are two possible outcomes : either the statement w be sufficient or it will not be sufficient. If the former is true then eith A or D is the correct answer and if the latter be true then either B or or E is the correct answer. Thus we have narrowed down the number possible answers from 5 to 2 or 3. Similarly this procedure can continued with the second statement. The complete step-by-step a proach is explained by the following diagram.

The step-by-step approach outlined here will be sufficient f you to lead to correct answers quickly. However there are some ad tional facts which you should keep in mind to have a still quick approach. These points are discussed below in the following section.



Note: If the sequence of choices (A, B, C, D, E) given in the direction is different then you should change your answer accordingly. Sometimes ....

Some Important Additional Points

(1) If a question involves two unknowns then (i) **two** (ii) **distinct** equations are required for it. If this condition is fulfilled then C is the answer; otherwise E is the answer. No other answers are possible. See the following examples :-

Ex 1. What is the value of x ?

(1) $x + y = 15$

(2) $3x - y = 1$

Soln : Since two unknowns are there and two distinct equations are given, the correct answer is C.

Ex 2. What is the volume of a rectangular box R ?

(1) The total surface area of R is 12 square metres.

(2) The height of R is 50 cms.

Soln : The volume of a rectangular box is given by volume = length × breadth × height. This involves three unknowns. Two pieces of information will never be sufficient for such a question. Answer is E.

Ex 3. What is the first term in a sequence of numbers ?

(1) The third term is 12.

(2) The second term is twice the first and the third term is three times the second.

Soln : To get the exact value of any term of a sequence we need to know at least two things: one, the exact value of any other term and two, the relation between that term and the required term. Since both are given, the answer is C. The sequence is 2, 4, 12, ...

Ex 4. What is the value of a two-digit number ?

(1) The sum of the two digits is 4.

(2) The difference between the two digits is 2.

Soln : A two-digit number has two digits, both unknown. To find the number we need to find the two digits which is possible only if two distinct equations are given. (1) and (2) provide two distinct equations. But the two unknowns give you two answers. The numbers may be either $10x + y$ or $10y + x$, hence we can't get the solution. Thus, the correct answer is E. In the above case the two answers are 13 and 31.

Ex 5. If $xy \neq 0$ what is the value of $\frac{x^4y^2-(xy)^2}{x^3y^2}$

(1) $x=2$
(2) $y=8$

Soln : $\frac{x^4y^2-(xy)^2}{x^3y^2} = x - \frac{1}{x}$

So, answer is A.

Ex 6. What is the value of $x+y$?

(1) $x-2y=5$
(2) $x^2-25=4xy-4y^2$

Soln : The question involves two unknowns. But the two equations are not distinct. The second equation can be rearranged to $x^2-4xy+4y^2=25$ which gets reduced to $(x-2y)^2 = 5^2$ or $x-2y=5$. Which is the same as statement (1).

Hence the answer will be E.

Ex 7. What is the value of x ?

(1) $x-2y=7$
(2) $4x-28=8y$.

Soln : The second equation is the same as the first equation.

$4x-28=8y$
or, $4x-8y=28$
or, $x-2y=7$ (on dividing by 4).

Hence, answer will be E.

Some exceptions and modifications

In some cases the rule of two equations being required for finding the value of an expression is relaxed and only one equation may give an answer (see Ex 8). In some other cases the rule may be modified in a slight way (see Ex 9).

Ex 8. What is the value of $(x-y)$?

(1) $x-y=y-x$
(2) $(x-y)=(x^2-y^2)$

Soln : The expression $x-y$ involves two unknowns. But the first equation is sufficient. To see this,

$(x-y)=y-x=-(x-y)$
$(x-y)=-(x-y)$ or, $2(x-y)=0$

This implies that $x-y=0$. Since a number is equal to its negative in one and only one possible way, that is, if the number is equal to zero. Hence either A or D is the answer. The second expression is not sufficient. Because, $x-y=x^2-y^2=(x-y)(x+y)$

$(x+y-1)(x-y)=0$

This leads to two possibilities : $x-y=0$ or $x+y=1$.

In this case the value of $(x-y)$ is not accurately determined. Hence A is the answer.

Ex 9. If $y=4$ what is the value of $y-x$?

(1) $x=4$ (2) $x+y=8$

Soln : This question asks about the value of an expression $y-x$. This expression has two unknowns and hence we need two distinct equations. But note that the given question itself has an equation: $y=4$. Hence we need only one more equation. Thus we should pick up the choice: either statement by itself suffices. Correct answer is D.

If you remain careful of this point of number of unknowns and number of equations you will have a quick sailing through many of the problems. Almost every one question out of three are of this type.

Some Questions based on inequality

Ex. 10 : Is x greater than y?

(1) x is greater than 145.
(2) y is greater than 140.

Soln : Both the statements even together are not enough to give the answer. For instance, x can be 146 and y can be 141 or 150. Thus we can't say whether x is greater than y or not. Hence our answer is E.

Ex. 11 : Is x greater than y?

(1) $x-y=25$ (2) $2x+y=9$

Soln : By the first statement x is greater than y because $x-y$ is a +ve value. The second statement is not sufficient because x may be greater or smaller than y. (Find those values.) Therefore, our answer is A.

Ex. 12 : Is x greater than y?

(1) x is a multiple of y
(2) $\frac{x}{6} = \frac{y}{3}$

Soln : The first statement is not sufficient because the multiple may be a whole number or a fraction. But the second statement clearly shows that $x > y$. Thus our answer is B.

Ex. 13 : Is $x > y$?

(1) $x^2 - 4x + 4 = 0$

(2) $y^2 - 6y + 12 = 2y - 4$

Soln : The first statement $(x-2)^2 = 0 \Rightarrow x = 2$; and the second statement $(y-4)^2 = 0 \Rightarrow y = 4$.

Thus we need both the statements to reach any result. Hence our answer is C.

Ex. 14 : If a and b are integers, is a + b an odd number?

(1) $8 < a < 11$ (2) $7 < b < 10$

Soln : By statement (1) 'a' may be 9 or 10. By statement (2) 'b' may be 8 or 9. Hence a + b may be either odd or even. Thus we can't answer the question even with the help of both the statements. Hence our answer is 'E'.

Based on Mathematical Chapters

Ex. 15 : What percent of all the marbles in the bag were black?

(1) The ratio of black to white marbles was 3 : 4.

(2) There were exactly 5 brown marbles in the bag.

Soln : No mention is made if black, white and brown are the only colours in the bag. Hence our answer is E.

Ex. 16 : The price of which of the two cars A and B was reduced by the largest amount ?

(1) The price of car A was reduced by 10%.

(2) The price of car B was reduced by 8%

Soln : The prices of both the cars should have been known before the question could be answered. None of the statements says about the prices. Thus answer is E.

Ex. 17 : How many people heard my joke ?

(1) I told the joke to 3 friends, each of whom repeated it to 4 friends who did not tell anybody else.

(2) No one heard the joke twice.

Soln : It seems that only statement (1) is sufficient to give the answer but friends may be common. Thus statement (2) is necessary to eliminate the common persons. Thus, both the statements are needed. Our answer is C.

Ex. 18 : Is a^2 an integer?

(1) a is a negative whole number.

(2) $4a^2$ is an integer.

Soln : Statement (1) gives the affirmative answer.

$$\text{Profit percentage} = \frac{\text{Profit}}{\text{CP } (= \text{SP} - \text{Profit})} \times 100$$

As the profit is already given, if either CP or SP is known, profit percentage can be obtained. So, the answer is (4).

Ex. 13: What is the gain or loss per cent of Seema who sells two chairs?

A. She sells one chair at 25% loss.

B. She sells the other chair at 25% gain.

C. She has bought her two chairs for Rs 2760.

1) All together are necessary
2) All even together are not sufficient
3) Only A and B together are sufficient
4) Only C and A together are sufficient
5) Any two statements are sufficient

Soln: (C) gives the cost price of each chair, which is $\frac{2760}{2} = \text{Rs } 1380$

(A) and (B) give the selling price of each chair. Hence, with the help of all the three statements, we can find the profit and hence the % profit. Hence, answer is (1).

Ex. 14: The compound interest on a sum of Rs 4000 is Rs 1324. Find the rate of interest.

A. The simple interest on the same sum at the same rate is Rs 1200.

B. Compound interest is compounded every four months.

C. The sum doubles itself in 25 years at the rate of 4% per annum.

1) Only B and C together are sufficient
2) Only A and C together are sufficient
3) All together are necessary
4) Either B and C together or A and C together are sufficient
5) All even together are not sufficient

Soln: C is not an informative statement because it is true in all cases. In order to find out the rate of interest, we need the time for which the sum has been deposited. But this has not been provided either in A or in B. So, answer is (5).

Ex. 15: A person deposited two amounts to a money lender at 5% simple interest for 3 years and 5 years. Find the two amounts.

A. Difference between the interests is Rs 600.

B. The two amounts are equal.

C. Had the amounts been deposited at 5% compound interest, the difference would have been Rs 71194.

1) Only A and B together are sufficient
2) Only A and C together are sufficient
3) Any two statements together are sufficient
4) B and either A or C is sufficient
5) All together are necessary

Soln: Any two statements are sufficient. The answer is (3).

Ex. 16: Two friends Sheela and Meena earned profit in a business. Fi out their shares.

A. Sheela had invested her capital for 9 months and Meena fo year.
B. The ratio of their capitals was 4 : 3.
C. The total profit was Rs 27500.

1) Only A and B together are sufficient
2) Only B and C together are sufficient
3) All together are necessary
4) Either B or A and C together are sufficient
5) All even together are not sufficient

Soln: From (A) and (B), we have

Ratio of profits = $9 \times 4 : 12 \times 3 = 36 : 36 = 1 : 1$

Now, with help of (C), shares of each of them $= \frac{27500}{1+1} \times 1$
$= \text{Rs } 13750$

Hence, the answer is (3).

Ex. 17: What is the relative speed of two trains running in oppos directions?

A. They take 15 seconds to cross each other.
B. The speed of one of the trains is 60 kmph.
C. The total length of the trains is 360 m.

1) Only A and B together are sufficient
2) Only B and C together are sufficient
3) Only A and C together are sufficient
4) Any two statements together are necessary
5) All together are necessary

Soln: Relative speed $= \frac{\text{Total length of the trains}}{\text{Time taken to cross each other}}$

So, from A and C, we get the relative speed $= \frac{360}{15} = 24$ m/s

Whereas from A and B, or from B and C the relative speed cann be obtained. Answer is (3).

Ex. 18: A man can row a distance of x km up the stream in 3.5 hours. Find his speed in still water.

A. He covers a distance of 84 km downstream in 6 hours.
B. He covers the distance of x km downstream in 2.5 hours.
C. The speed of the current is 2 kmph.

1) Only B and C together are sufficient
2) Only A and B together or A and C together are sufficient
3) Any two statements together are sufficient
4) All together are necessary
5) Either B alone or A and C together are sufficient

Soln: Upstream speed $= \frac{x}{3.5}$ km/hr (given in the question)

(A) + (B) $\Rightarrow$ Downstream speed $= \frac{84}{6} = 14$ km/hr

and also downstream speed $= \frac{x}{2.5}$

Now, $\frac{x}{2.5} = 14 \quad \therefore x = 2.5 \times 14 = 35$ km

From the given information,

Upstream speed $= \frac{x}{3.5} = \frac{35}{3.5} = 10$ km/hr

Thus, his speed in still water $= \frac{10+14}{2} = 12$ km/hr

(B) + (C) $\Rightarrow$ His speed in still water $= \frac{x}{3.5} + 2 = \frac{x}{2.5} - 2$

or, $\frac{x}{2.5} - \frac{x}{3.5} = 2 + 2 = 4$

or, $\frac{x}{2.5 \times 3.5} = 4 \quad \therefore x = 35$ km.

$\therefore$ His speed in still water $= \frac{x}{3.5} + 2 = \frac{35}{3.5} + 2 = 12$ km/hr

(A) + (C) $\Rightarrow$ His speed in still water $= \frac{84}{6} - 2 = 14 - 2 = 12$ km/hr

Hence answer is (3).

Ex. 19: P works for 4 days and leaves the job. In how many days can P alone finish the entire work?

A. Q finishes the remaining work in 8 days.
B. P and Q together can finish the work in $6\frac{2}{3}$ days.

C. The working efficiency of Q is double that of P.
1) Only B and C together are sufficient
2) Only A and B together or A and C together are sufficient
3) Any two statements together are sufficient
4) All together are necessary
5) Either B alone or A and C together are sufficient

Soln: Suppose P alone can finish the work in x days.

From A, we get P has worked for 4 days and done $\frac{4}{x}$ part of the work.

The remaining work = $1 - \frac{4}{x} = \frac{x-4}{x}$ part of the work that has been done by Q in 8 days. So, Q alone can finish the work in $\frac{8x}{x-4}$ days.

Now, from (A) and (B), we get,

$$\frac{\frac{8x}{8-4} \times x}{\frac{8x}{x-4} + x} = 6\frac{2}{3}$$

So, x can be obtained. (On solving, you will get x = 20.)

From C, we get Q alone can finish the work in $\frac{x}{2}$ days.

Now, from (A) and (C), we get

$$\frac{8x}{x-4} = \frac{x}{2}$$

So, x can be obtained (x = 20).

Now, from (B) and (C), we get

$$\frac{x \times \frac{x}{2}}{x + \frac{x}{2}} = 6\frac{2}{3}$$

So, x can be obtained (x = 20). Thus, the answer is 3.

Ex. 20: A water tank has been filled with two filler taps P and Q and a drain pipe R. Tap P and Q fill at the rate of 12 and 10 litre per minute respectively. What is the capacity of the tank?

A. Tap R drains out at the rate of 6 litres per minute.

B. If all the three taps are opened simultaneously, the tank is filled in 5 hours 45 minutes.

C. Tap R drains the filled tank in 15 hours 20 minutes.
1) Only A and B together are sufficient
2) Only either A and B together or A and C together are sufficient
3) All the three together are necessary
4) Any two statements are sufficient
5) Only either A and B together or B and C together are sufficient

Soln: With the help of (A) and (B):

All the three taps opened simulataneously fill 12 + 10 - 6 = 16 lt per minute

∴ Capacity of the tank = 16 × (5 × 60 + 45) = 5520 lt.

With the help of (A) and (C):

Capacity of the tank = 6 × (15 × 60 + 20) = 5520 lt.

With the help of (B) and (C):

Let the tap R drain out at the rate of x lt per minute.

Then, all the three taps opened simultaneously fill 12 + 10 - x = 22 - x litres per minute.

∴ Capacity of the tank = (22 - x) (5 × 60 + 20) and so x can be obtained.

Then substituting the value of x in any one of the above equations, the capacity of the tank can be obtained. Thus, the answer is (4).

Ex. 21: To find the total surface area of a hemisphere, we need which of the following informations?

A. Curved surface area of the hemisphere.

B. Volume of the hemisphere.

C. Radius of the hemisphere.
1) Only C alone is sufficient
2) Only B alone is sufficient
3) Only either C alone or A and B together are sufficient
4) Any one of the statements alone is sufficient
5) Only either C or B is sufficient

Soln: For a hemisphere we have, its curved surface area = $2\pi r^2$ sq. units

Total surface area = curved surface area + area of the base

$= 2\pi r^2 + \pi r^2 = 3\pi r^2$ sq. units and volume = $\frac{2}{3}\pi r^3$ cu. units.

(C) gives the value of r directly whereas (A) and (B) give the value of r indirectly.

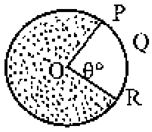
Thus, we conclude that any one of the statements is sufficient, i.e. answer is (4).

Ex. 22: To find out the area of the shaded portion of the figure, which of the given informations is needed?

A. Circumference of the circle.

B. The value of θ.

C. The length of the arc PQR.



1) Only A and B together are sufficient
2) Only A and C together are sufficient
3) Only A and either B or C are sufficient
4) Any two of A, B and C are sufficient
5) All together are necessary

Soln: Area of the shaded portion of the figure = (area of the circle = πr^2 - area of the sector PQRO.

Area of the sector PQRO = $\frac{\pi r^2 \theta°}{360} = \frac{1}{2} \times r \times$ arc PQR

(A) ⇒ radius of the circle

So, from (A) and (B) or (A) and (C), the required area can be determined.

From (B) and (C) and the equation $\frac{\pi r^2 \theta°}{360} = \frac{1}{2} \times r \times$ arc PQR

The value of r can be obtained first; consequently the area of the sector PQRO; and finally the reqd area can be obtained.

Thus, we conclude that any two statements together are sufficient. So, the answer is (4).

Ex. 23: To find the length of the carpet which covers the floor of rectangular hall, which of the following informations is needed

A. The length of the hall is 28 m.

B. The width of the carpet is 2.5 m.

C. The area of the hall is 490 sq. m.

1) Only A and B together are sufficient
2) Only B and C together are sufficient
3) Only A and C together are sufficient
4) Any two statements are sufficient
5) All together are necessary

Soln: Width of the carpet is needed, which is given in statement (B). Now, we need the area of the floor, which is given in statement (C). So, only B and C together are sufficient. Hence, the answer is (2).

Ex. 24: How many ice cubes can be accommodated in a container?

A. The length and breadth of the container are 16 cm and 12 cm respectively.

B. The edge of the ice cube is 2 cm.

C. The container is 384 times heavier as compared to a single ice cube.

1) Only A and B together are sufficient
2) Only B and C together are sufficient
3) Any two statements are sufficient
4) All together are necessary
5) All even together are not sufficient

Soln: To get the required number of ice cubes we need the volume of the container as well as that of an ice cube.

From (B), we have, the volume of an ice cube = (2^3 =) 8 cu. cm.

Still, as there is no information regarding the height of the container, so the volume of the container and consequently the no. of ice cubes cannot be determined. So, answer is (5).

EXERCISES (B)

(Asked in previous exams)

1. A trader sold an article at Rs 625. To find out his profit percentage which of the following statements is/are necessary/sufficient?

A. He gained Rs 600 by selling 8 such articles.

B. The cost price of the article is Rs 545 and transportation cost is Rs 5 each.

1) Only A 2) Only B
3) Both A and B are necessary/sufficient
4) Either A or B is sufficient
5) Neither A nor B is sufficient

2. Rohit got 700 marks in the examination. To find out his percentage of marks which of the following information is/are necessary/sufficient?

A. He appeared in all the nine papers.

B. The highest marks obtained is 90.

C. The maximum marks for each paper is 100.

1) Only A is sufficient 2) Only C is sufficient

3) Only A and B together are sufficient
4) Only A and C together are sufficient
5) All A, B and C are necessary

3. Two friends earned a profit in a business. To find out their shares which of the following statements is/are necessary/sufficient ?
A. The total profit was Rs 25000.
B. Rohan had invested his capital for 1 year against Sohan who had invested for 8 months.
C. The ratio of their capital was 2 : 3.
1) Only A and B together are sufficient
2) Only B and C together are sufficient
3) Only B alone is sufficient
4) All A, B and C together are necessary
5) None of these

Directions (Q. 4-8): Each of the questions below consists of a question and two statements numbered I and II given below it. You have to decide whether the data provided in the statements are sufficient to answer the question. Read both the statements and Give answer

1) if the data in **statement I alone** are **sufficient** to answer the question, while the data in **statement II alone** are **not sufficient** to answer the question.
2) if the data in **statement II alone** are **sufficient** to answer the question, while the data **in statement I alone** are **not sufficient** to answer the question.
3) if the data either **in statement I alone** or in **statement II alone** are sufficient to answer the question.
4) if the data even in **both the statements I and II** together are **not sufficient** to answer the question.
5) if the data in **both the statements I and II together** are **necessary** to answer the question.

4. What is the height of a triangle?
I. The area of the triangle is 20 times its base.
II. The perimeter of the triangle is equal to the perimeter of a square of 10 cm side.

5. What was the cost price of the suitcase purchased by Samir?
I. Samir got 20 per cent concession on the labelled price.
II. Samir sold the suitcase for Rs 2,000 with 25 per cent profit on the labelled price.

6. What was the speed of a running train?
I. The train crosses a signal post in 6 seconds.
II. The train crosses another train running in the opposite direction in 15 seconds.

7. What percentage rate of simple interest per annum did Ashok pay to Sudhir?
I. Ashok borrowed Rs 8,000 from Sudhir.
II. Ashok returned Rs 8,800 to Sudhir at the end of two years and settled the loan.

8. What was the ratio between the ages of P and Q four years ago?
I. The ratio between the present ages of P and Q is 3 : 4.
II. The ratio between the present ages of Q and R is 4 : 5.

9. A train crosses another train coming from the opposite direction in 18 sec. If the length of the train is 100 m, then to find out the speed of the train which of the following statements P, Q and R is sufficient/necessary?
P. The speed of the other train is 60 km/h.
Q. The train passes a platform in 14 sec.
R. The other train passes a telegraph pole in 6 sec.
1) Only P and Q together are sufficient.
2) Only Q and R together are sufficient.
3) Only P and R together are sufficient.
4) All P, Q and R together are necessary.
5) None of these

Directions (Q. 10-14): In each of the following questions, there is a question followed by two statements. You have to decide if the informations given in the statements is/are sufficient to answer the questions. Give answer

1) if the information given in **statement I alone** is **sufficient** to answer the question while information given in **II** is **not sufficient** to answer the question.
2) if the information given in **statement II alone** is **sufficient** to answer the question while the **statement I** is **not sufficient** to answer the question.
3) if either **I alone** or either **II alone is sufficient** to answer the question.
4) if both **statement I** and **II** are **necessary** to answer the question.
5) if **statements I** and **II** even together are **not sufficient** to answer the question.

10. What is the perimeter of a semi-circle?
I. The radius of the semi-circle is 7 m.
II. The area of the semi-circle is 154 cm^2.
11. Area of a right-angled triangle is equal to the area of a rectangle. What is the length of the rectangle?
I. The area of triangle is 100 m^2.
II. The base of triangle is equal to breadth of the rectangle.
12. What will be the difference of simple interest and compound interest of a sum at the same rate of interest after 3 yrs?
I. The rate of interest per annum is 5%.
II. The simple interest for 3 years on that sum is Rs 750.
13. What is the distance between A and B?
I. A scooterist covered the distance in half an hour.
II. The initial speed of the scooterist was 40 km/hr.
14. What profit did Rohit make if he sold the watch for Rs 600?
I. He bought the watch at 20% discount.
II. He sold the watch at 5% higher than the marked price.
15. Rs 16000 is divided among A, B and C. To find the share of C which of the following information(s) is/are necessary/sufficient?
M. C gets 4.5 times less than B.
N. A gets 2.5 times more than C.
O. B gets Rs 4000 more than A.
1) Only M and N together are sufficient.
2) Only M and O together are sufficient.
3) Only N and O together are sufficient.
4) All M, N and O are necessary.
5) Any two of the three are sufficient.
16. A triangle has hypotenuse measuring 42.42 cm. To find out its other sides which of the following information(s) is/are sufficient/necessary?
A. It is a right angled triangle.
B. Two of its sides are equal.
C. Hypotenuse is $\sqrt{2}$ times its each side.
1) Only A alone 2) Only B alone 3) Only C alone
4) Either C or B alone 5) None of these
17. P and Q worked together for 5 days. Then Q left the work and the work was finished by P alone in 8 days. How many days would Q alone take to finish the work if P had left? To answer the question, Which of the following information(s) is/are necessary/sufficient?

A. P and Q together can do the work in 10 days.
B. P alone can do the work in 16 days.
1) Only A alone 2) Only B alone
3) Either A or B alone 4) Both A and B together
5) Both together are even not sufficient
18. A product was sold at a profit of 15% after giving a discount of 20% on marked price. To find the cost price which of the following statement(s) is/are necessary/sufficient?
A. The discount given is Rs 350.
B. The marked price of the article is Rs 1750.
1) Only A alone 2) Only B alone
3) Both A and B together are necessary
4) Either A or B alone is sufficient
5) Neither A nor B alone is sufficient
19. An amount yields its $\frac{2}{5}$ as simple interest for 4 years. To find out its principal which of the following statement(s) is/are necessary/sufficient?
A. The rate of interest is 10%.
B. The interest for 2 years is Rs 450.
1) Only A alone 2) Only B alone 3) Either A or B alone
4) Both A and B together are necessary
5) Neither A nor B alone is sufficient
20. The ratio of the ages of Toni and Moni is 5 : 3. To find out the ratio of their ages after 5 years, which of the following is/are necessary/sufficient?
A) The sum of their ages is 48 years.
B) The difference of their ages is 12 years.
C) The ratio of their ages 6 years before was 2 : 1.
1) Either A or C only 2) Either B or C only
3) A, B and C together 4) Either A or B
5) Any one of the three
21. A scooter is sold for Rs 15000. To find out the profit percentage which of the following informations is/are sufficient/necessary?
A) If it would be sold at Rs 16,500 profit would be double.
B) The cost price was 15% less than the selling price.
1) Only A is sufficient 2) Only B is sufficient
3) Either A or B is sufficient 4) A and B together is necessary
5) Neither A nor B is sufficient

22. The walls of a room are to be mounted with wall paper 36 inches wide. To find out the length of paper, which of the following informations is/are necessary/sufficient ?
   A) The height of the room is 14 ft.
   B) The total area of doors and windows is 56 sq. ft.
   C) The perimeter of the room is 28 ft.
   1) Only B and C together
   2) Only A and B together
   3) The three even together are not sufficient
   4) B and either A or C
   5) All are necessary

23. The area of a rhombus is 1152 m^2. To find out its side which of the following is/are necessary/sufficient?
   A) One of its diagonal is 36 m.
   B) The other diagonal is 32 m.
   C) All the four sides are equal.
   1) A and B together are sufficient.
   2) Only A alone is sufficient.
   3) Only B alone is sufficient.
   4) Either A alone or B alone is sufficient.
   5) All A, B and C are necessary.

24. In how many days can Rajan finish the remaining work? To get the answer which of the following informations is/are necessary/sufficient?
   A) Rajan and Suman together can finish the work in 6 days.
   B) Suman alone can finish the work in 12 days.
   C) Rajan and Suman worked together for 2 days.
   1) Only A and B are sufficient.
   2) Only B and C are sufficient.
   3) Only A and C are sufficient.
   4) All A, B and C are necessary.
   5) All together are even not sufficient.

25. What is the age of Amit? To find out his age. Which of the following informations is necessary/sufficient?
   A) Amit is half the age of his father.
   B) His father is 25 yrs older than his brother.
   C) 10 yrs ago the ratio of his father's and his brother's age was 5 : 3.
   1) Only B and C are sufficient
   2) Only A and C are sufficient

   3) All A, B and C are necessary
   4) Only B and C are sufficient
   5) Any two of the three statements are sufficient

26. How much vote did the winner get in the election if the total electorate was 6 lakh. To get the answer which of the following informations is/are necessary/sufficient?
   A) He got 250% more than his rival.
   B) He defeated his rival by the margin of 1.5 lakh votes.
   C) Of the 58% votes polled the runner got only 13%.
   1) Only A and B together
   2) Only A and C together
   3) A, B and C together are necessary
   4) A and either B or C is sufficient
   5) Any two of the three statements are sufficient

27. Two friends started from two places to meet each other. When and where will they come across each other? To get the answer which of the following is/are necessary/sufficient?
   A) A started from Delhi at 60 km/hour for Panipat at 9 am.
   B) B started for Delhi from Panipat at 10 am.
   C) Panipat is 200 km from Delhi.
   1) Only A and B together are sufficient.
   2) Only A and C together are sufficient.
   3) Only B and C together are sufficient.
   4) All A, B and C are necessary.
   5) All together are even not sufficient.

28. A hawker sells 6 oranges for Rs 5. What percentage of profit does he get? To get the answer which of the following informations is/are necessary/sufficient?
   A) By selling at Rs 12 per dozen he would get 16% more.
   B) He bought the oranges at Rs 63 per hundred.
   1) Only A alone is sufficient
   2) Only B alone is sufficient
   3) Both A and B together alone are necessary
   4) Either A or B alone is sufficient
   5) Neither A alone nor B alone is sufficient

29. A goods train X crosses another train Y in 28 sec. To find the length of Y which of the following statements is/are sufficient/necessary?
   A. Speed of X is 45 km/hr
   B. Length of X is 130 m

C. Speed of Y is 54 km/hr
1) Only A and B together are sufficient
2) Only B and C together are sufficient
3) A, B and C together are necessary
4) Only A and C together are sufficient
5) All the three even together are not sufficient

30. On the sale of a cooler the dealer gets 6% profit. To get the CP which of the following statements is/are sufficient/necessary?
A) The sale price for the cooler is 3690.
B) By selling it at Rs 3795 he would get one-and-a-half times more profit.
1) Only A alone is sufficient
2) Only B alone is sufficient
3) A and B together are necessary
4) Either A alone or B alone is sufficient
5) A and B even together are not sufficient

31. A trader sells a homogeneous mixture of A and B at the rate of Rs 17 per kg. To find the profit percentage of the trader which of the following statements is/are sufficient/necessary?
A) He bought A at the rate of Rs 20 per kg.
B) He bought B at the rate of Rs 13 per kg.
1) Only A alone is sufficient
2) Only B alone is sufficient
3) Either A alone or B alone is sufficient
4) A and B together are sufficient
5) A and B even together are not sufficient

32. Ram's age is $\frac{2}{3}$ of Shyam's. To get Ram's age which of the following statements is/are sufficient/necessary?
A) Ram is 6 years older than Gopal.
B) 5 years ago ratio of Shyam's and Gopal's age was 3 : 5.
C) After 5 years the ratio of the ages of Ram and Shyam will be 1 : 2.
1) Only A and B together are sufficient
2) Only B and C together are sufficient
3) Only A and C together are sufficient
4) A, B and C together are necessary
5) Either C alone or A and B together are sufficient

33. A trapezium-shaped field is having parallel lines in the ratio of 5 : 3. To get the vertical distance between the lines, which of the following statements is/are sufficient/necessary?
A) The area of the field is 532 m^2.
B) The sum of parallel lines is 56 m.
1) Only A alone is sufficient
2) Only B alone is sufficient
3) A and B together are necessary
4) Either A alone or B alone is sufficient
5) A and B together even together are not sufficient

34. A person bought an article at 15% discount on marked price and sold it for Rs 1600. To get the percentage profit which of the following statements is/are sufficient/necessary?
A) The marked price of the article is Rs 1500.
B) By selling the article at Rs 153 more he would get 12% more.
1) Only A alone is sufficient
2) Only B alone is sufficient
3) Either A alone or B alone is sufficient
4) A and B together are necessary
5) Neither A alone nor B alone is sufficient

35. Mahesh got Rs 1200 as dividend from a finance co. What is the rate of interest given by the company? To get the answer which of the following is/are sufficent/necessary?
A. Mahesh has 960 shares of Rs 10 denomination.
B. The dividend paid last year was 9.5%
1) Only A is sufficient
2) Only B is sufficient
3) Either A or B is sufficient
4) A and B together are necessary
5) A and B even together are not sufficient

36. A boat sails 13 km upstream in 4 hrs. What is the speed of the stream? To get the answer which of the following informations is/are sufficient/necessary?
A. The boat goes downstream in 2 hrs.
B. The boat sails 6 km/hr in still water.
1) Only A alone is sufficient
2) Only B alone is sufficient
3) Either A alone or B alone is sufficient
4) A and B together are necessary

5) A and B even together are not sufficient

37. The ratio of boys and girls in a class is 5:3. To get the number of boys in the class which of the following informations is/are sufficient/necessary?

A) The number of girls is 60% of the number of boys.

B) If 12 more girls are brought the number of boys and girls will be equal.

C) The total number of students is 48.

1) Only A and B together are sufficient
2) Only B and C together are sufficient
3) Only B alone is sufficient
4) Only C alone is sufficient
5) Only either B alone or C alone is sufficient

38. A person lent Rs 2500 each to two of his friends. What is the rate of interest? To get the answer which of the following informations is sufficient/necessary?

A. Amount repaid by one of them was Rs 2700.

B. The other one repaid Rs 2900 after the due period.

C. Amount was given to one of them on simple interest and to another on compound interest.

1) Only A and B together are sufficient.
2) Only B and C together are sufficient.
3) A, B and C together are necessary.
4) Either A and B together or B and C together are sufficient
5) A, B and C even together are not sufficient

Answers

1. 4; If the sales price is given, profit percentage can be summed up if we know either CP or profit.

2. 4; To find out the percentage total marks has to be found out which is possible with A and C; that is, he got 700 marks out of 900.

3. 4

4. 4; From statement I If base = x then area = 20x

$$\text{height} = \frac{2 \times 20x}{x} = 40$$

But we don't get the unit of the height of the triangle. It may be in inches, cms, metres or kilometres. Then we can't get the correct value.

5. 5; From statement II,

$$\text{Labelled price} = 2000 \times \frac{100}{125} = \text{Rs } 1600$$

Statement (2) may give an integer or a fraction. Thus our answer is A.

Ex. 19 : Are two triangles congruent?

(1) They are both equilateral triangles.

(2) They both have equal bases and equal heights.

Soln : Equilateral triangles with same bases are congruent by the side-side-side postulate. Thus our answer is C.

Ex. 20 : What are the dimensions of a certain rectangle?

(1) The perimeter of the rectangle is 14.

(2) The diagonal of the rectangle is 5.

Soln : Statement (1) gives :

$2x + 2y = 14$ or, $x + y = 7$

Statement (2) gives : $x^2 + y^2 = 5^2$

We have two equations and two unknown; hence we can get the values of x and y. Hence our answer is C.

Ex. 21 : What % marks did he get in a test of 4 subjects?

(1) He got 90 in English and 84 in Maths.

(2) He got 75 in Hindi and 76 in Sanskrit.

Soln : From both the statements we get the total marks of the student but nothing is mentioned about the highest marks of any subject. So we can't answer the question at all. Our answer is E.

Ex. 22 : How long will it take two pipes A and B to empty or fill a tank that is 3/4 full?

(1) Pipe A can fill the tank in 12 minutes.

(2) Pipe B can empty it in 8 minutes.

Soln : Both the informations together are needed to find the answer. Our answer is C.

Ex. 23 : A table has a marked price of Rs 100. Discounts of 20% and 25% are allowed. What is the cost of the table to the dealer?

(1) The dealer's profit is 30% of the selling price.

(2) The dealer's cost of doing business is 10% of the selling cost.

Soln : Total discount is 40%. This brings the selling price down to Rs 60. Since the dealer's profit is 30% of Rs 60 (statement 1) he has bought the table for Rs $60 \times \frac{70}{100}$ = Rs 42. His cost of doing business is Rs $60 \times \frac{10}{100}$ = Rs 6 (from statement 2). Therefore, the cost of the table to him is Rs 42 - Rs 6 = Rs 36. Thus, we need both the statements to answer our question. Our answer is C.

Ex. 24 : How many letters can two typists complete in one day?

(1) A working day consists of 6 hours.

(2) Four typists can type 600 letters in 3 days.

Soln : The length of working day is irrelevant in this solution. Only statement (2) is sufficient to answer the question. Our answer is B.

Ex. 25 : What is the non-voting population of a country?

(1) Only males over 20 yrs of age are permitted.

(2) The country has a total population of 5362482.

Soln : We can't find the number of males who are over 20 yrs of age. Thus the answer is E.

EXERCISE (A)

Directions : Each of the following problems has a question and two statements which are labelled (1) and (2). Use the data given in (1) and (2) together to decide whether the statements are sufficient to answer the question. You mark

(A) if you can get the answer from (1) alone but not from (2) alone;

(B) if you can get the answer from (2) alone but not from (1) alone;

(C) if you can get the answer from (1) and (2) together, although neither statement by itself suffices;

(D) if statement (1) alone suffices and statement (2) alone suffices;

(E) if you cannot get the answer from statements (1) and (2) together, but need even more data.

(1) What was Mr. Mohan's income in 1990 ?

(1) His total income for 1988, 1989 and 1990 was Rs 50,000.

(2) He earned 20% more in 1989 than he earned in 1988.

(2) 50 students are taking at least one of the courses out of Chemistry and Physics. How many of the 50 students are taking Chemistry but not Physics ?

(1) 16 students are taking Chemistry and Physics.

(2) The number of students taking Physics but not Chemistry is the same as the number taking Chemistry but not Physics.

(3) How long will it take to travel from A to B ? It takes 4 hours to travel from A to B and back to A.

(1) It takes 25% more time to travel from A to B than it does to travel from B to A.

(2) C is midway between A and B, and it takes 2 hours to travel from A to C and back to A.

(4) Is a number divisible by 9 ?

(1) The number is divisible by 3.

(2) The number is divisible by 27.

(5) Is the integer 'K' odd or even?

(1) Square of K is odd.

(2) 2K is even.

(6) What percentage is Y's salary of X's salary ?

(1) X's salary is 80% of Z's salary.

(2) Y's salary is 120% of Z's salary.

(7) In a survey of 100 people, 70 people owned a T.V. or a telephone or both. If 30 people owned both a T.V. and a telephone, which group of surveyed people is larger: those who own a T.V. or those who own a telephone ?

(1) 25 people own a television but do not own a telephone.

(2) 45 people own a telephone.

(8) Train Y leaves N. Delhi at 1 a.m. and travels east at a constant speed of y m.p.h. Train Z leaves N. Delhi at 2 a.m. and travels east at a constant speed of z m.p.h. Which train will travel farther by 4 a.m.?

(1) $y > z$

(2) $y=1.2z$

(9) A square originally had sides with length 's'. The length of the side is increased by x%. Did the area of the square increase by more than 10% ?

(1) x is greater than 5.

(2) x is less than 10.

(10) There are 450 boxes to load on a truck. A and B working independently but at the same time take 30 minutes to load the truck. How long should it take B working by himself to load the truck ?

(1) A loads twice as many boxes as B.

(2) A would take 45 minutes by himself.

(11) A worker is hired for five days. He is paid Rs. 5 more for each day of work than he was paid for the preceding day of work. What was the total amount he was paid for the five days of work ?

(1) He had made 50% of the total by the end of the third day.

(2) He was paid twice as much for the last day as he was for the first day.

(12) How far is it from town A to town B ? Town C is 12 miles east of Town A.

(1) Town C is south of town B.

(2) It is 9 miles from town B to town C.

(13) Mohan must work 15 hours to make in wages the cost of a set of luggage. How many rupees does the set of luggage cost ?

(1) Shyam must work 12 hours to make in wages the cost of the set of luggage.

(2) Shyam's hourly wage is 125% of Mohan's hourly wages.

(14) A cylindrical tank has a radius of 10 feet and its height is 20 feet. How many gallons of a liquid can be stored in the tank ?

(1) A gallon of the liquid occupies about 0.13 cubic foot of space.

(2) The diameter of the tank is 20 feet.

(15) Two different holes, hole A and hole B, are made in the bottom of a full water tank. If the water drains out through the holes, how long will it take to empty the tank?

(1) If only hole A is put in the bottom, the tank will be empty in 24 minutes.

(2) If only hole B is put in the bottom, the tank will be empty in 42 minutes.

(16) A crate of oranges costs Rs. 25. What per cent of the cost of an orange is the selling price of an orange ?

(1) The oranges are sold for Rs. 1.30 each.

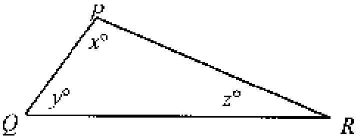
(2) There are 20 oranges in a crate.

(17) Ram and Sita are standing together on a sunny day. Ram's shadow is 3 metres long. Sita's shadow is 2.5 metres long. How tall is Sita?

(1) Ram is 2 metres tall.

(2) Ram is standing 0.75 metre away from Sita.

(18) In Δ PQR, what is the value of x?



(1) PQ = PR

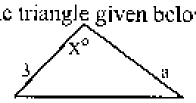
(2) y = 40

(19) Is x > y ?

(1) $0 < x < 0.75$

(2) $0.25 < y < 1.0$

(20) What is the area of the triangle given below?



(1) $a^2 + 9 = 25$

(2) x = 90

(21) Is x > y ?

(1) $x^2 > y^2$ (2) $x - y > 0$

(22) What is the value of the two-digit number x ?

(1) The sum of the two digits is 4.

(2) The difference between the two digits is 2.

(23) Is xy < 0 ?

(1) $x^2y^3 < 0$ (2) $xy^2 > 0$

(24) If $x = y^2$, what is the value of y - x ?

(1) x = 4 (2) x + y = 2

(25) How many minutes long is time period x ?

(1) Time period x is 3 hours long.

(2) Time period x starts at 11 p.m. and ends at 2 a.m.

(26) If the price of potatoes is 20 P per kg, what is the maximum number of potatoes that can be bought for Re 1.

(1) The price of a bag of potatoes is Rs 2.80.

(2) There are 15 to 18 potatoes in every 5 kg.

(27) A certain alloy contains only lead, copper and tin. How many pounds of tin are contained in 56 kg of the alloy?

(1) By weight the alloy is $\frac{3}{7}$ lead and $\frac{5}{14}$ copper.

(2) By weight the alloy contains 6 parts lead and 5 parts copper.

(28) If n is a positive integer, are n and 1 the only positive divisors of n?

(1) n is less than 14.

(2) If n is doubled, the result is less than 27.

(29) If φ is an operation, is the value of b φ c greater than 10?

(1) $x \varphi y = x^2 + y^2$ for all x and y.

(2) b = 3 and c = 2

(30) If today the price of an item is Rs 3,600, what was the price of the item exactly 2 years ago?

(1) The price of the item increased by 10% per year during this 2-year period.

(2) Today the price is 1.21 times its price exactly 2 years ago.

(31) If x is an integer, what is the value of x?

(1) $\frac{1}{5} < \frac{1}{x+1} < \frac{1}{2}$

(2) $(x - 3)(x - 4) = 0$

(32) Is $x^2 - y^2$ a positive number?

(1) (x - y) is a positive number.

(2) (x + y) is a positive number.

(33) If * is one of the operations — addition or multiplication—, which is it?

(1) $0 * 0 = 0$ (2) $0 * 1 = 1$

(34) What is the annual interest which a bank will pay on a principal o Rs 10,000?

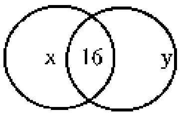
(1) The interest is to be paid every six months.

(2) The interest rate is 4%

SOLUTION

(1) E. Using (1) we can find the income for 1990 if we know the income for 1988 and 1989, but (1) gives no more information about the incomes for 1988 and 1989. If we also use (2) we can get the income in 1989 if we know the income for 1988. Therefore, both together are not sufficient.

(2) C



In the figure, x denotes the number taking Chemistry but not Physics, and y denotes the number taking Physics but not Chemistry. From (1) we know that $x + 16 + y = 50$; from (2), $x = y$. To know the value of x or y, we need both the equations (1) and (2). Thus neither statement alone can be solved for x, but both together are sufficient (and yield $x = 17$).

(3) A. Let x be the time it takes to travel from A to B and let y be the time it takes to travel from B to A. We know that $x + y = 4$. (1) says x is 25% more of y i.e. $x = 125\%$ of y or, $x = 1.25y$. So using (1) only we can get the value of x. Thus (1) alone is sufficient. (2) alone is not sufficient since we need information about the relation of x to y to solve the problem and (2) says nothing about the relation between x and y.

(4) B. Statement (1) is not sufficient, since 12 is divisible by 3 but 12 is not divisible by 9. Statement (2) alone is sufficient, since if a number is divisible by 27 then, because $27 = 9 \times 3$, the number must be divisible by 9.

(5) A. The square of an even integer is always even. So if square of K is odd, K cannot be even. Therefore, K is odd and (1) alone is sufficient. Statement (2) alone is not sufficient, since 2K is even for every integer K.

(6) C. To get the relation between the salaries of X and Y we need both the statements (1) and (2) at a time. Neither of the statements gives the relation of X's and Y's salaries independently.

(7) D. The people can be divided into three distinct groups which do not overlap :

X= people who own a T. V. but do not own a telephone;

Y= people who own both a T.V. and a telephone;

Z= people who own a telephone but do not own a T.V.

You are given that $X + Y + Z = 70$ and $Y = 30$. Since the groups you need to compare are those with a T.V. (i.e., those in X or Y) and those with a telephone (i.e., those in Y or Z), it is sufficient to know whether X or Z has more numbers. By the above equation, if you know X, then you can determine Z, and vice versa. Statement (1) is sufficient, since it tells you how many people are in group X. Statement (2) is sufficient, since it tells you how many people are in group Y or in group Z. Since there are 30 people in Y, you can determine how many are in Z.

(8) D. Since train Y travels for 3 hrs and train Z travels for 2 hrs, the distance train Y travels is 3y, and the distance train Z travels is 2z. Both the statements say that y is larger than z. Hence with the help of any of the statements you can get the answer.

(9) A. Statement (1) alone is sufficient. If x is equal to 5, then the area increases by 10.25%. As the statement (1) says that x is greater than 5, in this case area increased must be more than 10%. Statement (2) alone is not sufficient. As x is less than 10, it might be 1, 2, 3 or 4. In that case area cannot increase by more than 10%.

(10) D. Statement (1) is sufficient since it implies that A loaded 300 boxes in 30 minutes and B loaded 150 boxes in 30 minutes. So B should take 90 minutes to load the 450 boxes by himself.

Statement (2) is also sufficient since it implies A loads 10 boxes per minute; hence A loads 300 boxes in 30 minutes, and by the above argument we can deduce that B will take 90 minutes to load all the 450 boxes.

(11) D. Let x be the amount he was paid on the first day; then he was paid x+5, x+10, x+15 and x+20 for the remaining days of work. The total amount he was paid is 5x+50. Thus if we find x, we can find the total amount he was paid. Statement (1) is sufficient since after 3 days his total pay was 3x+15; that is equal to half of 5x+50. Thus we get the value of x. Statement (2) is sufficient since he was paid x+20 on the last day and so x+20=2x which also gives the value of x.

(12) C. Statement (2) alone is insufficient since you need to know what

direction town B is from town C. Statement (1) alone is insufficient since you need to know how far it is from town B to town C. Using both (1) and (2), A, B and C form a right triangle with legs of 9 miles and 12 miles. You need to find the hypotenuse.

(13) E. Statements (1) and (2) only give relations between Mohan's wages and Shyam's wages and tell you the cost of the set of luggage in terms of hours of wages. Since there is no information about the value of the hourly wages in rupees, statements (1) and (2) together are not sufficient.

(14) A. To find how many gallons the tank will hold, we need to calculate the volume of the tank and then divide this by the volume of one gallon of the liquid. Therefore statement (1) alone is sufficient. Statement (2) alone is not sufficient since it gives no further information about the tank.

(15) C. According to statement (1) hole A drains 1/24 of the tank in each minute. Since we have no information about B, statement (1) alone is not sufficient. Similarly, statement (2) alone is not sufficient. But if we use both the statements, we get that a certain part of the tank is drained out each minute which further gives the required answer.

(16) C. To know the relation between the cost and the selling prices of an orange we need both the informations, i.e. the number of oranges in a crate and the selling price of an orange. The cost of an orange can be gotten with the help of statement (2). And the selling price of an orange is given in statement (1). Hence both the statements together are sufficient and not independently.

(17) A. Statement (1) alone is sufficient. Since the shadows are proportional to their heights we can get the height of Sita by the law of proportionality. Statement (2) alone is not sufficient. The distance they are apart does not give us any information about their heights.

(18) C; By statement 1, PQ = PR; therefore, it is an isosceles triangle and $y = z$.

We know that $x + y + z = 180$;

or, $x + 2y = 180$.

Now, to know the value of x, we need the value of y. Which is given in statement 2. Hence we need both the statements.

(19) E; Clearly neither (1) nor (2) alone is sufficient to determine whether $x > y$. Thus, the answer must be C or E. Statement (1) and (2) together are also not sufficient to answer the question. For example if $x = 0.6$ and $y = 0.5$, $x > y$; but, if $x = 0.6$ and $y = 0.9$, $x < y$. Therefore, we can't conclude the result. Thus, answer is E.

(20) D; Statement (1) implies that $a = 4$. Thus, the given triangle is a 3-4-5 right-angle triangle and so $x = 90$. Therefore, the area of the triangle is $\frac{3 \times 4}{2}$. Statement (2) also indicates that the triangle is a right-angle triangle and hence $a = 4$ and area $= \frac{3 \times 4}{2}$. Thus our answer is D.

(21) B; Statement (1) is not sufficient to determine whether $x > y$ because it does not imply anything about the signs of x and y. For example, if $x = 3$ and $y = 2$, $x > y$, but if $x = -3$ and $y = 2$, $x < y$. Statement (2) clearly says that $x > y$. Thus is gives affirmative answer. Our answer should be B.

(22) E; **This is an unique example :** If we move with the help of method of equation:

Let the number be $10x + y$.

We are given that $x + y = 4$ ---- (1)

and $x - y = 2$ ------ (2)

Solving these two equations we get $x = 3$, $y = 1$

$\therefore$ number is 31.

But, statement (2) does not say that ten-digit is greater than unit-digit. It may be reverse, and hence equation (2) may be $y - x = 2$. In this case $x = 1$ and $y = 3$ and the number is 13.

(You are expected to mark this point in future)

(23) C; Statement (1) implies that $x \neq 0$ and $y \neq 0$ since the product is not equal to zero. x^2 must be greater than zero (as square value is always positive). Thus, $y^3 < 0$ or, $y < 0$. But still only statement (1) is not sufficient because we havn't got any inequality about x.

Statement (2) gives that $x > 0$ (because y^2 is always a +ve value), but it doesn't imply that $y > 0$ or $y < 0$.

Combining the two statements (1) & (2) we know that $y < 0$ and $x > 0$; so $xy < 0$. Thus, answer is C.

(24) C; From statement (1) we find $y = 2$ or -2. Therefore (1) alone is not a sufficient statement. From statement (2) we find that

$y^2 + y = 2 \Rightarrow y = 1$ or -2. Therefore,

(2) alone is also not sufficient to answer the question. Using (1) & (2) together we find that $x = 4$ and $y = -2$ (common value of y) and hence $y - x = -6$. Therefore, answer is C.

(25) A; Statement (1) is sufficient because from (1) we can determine that time period X is 180 minutes long.

Statement (2) alone is not sufficient to answer the question because we don't know whether the two given times are for consecutive days. This is the question that depends not on calculation but on your analysis of the assumptions made or not made by the statements.

(26) B; Clearly statement (1) alone is not sufficient to answer the question. Statement (2) alone is sufficient because 5 kg of potatoes can be bought for Re 1.00 and the maximum number of potatoes in 5 kg is 18. Therefore, B is the best answer.

(27) A; From statement (1) we know that $\frac{3}{14}$ part of alloy is tin by weight; so quantity of tin $= \frac{3}{14} \times 56 = 12$ kg. Since statement (2) does not tell how many parts tin are there in the alloy, we can't answer the question. Therefore, our answer is A.

(28) E; From statement (1) we don't know whether or not n is a prime number, and thus whether n and 1 are the only positive divisors of n. For example, 1 and 5 are the only positive divisors of 5, but 2 and 3 as well as 1 and 6 are positive divisors of 6. Since statement (2) is no more restrictive than (1), the answer is E.

(29) C; Statement (1) is not sufficient to answer the question because we don't know the values of b and c. Statement (2) alone is not sufficient because we don't know what operation φ represents. From (1) and (2) together we know that $3 \varphi 2 = 3^2 + 2^2 = 13$, therefore, answer is C.

(30) D; Statement (1) alone is sufficient because by the rule of compound interest,

If the price two years earlier be Rs x, then $x\left(1 + \frac{10}{100}\right)^2 = 3600$

$\Rightarrow x = 3600\left(\frac{100}{121}\right)$

Statement (2) also is sufficient because,

$x = \frac{3600}{1.21}$

Thus, both the statements independently are sufficient to answer the question.

(31) C; From (1) it can be concluded that $x + 1 = 3$ or $x + 1 = 4$; thus $x = 2$ or 3. From (2) it can be concluded that $x = 3$ or 4. Since the precise value of x can't be determined from either (1) or (2) taken alone, the answer must be C or E. If (1) and (2) are considered together, the only value of x that satisfies both conditions is $x = 3$. Therefore the answer is C.

(32) C; The expression $x^2 - y^2$ is a +ve number if and only if both its factors $(x + y)$ and $(x - y)$ are positive or both are negative. From (1) alone it can't be determined whether $x + y$ is positive. (Try to check it.) Similarly from (2) alone it can't be determined whether $x - y$ is positive.

Since both (1) and (2) are needed to establish that both the factors have the same sign, our answer is C.

(33) B; By statement (1) the operations could be either addition or multiplication.

By statement (2) it could only be addition.

(34) B; To get the annual interest, we need the rate of interest which is given only in statement (2).

Note : Don't combine the two statements. One may be confused by the two statements and conclude wrongly after combining the two. In that case the rate of interest becomes 2% and time becomes 2 yrs and hence you get a different answer.

Three-Statement Data Sufficiency (Type - I)

Directions: The following questions are accompanied by three statements A, B and C. You have to determine which statement(s) is/are sufficient/necessary to answer the questions.

Ex. 1: Find three positive consecutive even numbers.

A. The average of four consecutive even numbers starting from the last of the given numbers is 17.

B. The difference of the highest and the lowest number is 4.

C. The sum of the squares of the three numbers is 440.

1) A alone is sufficient   2) A and B are sufficient
3) C is sufficient   4) Either A or C is sufficient
5) All together are necessary

Soln: Let the three consecutive even numbers be $x - 2$, x and $x + 2$.

$A \Rightarrow \frac{(x+2)+(x+4)+(x+6)+(x+8)}{4} = 17$

or, $4x + 20 = 4 \times 17 \quad \therefore x = 12$

So, the numbers are 10, 12 and 14.

$C \Rightarrow (x-2)^2 + x^2 + (x+2)^2 = 440$

or, $2(x^2 + 4) + x^2 = 440$

or, $3x^2 = 432$ or, $x^2 = 144 \quad \therefore x = \pm 12$.

Neglecting the -ve value, we have $x = 12$ and the numbers are 10, 12 and 14.

Thus, we conclude that either A or C is sufficient to find out the numbers. Hence, the answer is 4.

Ex. 2: Find the number of sheep in a group of sheep and pigeons.

A. The total number of pigeons is one-third that of sheep.

B. In the group, the no. of legs is 36 more than twice the number of heads.

C. If 4 rabbits, which is two-thirds of the number of pigeons, be included in the group, the total number rises to 28.

1) Only B and C together are sufficient
2) Only A and C together are sufficient
3) Only B alone is sufficient
4) All even together are not sufficient
5) Either B alone or A and C together are sufficient

Soln: From A and C,

No. of pigeons $= 4 \times \frac{3}{2} = 6$

Then no. of sheep $= 6 \times 3 = 18$.

From B alone: no. of legs $= 2 \times$ no. of heads $+ 36$

or, $4S + 2P = 2(S + P) + 36$

Where S = no. of sheep and P = no. of pigeons.

(A sheep has 4 legs while a pigeon has 2, but each of them has only one head.)

Or, $2S = 36 \quad \therefore S = 18$

Thus our answer is 5.

Ex. 3: Sonu's income is how much more than Monu's?

A. Sonu's income is 30% less than her husband's whose provident fund deduction at the rate of 5% is Rs 975 per month.

B. Monu spends 30% of her income on house rent, 15% of which is electricity bill.

C. Sonu's expenditure on house rent is Rs 4500 more than that of Monu's.

1) Only B and C are sufficient 2) Only A and C are sufficient
3) Any two statements are sufficient 4) All together are necessary
5) All even together are not sufficient

Soln: From A, Sonu's income can be obtained. But Monu's income can't be obtained even with the help of B and C together. So, our answer is (5).

Ex. 4: Rs 310 is divided among three persons A, B and C. Find A's share.

A. B gets Rs 16 more than C.

B. A gets Rs 3 more than C.

C. A gets Rs 13 less than B.

1) Only A and C together are sufficient
2) Only B and C together are sufficient
3) All together are necessary
4) C and either A or B are sufficient
5) Any two of the three statements are sufficient

Soln: We have $A + B + C = 310$.

$(A) \Rightarrow B = 16 + C$; $(B) \Rightarrow A = 3 + C$; $(C) \Rightarrow A = B - 13$

So, from any two of the statements A, B and C the share of any person can be obtained. Thus, answer is (5).

Ex. 5: Find out the share of B out of the combined share of A, B and C of Rs 946.

A. The share of A is $\frac{2}{9}$ of the combined share of B and C.

B. The share of B is $\frac{3}{19}$ of the combined share of A and C.

C. The share of C is 2.143 times the combined share of B and A.

1) Only statements A and C are sufficient
2) Any two statements are sufficient
3) Only statement B alone is sufficient
4) Either statements A and C together or B alone is sufficient
5) All even together are not sufficient

Soln: Statement B alone is sufficient

$B : (A + C) = \frac{3}{19} = 3 : 19$

$\therefore B = \frac{3}{(3 + 19 =) 22} \times 946 = 3 \times 43 = \text{Rs } 129$

$(A) \Rightarrow A : (B + C) = \frac{2}{9} = 2 : 9$

$\therefore A = \frac{2}{(2 + 9 =) 11} \times 946 = \text{Rs } 172$

$(C) \Rightarrow C : (A + B) = 2.143 = \frac{2143}{1000} = 2143 : 1000$

$\therefore C = \frac{2143}{3143} \times 946 = \text{Rs } 645$

So, $(A) + (C) \Rightarrow A + C = 172 + 645 = 817$

We have, $A + B + C = 946$
Hence $B = 946 - 817 = $ Rs 129.
Thus A and C together are also sufficient. Thus, answer is (4).

Ex. 6: Income of P is $1\frac{2}{3}$ of the income of Q. The expenses of P, Q and R are in the ratio of 6 : 4 : 5. Find the expenses of Q.
A. Expenses of R is Rs 2000 less than that of P.
B. P's saving is Rs 3000.
C. Income of R is one-third of the total incomes of P, Q and R of Rs 36000.
1) Only B and C together are sufficient
2) Only A alone is sufficient
3) All together are necessary
4) Either B and C together or A alone is sufficient
5) All even together are not sufficient

Soln: (A) $\Rightarrow$ expenses of (P - R) = 6 - 5 = 1 ≡ Rs 2000
$\therefore$ Expense of Q = 4 ≡ Rs 8000
(C) $\Rightarrow$ Income of R = $\frac{36000}{3}$ = Rs 12000
$\therefore$ Income of (P + Q) = Rs 24000
Income of P + $\frac{3}{5}$ income of P = Rs 24000
$\therefore$ Income of P = $24000 \times \frac{5}{8}$ = Rs 15000
(B) $\Rightarrow$ P's saving = P's (income - expense) = Rs 3000
Now, (B) + (C) $\Rightarrow$ P's expense = 15000 - 3000 = Rs 12000
$\therefore$ Expense of Q = $\frac{1200}{6} \times 4$ = Rs 8000
Hence our answer is (4).

Ex. 7: Mohan is 6 years older than Sohan. What will be the sum of their present ages?
A. After 6 years the ratio of their ages will be 6 : 5.
B. The ratio of their present ages is 5 : 4.
C. 6 years ago the ratio of their ages was 4 : 3.
1) Only B alone is sufficient
2) Only A and C together are sufficient
3) Only A alone is sufficient
4) Any one of A, B and C is sufficient
5) All even together are not sufficient

Soln: Our answer is (4). Check it yourself.

Ex. 8: What will be the average of three numbers?
A. The difference of the first two numbers is 2.
B. The largest no. is greater than the smallest no. by 10.
C. The difference of the last two numbers is 8.
1) Only A and B together are sufficient
2) Only B and C together are sufficient
3) Any two statements together are sufficient
4) All together are necessary
5) All even together are not sufficient

Soln: Answer is (5). Try it yourself.

Ex. 9: Find the value of the integer a.
A. $a^2 \le 61$ **B.** $a < 5$ **C.** $a^2 > 31$
1) Only A and B together are sufficient
2) Only A and C together are sufficient
3) Any two of the three together are sufficient
4) All together are necessary
5) All even together are not sufficient

Soln: We have, $x^2 = k \Rightarrow x = \pm\sqrt{k}$
$x^2 < k \Rightarrow -\sqrt{k} < x < \sqrt{k}$
and $x^2 > k \Rightarrow x < -\sqrt{k}$ or $x > \sqrt{k}$
(A) $\Rightarrow a^2 < 61 \Rightarrow -\sqrt{61} \le a \le \sqrt{61}$
Since $(7^2 =)\ 49 < 61 < (8^2 =)\ 64$ and a is an integer so we have, $-7 \le a \le 7$.
(B) $\Rightarrow a < 5$
(C) $\Rightarrow a^2 > 31 \Rightarrow a < -\sqrt{31}$ or $a > \sqrt{31}$
Since $(5^2 =)\ 25 < 31 < (6^2 =)\ 36$ and a is an integer, so we have $a < -5$ or $a > 5$
Combining all these, we get a = -6, -7.
No single value of a is obtained. Hence our answer is (5).

Ex. 10: If m and n are integers, is m + n an odd number?
A. $m \le n$ **B.** $11 < m \le 13$ **C.** $12 < n \le 14$
1) Only A and B together are sufficient
2) Only B and C together are sufficient
3) Any two of the three together are sufficient
4) All together are necessary
5) All even together are not sufficient

Soln: (B) $\Rightarrow$ m = 12, 13
(C) $\Rightarrow$ n = 13, 14

(A) ⇒ $m \le n$

We see that no combination of statements gives the certain value of $m + n$.

Even after combining the three statements, we get $m = 12, 13$ and $n = 13$ ($n = 14$ is not acceptable).

Still when $m = 12$ then $m + n = 12 + 13 = 25$, an odd no.

When $m = 13$ then $m + n = 13 + 13 = 26$, an even no.

Hence our answer is (5).

Ex. 11: A customer is given two successive discounts on an article. To find the second discount, which of the following informations is/are necessary/sufficient?

A. The cost price of the article.
B. The selling price of the article.
C. The first discount percentage is 75% of the second discount percentage.

1) Only A and B together are sufficient
2) Only B and C together are sufficient
3) Any two statements together are sufficient
4) All the three together are necessary
5) All even together are not sufficient

Soln: Suppose second discount percentage is x%. Then with the help of (C), first discount % $= \frac{3}{4}x\% = 0.75x\%$

$$SP = CP\left(\frac{100 - 0.75x}{100}\right)\left(\frac{100 - x}{100}\right)$$

Since SP and CP are given in statements (A) and (B), we can find the value of x. Thus, answer is (4).

Ex. 12: A shopkeeper sold a watch and got Rs 225 as profit. Find the profit percentage.

A. Selling price of the watch is Rs 650.
B. He gave 20% discount on the labelled price, which is Rs 812.50.
C. Cost price of the watch is Rs 425.

1) Only either B or C is sufficient
2) Only either A or C is sufficient
3) Only A and C together are sufficient
4) Any one of A, B and C is sufficient
5) Any two of A, B and C are sufficient

Soln: (B) ⇒ Selling price = (100 - 20 =) 80% of Rs 812.50 = Rs 650

$$\text{Profit percentage} = \frac{\text{Profit}}{\text{CP } (= \text{SP} - \text{Profit})} \times 100$$

As the profit is already given, if either CP or SP is known, profit percentage can be obtained. So, the answer is (4).

Ex. 13: What is the gain or loss per cent of Seema who sells two chairs?

A. She sells one chair at 25% loss.
B. She sells the other chair at 25% gain.
C. She has bought her two chairs for Rs 2760.

1) All together are necessary
2) All even together are not sufficient
3) Only A and B together are sufficient
4) Only C and A together are sufficient
5) Any two statements are sufficient

Soln: (C) gives the cost price of each chair, which is $\frac{2760}{2}$ = Rs 1380

(A) and (B) give the selling price of each chair. Hence, with the help of all the three statements, we can find the profit and hence the % profit. Hence, answer is (1).

Ex. 14: The compound interest on a sum of Rs 4000 is Rs 1324. Find the rate of interest.

A. The simple interest on the same sum at the same rate is Rs 1200.
B. Compound interest is compounded every four months.
C. The sum doubles itself in 25 years at the rate of 4% per annum.

1) Only B and C together are sufficient
2) Only A and C together are sufficient
3) All together are necessary
4) Either B and C together or A and C together are sufficient
5) All even together are not sufficient

Soln: C is not an informative statement because it is true in all cases. In order to find out the rate of interest, we need the time for which the sum has been deposited. But this has not been provided either in A or in B. So, answer is (5).

Ex. 15: A person deposited two amounts to a money lender at 5% simple interest for 3 years and 5 years. Find the two amounts.

A. Difference between the interests is Rs 600.
B. The two amounts are equal.
C. Had the amounts been deposited at 5% compound interest, the difference would have been Rs 71194.

1) Only A and B together are sufficient
2) Only A and C together are sufficient
3) Any two statements together are sufficient
4) B and either A or C is sufficient
5) All together are necessary

Soln: Any two statements are sufficient. The answer is (3).

Ex. 16: Two friends Sheela and Meena earned profit in a business. Find out their shares.

A. Sheela had invested her capital for 9 months and Meena for 1 year.
B. The ratio of their capitals was 4 : 3.
C. The total profit was Rs 27500.

1) Only A and B together are sufficient
2) Only B and C together are sufficient
3) All together are necessary
4) Either B or A and C together are sufficient
5) All even together are not sufficient

Soln: From (A) and (B), we have

Ratio of profits = $9 \times 4 : 12 \times 3 = 36 : 36 = 1 : 1$

Now, with help of (C), shares of each of them $= \frac{27500}{1+1} \times 1$
$= \text{Rs } 13750$

Hence, the answer is (3).

Ex. 17: What is the relative speed of two trains running in opposite directions?

A. They take 15 seconds to cross each other.
B. The speed of one of the trains is 60 kmph.
C. The total length of the trains is 360 m.

1) Only A and B together are sufficient
2) Only B and C together are sufficient
3) Only A and C together are sufficient
4) Any two statements together are necessary
5) All together are necessary

Soln: Relative speed $= \frac{\text{Total length of the trains}}{\text{Time taken to cross each other}}$

So, from A and C, we get the relative speed $= \frac{360}{15} = 24$ m/s

Whereas from A and B, or from B and C the relative speed cannot be obtained. Answer is (3).

Ex. 18: A man can row a distance of x km up the stream in 3.5 hours. Find his speed in still water.

A. He covers a distance of 84 km downstream in 6 hours.
B. He covers the distance of x km downstream in 2.5 hours.
C. The speed of the current is 2 kmph.

1) Only B and C together are sufficient
2) Only A and B together or A and C together are sufficient
3) Any two statements together are sufficient
4) All together are necessary
5) Either B alone or A and C together are sufficient

Soln: Upstream speed $= \frac{x}{3.5}$ km/hr (given in the question)

(A) + (B) $\Rightarrow$ Downstream speed $= \frac{84}{6} = 14$ km/hr

and also downstream speed $= \frac{x}{2.5}$

Now, $\frac{x}{2.5} = 14 \quad \therefore x = 2.5 \times 14 = 35$ km

From the given information,

Upstream speed $= \frac{x}{3.5} = \frac{35}{3.5} = 10$ km/hr

Thus, his speed in still water $= \frac{10+14}{2} = 12$ km/hr

(B) + (C) $\Rightarrow$ His speed in still water $= \frac{x}{3.5} + 2 = \frac{x}{2.5} - 2$

or, $\frac{x}{2.5} - \frac{x}{3.5} = 2 + 2 = 4$

or, $\frac{x}{2.5 \times 3.5} = 4 \qquad \therefore x = 35$ km.

$\therefore$ His speed in still water $= \frac{x}{3.5} + 2 = \frac{35}{3.5} + 2 = 12$ km/hr

(A) + (C) $\Rightarrow$ His speed in still water $= \frac{84}{6} - 2 = 14 - 2 = 12$ km/hr

Hence answer is (3).

Ex. 19: P works for 4 days and leaves the job. In how many days can P alone finish the entire work?

A. Q finishes the remaining work in 8 days.

B. P and Q together can finish the work in $6\frac{2}{3}$ days.

C. The working efficiency of Q is double that of P.
1) Only B and C together are sufficient
2) Only A and B together or A and C together are sufficient
3) Any two statements together are sufficient
4) All together are necessary
5) Either B alone or A and C together are sufficient

Soln: Suppose P alone can finish the work in x days.

From A, we get P has worked for 4 days and done $\frac{4}{x}$ part of the work.

The remaining work = $1 - \frac{4}{x} = \frac{x-4}{x}$ part of the work that has been done by Q in 8 days. So, Q alone can finish the work in $\frac{8x}{x-4}$ days.

Now, from (A) and (B), we get,

$$\frac{\frac{8x}{8-4} \times x}{\frac{8x}{x-4} + x} = 6\frac{2}{3}$$

So, x can be obtained. (On solving, you will get x = 20.)

From C, we get Q alone can finish the work in $\frac{x}{2}$ days.

Now, from (A) and (C), we get

$$\frac{8x}{x-4} = \frac{x}{2}$$

So, x can be obtained (x = 20).
Now, from (B) and (C), we get

$$\frac{x \times \frac{x}{2}}{x + \frac{x}{2}} = 6\frac{2}{3}$$

So, x can be obtained (x = 20). Thus, the answer is 3.

Ex. 20: A water tank has been filled with two filler taps P and Q and a drain pipe R. Tap P and Q fill at the rate of 12 and 10 litre per minute respectively. What is the capacity of the tank?

A. Tap R drains out at the rate of 6 litres per minute.

B. If all the three taps are opened simultaneously, the tank is filled in 5 hours 45 minutes.

C. Tap R drains the filled tank in 15 hours 20 minutes.
1) Only A and B together are sufficient
2) Only either A and B together or A and C together are sufficient
3) All the three together are necessary
4) Any two statements are sufficient
5) Only either A and B together or B and C together are sufficient

Soln: With the help of (A) and (B):

All the three taps opened simulataneously fill 12 + 10 - 6 = 16 lt per minute

∴ Capacity of the tank = 16 × (5 × 60 + 45) = 5520 lt.

With the help of (A) and (C):

Capacity of the tank = 6 × (15 × 60 + 20) = 5520 lt.

With the help of (B) and (C):

Let the tap R drain out at the rate of x lt per minute.
Then, all the three taps opened simultaneously fill 12 + 10 - x = 22 - x litres per minute.

∴ Capacity of the tank = (22 - x) (5 × 60 + 20) and so x can be obtained.

Then substituting the value of x in any one of the above equations, the capacity of the tank can be obtained. Thus, the answer is (4).

Ex. 21: To find the total surface area of a hemisphere, we need which of the following informations?

A. Curved surface area of the hemisphere.
B. Volume of the hemisphere.
C. Radius of the hemisphere.
1) Only C alone is sufficient
2) Only B alone is sufficient
3) Only either C alone or A and B together are sufficient
4) Any one of the statements alone is sufficient
5) Only either C or B is sufficient

Soln: For a hemisphere we have, its curved surface area = $2\pi r^2$ sq. units

Total surface area = curved surface area + area of the base

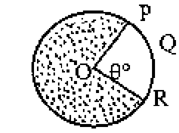
$= 2\pi r^2 + \pi r^2 = 3\pi r^2$ sq. units and volume $= \frac{2}{3}\pi r^3$ cu. units.

(C) gives the value of r directly whereas (A) and (B) give the value of r indirectly.

Thus, we conclude that any one of the statements is sufficient, i.e. answer is (4).

Ex. 22: To find out the area of the shaded portion of the figure, which of the given informations is needed?

A. Circumference of the circle.
B. The value of θ.
C. The length of the arc PQR.



1) Only A and B together are sufficient
2) Only A and C together are sufficient
3) Only A and either B or C are sufficient
4) Any two of A, B and C are sufficient
5) All together are necessary

Soln: Area of the shaded portion of the figure = (area of the circle =) πr^2 - area of the sector PQRO.

Area of the sector PQRO $= \frac{\pi r^2 \theta^\circ}{360} = \frac{1}{2} \times r \times$ arc PQR

(A) $\Rightarrow$ radius of the circle

So, from (A) and (B) or (A) and (C), the required area can be determined.

From (B) and (C) and the equation $\frac{\pi r^2 \theta^\circ}{360} = \frac{1}{2} \times r \times$ arc PQR

The value of r can be obtained first; consequently the area of the sector PQRO; and finally the reqd area can be obtained.

Thus, we conclude that any two statements together are sufficient. So, the answer is (4).

Ex. 23: To find the length of the carpet which covers the floor of a rectangular hall, which of the following informations is needed?

A. The length of the hall is 28 m.
B. The width of the carpet is 2.5 m.
C. The area of the hall is 490 sq. m.

1) Only A and B together are sufficient
2) Only B and C together are sufficient
3) Only A and C together are sufficient
4) Any two statements are sufficient
5) All together are necessary

Soln: Width of the carpet is needed, which is given in statement (B). Now, we need the area of the floor, which is given in statement (C). So, only B and C together are sufficient. Hence, the answer is (2).

Ex. 24: How many ice cubes can be accommodated in a container?

A. The length and breadth of the container are 16 cm and 12 cm respectively.
B. The edge of the ice cube is 2 cm.
C. The container is 384 times heavier as compared to a single ice cube.

1) Only A and B together are sufficient
2) Only B and C together are sufficient
3) Any two statements are sufficient
4) All together are necessary
5) All even together are not sufficient

Soln: To get the required number of ice cubes we need the volume of the container as well as that of an ice cube.

From (B), we have, the volume of an ice cube = (2^3 =) 8 cu. cm.

Still, as there is no information regarding the height of the container, so the volume of the container and consequently the no. of ice cubes cannot be determined. So, answer is (5).

EXERCISES (B)

(Asked in previous exams)

1. A trader sold an article at Rs 625. To find out his profit percentage which of the following statements is/are necessary/sufficient?

A. He gained Rs 600 by selling 8 such articles.
B. The cost price of the article is Rs 545 and transportation cost is Rs 5 each.

1) Only A 2) Only B
3) Both A and B are necessary/sufficient
4) Either A or B is sufficient
5) Neither A nor B is sufficient

2. Rohit got 700 marks in the examination. To find out his percentage of marks which of the following information is/are necessary/sufficient?

A. He appeared in all the nine papers.
B. The highest marks obtained is 90.
C. The maximum marks for each paper is 100.

1) Only A is sufficient 2) Only C is sufficient

3) Only A and B together are sufficient
4) Only A and C together are sufficient
5) All A, B and C are necessary

3. Two friends earned a profit in a business. To find out their shares which of the following statements is/are necessary/sufficient ?
A. The total profit was Rs 25000.
B. Rohan had invested his capital for 1 year against Sohan who had invested for 8 months.
C. The ratio of their capital was 2 : 3.
1) Only A and B together are sufficient
2) Only B and C together are sufficient
3) Only B alone is sufficient
4) All A, B and C together are necessary
5) None of these

Directions (Q. 4-8): Each of the questions below consists of a question and two statements numbered I and II given below it. You have to decide whether the data provided in the statements are sufficient to answer the question. Read both the statements and Give answer

1) if the data in **statement I alone** are **sufficient** to answer the question, while the data in **statement II alone** are **not sufficient** to answer the question.
2) if the data in **statement II alone** are **sufficient** to answer the question, while the data in **statement I alone** are **not sufficient** to answer the question.
3) if the data either in **statement I alone** or in **statement II alone** are sufficient to answer the question.
4) if the data even in **both the statements I and II** together are **not sufficient** to answer the question.
5) if the data in **both the statements I and II together** are **necessary** to answer the question.

4. What is the height of a triangle?
I. The area of the triangle is 20 times its base.
II. The perimeter of the triangle is equal to the perimeter of a square of 10 cm side.

5. What was the cost price of the suitcase purchased by Samir?
I. Samir got 20 per cent concession on the labelled price.
II. Samir sold the suitcase for Rs 2,000 with 25 per cent profit on the labelled price.

6. What was the speed of a running train?
I. The train crosses a signal post in 6 seconds.
II. The train crosses another train running in the opposite direction in 15 seconds.

7. What percentage rate of simple interest per annum did Ashok pay to Sudhir?
I. Ashok borrowed Rs 8,000 from Sudhir.
II. Ashok returned Rs 8,800 to Sudhir at the end of two years and settled the loan.

8. What was the ratio between the ages of P and Q four years ago?
I. The ratio between the present ages of P and Q is 3 : 4.
II. The ratio between the present ages of Q and R is 4 : 5.

9. A train crosses another train coming from the opposite direction in 18 sec. If the length of the train is 100 m, then to find out the speed of the train which of the following statements P, Q and R is sufficient/necessary?
P. The speed of the other train is 60 km/h.
Q. The train passes a platform in 14 sec.
R. The other train passes a telegraph pole in 6 sec.
1) Only P and Q together are sufficient.
2) Only Q and R together are sufficient.
3) Only P and R together are sufficient.
4) All P, Q and R together are necessary.
5) None of these

Directions (Q. 10-14): In each of the following questions, there is a question followed by two statements. You have to decide if the informations given in the statements is/are sufficient to answer the questions. Give answer

1) if the information given in **statement I alone is sufficient** to answer the question while information given in **II** is **not sufficient** to answer the question.
2) if the information given in **statement II alone is sufficient** to answer the question while the **statement I** is **not sufficient** to answer the question.
3) if either **I alone** or either **II alone** is **sufficient** to answer the question.
4) if both **statement** I and II are **necessary** to answer the question.
5) if **statements** I and II even together are **not sufficient** to answer the question.

10. What is the perimeter of a semi-circle?
I. The radius of the semi-circle is 7 m.
II. The area of the semi-circle is 154 cm^2.
11. Area of a right-angled triangle is equal to the area of a rectangle. What is the length of the rectangle?
I. The area of triangle is 100 m^2.
II. The base of triangle is equal to breadth of the rectangle.
12. What will be the difference of simple interest and compound interest of a sum at the same rate of interest after 3 yrs?
I. The rate of interest per annum is 5%.
II. The simple interest for 3 years on that sum is Rs 750.
13. What is the distance between A and B?
I. A scooterist covered the distance in half an hour.
II. The initial speed of the scooterist was 40 km/hr.
14. What profit did Rohit make if he sold the watch for Rs 600?
I. He bought the watch at 20% discount.
II. He sold the watch at 5% higher than the marked price.
15. Rs 16000 is divided among A, B and C. To find the share of C which of the following information(s) is/are necessary/sufficient?
M. C gets 4.5 times less than B.
N. A gets 2.5 times more than C.
O. B gets Rs 4000 more than A.
1) Only M and N together are sufficient.
2) Only M and O together are sufficient.
3) Only N and O together are sufficient.
4) All M, N and O are necessary.
5) Any two of the three are sufficient.
16. A triangle has hypotenuse measuring 42.42 cm. To find out its other sides which of the following information(s) is/are sufficient/necessary?
A. It is a right angled triangle.
B. Two of its sides are equal.
C. Hypotenuse is $\sqrt{2}$ times its each side.
1) Only A alone 2) Only B alone 3) Only C alone
4) Either C or B alone 5) None of these
17. P and Q worked together for 5 days. Then Q left the work and the work was finished by P alone in 8 days. How many days would Q alone take to finish the work if P had left? To answer the question, Which of the following information(s) is/are necessary/sufficient?

A. P and Q together can do the work in 10 days.
B. P alone can do the work in 16 days.
1) Only A alone 2) Only B alone
3) Either A or B alone 4) Both A and B together
5) Both together are even not sufficient
18. A product was sold at a profit of 15% after giving a discount of 20% on marked price. To find the cost price which of the following statement(s) is/are necessary/sufficient?
A. The discount given is Rs 350.
B. The marked price of the article is Rs 1750.
1) Only A alone 2) Only B alone
3) Both A and B together are necessary
4) Either A or B alone is sufficient
5) Neither A nor B alone is sufficient
19. An amount yields its $\frac{2}{5}$ as simple interest for 4 years. To find out its principal which of the following statement(s) is/are necessary/sufficient?
A. The rate of interest is 10%.
B. The interest for 2 years is Rs 450.
1) Only A alone 2) Only B alone 3) Either A or B alone
4) Both A and B together are necessary
5) Neither A nor B alone is sufficient
20. The ratio of the ages of Toni and Moni is 5 : 3. To find out the ratio of their ages after 5 years, which of the following is/are necessary/sufficient?
A) The sum of their ages is 48 years.
B) The difference of their ages is 12 years.
C) The ratio of their ages 6 years before was 2 : 1.
1) Either A or C only 2) Either B or C only
3) A, B and C together 4) Either A or B
5) Any one of the three
21. A scooter is sold for Rs 15000. To find out the profit percentage which of the following informations is/are sufficient/necessary?
A) If it would be sold at Rs 16,500 profit would be double.
B) The cost price was 15% less than the selling price.
1) Only A is sufficient 2) Only B is sufficient
3) Either A or B is sufficient 4) A and B together is necessary
5) Neither A nor B is sufficient

22. The walls of a room are to be mounted with wall paper 36 inches wide. To find out the length of paper, which of the following informations is/are necessary/sufficient ?
   A) The height of the room is 14 ft.
   B) The total area of doors and windows is 56 sq. ft.
   C) The perimeter of the room is 28 ft.
   1) Only B and C together
   2) Only A and B together
   3) The three even together are not sufficient
   4) B and either A or C
   5) All are necessary
23. The area of a rhombus is 1152 m^2. To find out its side which of the following is/are necessary/sufficient?
   A) One of its diagonal is 36 m.
   B) The other diagonal is 32 m.
   C) All the four sides are equal.
   1) A and B together are sufficient.
   2) Only A alone is sufficient.
   3) Only B alone is sufficient.
   4) Either A alone or B alone is sufficient.
   5) All A, B and C are necessary.
24. In how many days can Rajan finish the remaining work? To get the answer which of the following informations is/are necessary/sufficient?
   A) Rajan and Suman together can finish the work in 6 days.
   B) Suman alone can finish the work in 12 days.
   C) Rajan and Suman worked together for 2 days.
   1) Only A and B are sufficient.
   2) Only B and C are sufficient.
   3) Only A and C are sufficient.
   4) All A, B and C are necessary.
   5) All together are even not sufficient.
25. What is the age of Amit? To find out his age. Which of the following informations is necessary/sufficient?
   A) Amit is half the age of his father.
   B) His father is 25 yrs older than his brother.
   C) 10 yrs ago the ratio of his father's and his brother's age was 5 : 3.
   1) Only B and C are sufficient
   2) Only A and C are sufficient

   3) All A, B and C are necessary
   4) Only B and C are sufficient
   5) Any two of the three statements are sufficient
26. How much vote did the winner get in the election if the total electorate was 6 lakh. To get the answer which of the following informations is/are necessary/sufficient?
   A) He got 250% more than his rival.
   B) He defeated his rival by the margin of 1.5 lakh votes.
   C) Of the 58% votes polled the runner got only 13%.
   1) Only A and B together
   2) Only A and C together
   3) A, B and C together are necessary
   4) A and either B or C is sufficient
   5) Any two of the three statements are sufficient
27. Two friends started from two places to meet each other. When and where will they come across each other? To get the answer which of the following is/are necessary/sufficient?
   A) A started from Delhi at 60 km/hour for Panipat at 9 am.
   B) B started for Delhi from Panipat at 10 am.
   C) Panipat is 200 km from Delhi.
   1) Only A and B together are sufficient.
   2) Only A and C together are sufficient.
   3) Only B and C together are sufficient.
   4) All A, B and C are necessary.
   5) All together are even not sufficient.
28. A hawker sells 6 oranges for Rs 5. What percentage of profit does he get? To get the answer which of the following informations is/are necessary/sufficient?
   A) By selling at Rs 12 per dozen he would get 16% more.
   B) He bought the oranges at Rs 63 per hundred.
   1) Only A alone is sufficient
   2) Only B alone is sufficient
   3) Both A and B together alone are necessary
   4) Either A or B alone is sufficient
   5) Neither A alone nor B alone is sufficient
29. A goods train X crosses another train Y in 28 sec. To find the length of Y which of the following statements is/are sufficient/necessary?
   A. Speed of X is 45 km/hr
   B. Length of X is 130 m

C. Speed of Y is 54 km/hr
1) Only A and B together are sufficient
2) Only B and C together are sufficient
3) A, B and C together are necessary
4) Only A and C together are sufficient
5) All the three even together are not sufficient

30. On the sale of a cooler the dealer gets 6% profit. To get the CP which of the following statements is/are sufficient/necessary?
A) The sale price for the cooler is 3690.
B) By selling it at Rs 3795 he would get one-and-a-half times more profit.
1) Only A alone is sufficient
2) Only B alone is sufficient
3) A and B together are necessary
4) Either A alone or B alone is sufficient
5) A and B even together are not sufficient

31. A trader sells a homogeneous mixture of A and B at the rate of Rs 17 per kg. To find the profit percentage of the trader which of the following statements is/are sufficient/necessary?
A) He bought A at the rate of Rs 20 per kg.
B) He bought B at the rate of Rs 13 per kg.
1) Only A alone is sufficient
2) Only B alone is sufficient
3) Either A alone or B alone is sufficient
4) A and B together are sufficient
5) A and B even together are not sufficient

32. Ram's age is $\frac{2}{3}$ of Shyam's. To get Ram's age which of the following statements is/are sufficient/necessary?
A) Ram is 6 years older than Gopal.
B) 5 years ago ratio of Shyam's and Gopal's age was 3 : 5.
C) After 5 years the ratio of the ages of Ram and Shyam will be 1 : 2.
1) Only A and B together are sufficient
2) Only B and C together are sufficient
3) Only A and C together are sufficient
4) A, B and C together are necessary
5) Either C alone or A and B together are sufficient

33. A trapezium-shaped field is having parallel lines in the ratio of 5 : 3. To get the vertical distance between the lines, which of the following statements is/are sufficient/necessary?
A) The area of the field is 532 m^2.
B) The sum of parallel lines is 56 m.
1) Only A alone is sufficient
2) Only B alone is sufficient
3) A and B together are necessary
4) Either A alone or B alone is sufficient
5) A and B together even together are not sufficient

34. A person bought an article at 15% discount on marked price and sold it for Rs 1600. To get the percentage profit which of the following statements is/are sufficient/necessary?
A) The marked price of the article is Rs 1500.
B) By selling the article at Rs 153 more he would get 12% more.
1) Only A alone is sufficient
2) Only B alone is sufficient
3) Either A alone or B alone is sufficient
4) A and B together are necessary
5) Neither A alone nor B alone is sufficient

35. Mahesh got Rs 1200 as dividend from a finance co. What is the rate of interest given by the company? To get the answer which of the following is/are sufficent/necessary?
A. Mahesh has 960 shares of Rs 10 denomination.
B. The dividend paid last year was 9.5%
1) Only A is sufficient
2) Only B is sufficient
3) Either A or B is sufficient
4) A and B together are necessary
5) A and B even together are not sufficient

36. A boat sails 13 km upstream in 4 hrs. What is the speed of the stream? To get the answer which of the following informations is/are sufficient/necessary?
A. The boat goes downstream in 2 hrs.
B. The boat sails 6 km/hr in still water.
1) Only A alone is sufficient
2) Only B alone is sufficient
3) Either A alone or B alone is sufficient
4) A and B together are necessary

5) A and B even together are not sufficient

37. The ratio of boys and girls in a class is 5:3. To get the number of boys in the class which of the following informations is/are sufficient/necessary?

A) The number of girls is 60% of the number of boys.

B) If 12 more girls are brought the number of boys and girls will be equal.

C) The total number of students is 48.

1) Only A and B together are sufficient
2) Only B and C together are sufficient
3) Only B alone is sufficient
4) Only C alone is sufficient
5) Only either B alone or C alone is sufficient

38. A person lent Rs 2500 each to two of his friends. What is the rate of interest? To get the answer which of the following informations is sufficient/necessary?

A. Amount repaid by one of them was Rs 2700.

B. The other one repaid Rs 2900 after the due period.

C. Amount was given to one of them on simple interest and to another on compound interest.

1) Only A and B together are sufficient.
2) Only B and C together are sufficient.
3) A, B and C together are necessary.
4) Either A and B together or B and C together are sufficient
5) A, B and C even together are not sufficient

Answers

1. 4; If the sales price is given, profit percentage can be summed up if we know either CP or profit.

2. 4; To find out the percentage total marks has to be found out which is possible with A and C; that is, he got 700 marks out of 900.

3. 4

4. 4; From statement I If base = x then area = 20x

$$\text{height} = \frac{2 \times 20x}{x} = 40$$

But we don't get the unit of the height of the triangle. It may be in inches, cms, metres or kilometres. Then we can't get the correct value.

5. 5; From statement II,

$$\text{Labelled price} = 2000 \times \frac{100}{125} = \text{Rs } 1600$$

Now, with help of statement I,

$$\text{Cost price} = 1600\left(\frac{80}{100}\right) = \text{Rs } 1280$$

4; We can't find out the speed unless we know the length of the train(s).

5; $R = \frac{800 \times 100}{8000 \times 2} = 5\%$

4

1; From P and R we can find out the length of the other train and thus the distance covered in the given period, which is necessary to find out the speed.

3; Perimeter of the semicircle can be found out with either of the statement.

5

4; With the help of I and II we can find out the principal and hence the difference.

5

4; From II we can find out the tag price and from I the purchase price, and thus the profit.

5; We have three unknowns (A, B & C). From the question we find one equation as: A + B + C = 1600.
We need two more equations to find the solution. These two equation may be found from any two of the given three statements.

4; With either of the informations we can find out the other side. (A) is a repetition of what is already given in the question: a hypotenuse can be there *only* in a right-angled tringle.

3; We can find this out with any of the two statements.

4; We can find out the cost price with the help of either of the statements.

2; Principal can be found out with the help of B only. Statement A is true for any principal.

5; Ratio of their ages can be found out with help of any of the three informations.

3; The cost price can be found out with either of the two informations.

5; All the informations are necessary to find out total area of walls to be mounted with paper.

4; Area of rhombus = Product of diagonals. Thus, with the help of either (A) or (B) we can get the other diagonal. Now, we have both the diagonals which provide us the length of side.

4

25. 3; From B and C, we get the father's age and then from A we get A
age.
26. 4; From B, we get the difference of votes of the winner and the ru
ie, 1.5 lakh. By using A we can calculate the votes secured by wi
From C we get the number of votes the runner won and by usi
we get the votes won by the winner.
27. 5; We can't get the time since speed of B is not given.
28. 4
29. 5; We can't find out the length of another train unless we know direc
of both the trains.
30. 4; From either of the two statements we can find out CP.
31. 5; We can't find out the profit/loss unless we know the ratio of A &
in the mixture.
32. 5; From A & B we can find out Gopal's age and hence Ram's age. Fro
C we can get the answer directly.
33. 3; From A we get the area and from B sum of parallel lines. So bot
necessary.

$$\text{Area} = \frac{1}{2} \times \text{height} \times \text{sum of parallel lines}$$

34. 3
35. 1; Dividend of last year and this year may not be the same.
36. 2; A does not state what distance it covers in 2 hrs. From B we ca
the answer.
37. 5; We can get the answer either from B or C. A is a mere repetiti
the question.
38. 5; Since no time is given, we can't find the rate.

Three-Statement Data Sufficiency (Type - II)

BSRB Bangalore PO held on 7th March 1999

Directions: (Ex. 1-5): *In each of the following questio question is asked followed by three statements. You have to stud questions and all the three statements given and decide whethe information provided in the statement(s) are redundant and ca dispensed with while answering the questions?*

1. What is the area of the given rectangle?
   A. Perimeter of the rectangle is 60 cms.
   B. Breadth of the rectangle is 12 cms.
   C. Sum of two adjacent sides is 30 cms.
   1) A or B only 2) A only 3) B only

4) C only 5) A or C only

2. Who is the tallest among M, P, Q and R?
   A. P is taller than Q but not as tall as R.
   B. R is taller than M.
   C. M is taller than P but not as tall as R.
   1) C only 2) B only 3) B or C only
   4) A or B only 5) A or C only
3. What will be the ratio between the ages of Samir and Anil after five years?
   A. Samir's present age is more than Anil's present by 4 years.
   B. Anil's present age is 20 years.
   C. Anil and Samir's present ages are in the ratio 3 : 4.
   1) A or B or C only 2) B only 3) C only
   4) A or C only 5) B or C only
4. Mr X borrowed a sum of money on compound interest. What will be the amount to be repaid if he is repaying the entire amount at the end of two years?
   A. The rate of interest is 5 p.c.p.a.
   B. Simple interest fetched on the same amount in one year is Rs 600.
   C. The amount borrowed is 10 times the simple interest in two years.
   1) C only 2) A only
   3) A or B only 4) A or C only
   5) All A, B and C are required to answer the question.
5. A boat will take how much time to cross the river against the stream of the river?
   A. In still water the speed of the boat is 15 km/hour.
   B. The width of the river is 8 km.
   C. The speed of the stream is 2 km/hr.
   1) Only A 2) Only B 3) Only C
   4) All A, B, C are required to answer the question
   5) It is not possible to answer the question with the help of all the three statements A, B and C.

Solutions:

1. 5; Clearly, A and C imply the same thing and the area can be known by either A and B or B and C.
When we take the statements A and B, then C is superfluous. When we take B and C, then A is superfluous. So, answer is 5.

2. 3; A ⇒ R > P > Q B ⇒ R > M C ⇒ R > M > P
From A and B or from A and C we find that R is the tallest among M, P, Q and R. So, answer is 3.
3. 1; (A) ⇒ S = A - 4
(B) ⇒ A = 20
(C) ⇒ A : S = 3 : 4
Since, to solve two linear equations, we need only two equations, so any one of the three equations can be dispensed with.
4. 4; With the help of A and B, we can find the amount borrowed. Hence, C can be dispensed with.
Again, with the help of B and C, the amount = 2 × 600 × 10 = 12000. Hence, A can also be dispensed with.
Thus our answer is (4).
5. 5; Our answer is (5). Try yourself.

SBI Associates PO held on 18th July, 1999

Directions (Ex. 6-10): *Each question is followed by three statements. You have to study the question and all the three statements given and decide whether any information provided in the statement(s) is redundant and can be dispensed with while answering the questions.*

6. At what time will the train reach city 'X' from city 'Y'?
A. The train crosses another train of equal length of 200 metres and running in opposite direction in 15 secs.
B. The train leaves city 'Y' at 7.15 a.m. for city 'X', situated at a distance of 560 kms.
C. The 300-metre-long train crosses a signal pole in 10 secs.
1) A only. 2) B only
3) C only 4) B and C only
5) All A, B and C are required to answer the question
7. What is the amount saved by Sahil per month from his salary?
A. Sahil spends 25% of his salary on food, 35% on medicine and education.
B. Sahil spends Rs 4000 per month on food and 15% on entertainment and saves the remaining amount.
C. Sahil spends Rs 2,500 per month on medicine and education and saves the remaining amount.
1) B only 2) C only
3) B & C both 4) B or C only

5) Question cannot be answered even with the information given in all three statements.
8. What is the rate of interest pcpa?
A. The amount becomes Rs 11,025 at compound interest after 2 years.
B. The same amount with simple interest becomes Rs 11,000 after two years.
C. The amount invested is Rs 10,000.
1) A or B or C only 2) A or B only
3) B and C only 4) A or C only
5) All A, B & C are required to answer the question
9. What is the ratio of the present ages of Rohan and his father?
A. Five years ago Rohan's age was one-fifth of his father's age that time.
B. Two years ago the sum of the ages of Rohan and his father was 36.
C. The sum of the ages of Rohan, his mother and his father is 62.
1) A only 2) A and B only 3) C only
4) B or C only 5) A or C only
10. What will be the share of P in the profit earned by P, Q & R together?
A. P, Q & R invested total amount of Rs 25,000 for a period of two years.
B. The profit earned at the end of two years is 30%.
C. The amount invested by Q is equal to the amount invested by P & R together.
1) A only 2) B only 4) C only
4) All A, B & C are required to answer the question
5) Question cannot be answered even with the information given in all three statements.

Solutions:

6. 1; (C) gives speed of the train. (B) gives the distance between x and y and also the starting time. Hence (B) & (C) are sufficient to answer the question. Therefore (A) is redundant and can be dispensed with.
7. 4; When we combine (A) and (B), we can get the answer. His total salary is Rs 16000, out of which he saves {100 - (25 + 35 + 15) =} 25%. Therefore he saves 25% of Rs 16000 = Rs 4000.
So, we can dispense with (C).
In second case, when we combine (A) & (C), we also get another answer which is different from the first one.

His salary = Rs $\frac{2500}{35} \times 100$

He saves 100 - (25 + 35) = 40% of his salary

$\therefore$ he saves 40% of Rs $\frac{2500}{35} \times 100$

So, we can dispense with (B).

Combining the two cases our answer is (4), i.e. B or C only.

8. 1; We can solve the question with the help of any of the two informations. Thus, we can dispense with any one of A, B and C. Hence answer is (1).

With help of (A) and (B):

Suppose principal = P.

Rate of interest = r%

Then, (A) $\Rightarrow P\left(1+\frac{r}{100}\right)^2 - 11025$

$\therefore$ (B) $\Rightarrow P + \frac{2rp}{100} = 11000$

In the above two equations we have two unknowns, so we can solve them. (You don't need to proceed further for geting soln.)

With help of (A) and (C):

$11025 = 10000\left(1+\frac{r}{100}\right)^2$

or, $\left(1+\frac{r}{100}\right)^2 = \frac{11025}{10000} = \left(\frac{105}{100}\right)^2 = \left(1+\frac{5}{100}\right)^2$

$\therefore r = 5\%$

With help of (B) and (C):

$11000 = \frac{2 \times r \times 10000}{100} + 10000$

or, $1000 = 200r \quad \therefore r = 5\%$

9. 3; Information given in (C) includes the age of his mother. So, (C) is useless. Only with the help of (A) and (B) we can get the answer.

10. 5; Even after using all the statements we cannot separate the combined profit of P and R.

BSRB Guwahati PO held on 8th August, 1999

Directions (Ex. 11-15): *In each of the following questions a question is asked followed by three statements. You have to study the questions and all the three statements given and decide whether any*

*formation provided in the statement(s) is/are **redundant** and can be spensed with while answering the questions.*

1. What will be the cost of fencing a circular plot? $\left(\pi = \frac{22}{7}\right)$

A. Area of the plot is 616 square metres.

B. Cost of fencing a rectangular plot whose perimeter is 120 metres is Rs 780.

C. Area of a square plot with side equal to the radius of the circular plot is 196 sq metres.

1) A only 2) C only
3) A or C only 4) B only
5) Question can not be answered even with information in all three statements

2. What will be the sum of the ages of father and the son after five years?

A. Father's present age is twice son's present age.

B. After ten years the ratio of father's age to the son's age will become 12 : 7.

C. Five years ago the difference between the father's age and son's age was equal to the son's present age.

1) A or B only 2) B or C only 3) A or C only
4) C only 5) A or B or C only

3. The difference between the compound interest and the simple interest at the same rate on a certain amount at the end of two years is Rs 12.50. What is the rate of interest?

A. Simple interest for two years is Rs 500.

B. Compound interest for two years is Rs 512.50.

C. Amount on simple interest after two years becomes Rs 5,500.

1) A or B only 2) A or C only
3) C only 4) C and either A or B
5) Any two of (A), (B) and (C)

4. 12 men and 8 women can complete a piece of work in 10 days. How many days will it take for 15 men and 4 women to complete the same work?

A. 15 men can complete the work in 12 days.

B. 15 women can complete the work in 16 days.

C. The amount of work done by a woman is three-fourth of the work done by a man in one day.

1) A or B or C only 2) B or C only
3) C only 4) Any two of the three

5) B only

15. P, Q and R together invested an amount of Rs 20,000 in the ratio 5 : 3 : 2. What was the percent profit earned by them at the end of one
   A. Q's share in the profit is Rs 2,400.
   B. The amount of profit received by P is equal to the amou profit received by Q and R together.
   C. The amount of profit received by Q and R together is Rs.4
   1) B and A or C only 2) A or C only
   3) A and B both 4) B and C both
   5) Information in all the three statements is required to answer th question

Solutions:

11. 3; (B) is necessary because only this statement gives the rate of fen Any one of (A) or (C) gives the value of radius, which enables find the circumference. Hence either (A) or (C) can be dispensed
12. 5; Any two of the three statements are sufficient to answer the que (As to find the two unknowns we need two equations). Hence any of the statements can be dispensed with.
13. 5; Any one of the three statements is alone sufficient to answe question. So any two can be dispensed with.

From (A) alone

$$\text{Rate} = \frac{\text{Diff.} \times 2}{\text{SI}} \times 100 = \frac{25}{500} \times 100 = 5\%$$

From (B) alone:

CI = Rs 512.5 $\therefore$ SI = Rs 512.5 - Rs 12.5 = Rs 500

$$\text{Again, Rate} = \frac{\text{Diff.} \times 2}{\text{SI}} \times 100 = \frac{25}{500} \times 100 = 5\%$$

From (C) alone:

Suppose Principal = P and Rate of Interest = r%

$$\text{Then, } P\left(1 + \frac{r}{100}\right)^2 = 5500 + 12.5 = 5512.5 \text{ .... (1)}$$

$$\text{and } P + \frac{2rP}{100} = 5500$$

$$\text{or, } P\left[1 + \frac{2r}{100}\right] = 5500 \text{ .... (2)}$$

Dividing (1) by (2) we have

$$\frac{\left(1 + \frac{r}{100}\right)^2}{1 + \frac{2r}{100}} = \frac{5512.5}{5500} \text{....(*)}$$

This is a quadratic equation which has only one variable, r. It can be solved. Hence value of r can be obtained.

Note: (*) is satisfied with the value r = 5.
So, it confirms that equation is solvable.

14. 4
15. 1; Statement (B) is useless because it is the same as the given statement. [Profit is distributed in the same ratio as their investment. Since their investments are in ratio 5 : 3 : 2, the profit of P(=5) is equal to the profit of Q and R together (3 + 2 = 5)]
   Statement (A) alone is sufficient to answer.
   Q's share = Rs 2400

$$\text{Total profit of P + Q + R} = \frac{2400}{3} \times (5 + 3 + 2) = \text{Rs } 8000$$

$$\therefore \text{ \% profit} = \frac{8000}{20000} \times 100 = 40\%$$

Similarly, statement (C) alone is sufficient to answer the question. Hence (B) and (A) or (C) can be dispensed with.

Data Analysis
(TABLES AND GRAPHS)

roduction :- Data analysis is an important aspect of almost every npetitive examination today. Usually, a table or a bar diagram or a -chart or a sub-divided bar diagram or a graph is given and candidates asked questions that test their ability to analyse the data given in those ms.

In order to solve such questions quickly, you should try to have following things in mind:

) First have a cursory glance at the given diagram. Try to digest quickly at the diagram represents. **Take special care of units** because in some minations two lines in a single graph have been found to be in ferent units. If you find anything striking, or odd, make a note of it mewhere alongside the answer book.

i) If you are doing a table always make sure what the sum of all entries one row represents and what the sum of all entries in a column presents.

For example in Ex 1 below, you should notice that the sum of y row represents the total amount of loan disbursed by a bank over the ears 1982 to 1986. In the same table the sum of all entries of any column presents the amount of loan disbursed by the five banks in any year.

ii) Some questions may try to make you perform unnecessary steps. Do t get trapped into it. **Never do anything that is unnecessary.**

A good example of such a question is Q 1 of Ex 1 below. In s question you are asked to find out the year in which the disbursement loans of all the banks (it means the sum of all entries in one column, member) is the least compared to the average disbursement of loans ver the years. If you are reading this question like any other student you ill immediately start calculating the average disbursement of loans over e years. But as a student of our Quicker Mathematics you should ways think before you act..... The year in which disbursement was inimum will remain so whatever be the quantity you compare with. ence to answer this question you should simply find out the year in hich the disbursement of loans was the least.

v) Questions having diagrams give you the facility of having an idea things by just looking at it. Many questions are there which do not quire the candidate to do anything more than just looking at the

diagram. If a candidate starts using his pen ... ins calculat will only be wasting his time. **Never use your pe ... question your eyes can solve.**

For example, consider Q 6 of **Ex 4.** In this question y asked to find out the faculty in which there is a regular decrease. In words you are asked to see which type of bar continuously decr Obviously it is the bar representing 'Arts', only a look can tell. It w foolish to use a pen for this question.

As another example, consider Q 5, **Ex 7.** In this questio are asked to judge the company which has a surplus, adequate en to cater to the demand of company A. In other words you have to the company having the maximum surplus. This means that since su implies the difference between demand and production you have to the company whose dark bar is taller than the shaded bar by the maxi level. Only a look can tell that it is the company D. You do not ne use your pen for this type of question.

Whenever you read a question and you proceed to solve way of calculations etc., pause. Ask yourself this question: "Can't it without pen?". We very strongly recommend this approach, espec in your practice sessions at home. This would help inculcate a hab time-saving in data analysis which will prove very rewarding in the l run.

(v) However, there may be some questions where calculations wil unavoidable. But you should always be on the look-out for some sh cuts. Of course, there will be many questions where you will *have* perform all the calculations without any short-cuts, but there may many questions where short-cuts can and should be made.

A good way of quicker calculation and short cut is **appro mation.** Always use approximate values if this may save time and t does not lead to wrong answers. For example, see Q 1, **Ex 6.** Here this question, you have to find out the year in which the impo registered the highest increase. In other words, you are required to fi the year in which the bar increases in length by the maximum. A look the diagrams suggests that the answer is either 1974 (from 1973) or 19 (from 1974). Now we have to choose between those two choices a here we will have to use pen and calculate. To calculate the increase w will have to substract the value of imports in the previous year from t value of imports in that year. The increases are : 4203 - 2413, 7016 4203. To calculate these values we can make a safe approximation : 4 - 24, 70 - 42. Obviously, the second of these should be chosen.

As another example, consider Q 2 of the same example. Here have to simply divide 5832 by 1811. But to get an answer it would ufficient if you divide 58 by 18 and note that this would be only htly more than 3. This would be sufficient for you to choose (d) as r answer.

ne Solved Examples

We are now presenting some solved examples. We would ise that you try them first, keeping the suggestions given above, in r mind. Then you should compare your approach with ours. Also, p a record of the time taken by you and also of the type of questions prove time-taking.

1. Study the following table carefully and answer the questions given below :

Loan disbursed by 5 banks
(Rupees in Crores)

	Years				
Banks	1982	1983	1984	1985	1986
A	18	23	45	30	70
B	27	33	18	41	37
C	29	29	22	17	11
D	31	16	28	32	43
E	13	19	27	34	42
Total	118	120	140	154	203

1. In which year was the disbursement of loans of all the banks put together least compared to the average disbursement of loans over the years ?
   a) 1982 b) 1983 c) 1984 d) 1985 e) 1986
2. What was the percentage increase of disbursement of loans of all banks together from 1984 to 1985?
   a) 110 b) 14 c) $90\frac{10}{11}$
   d) 10 e) None of these
3. In which year was the total disbursement of loans of banks A & B exactly equal to the total disbursement of banks D and E ?
   a) 1983 b) 1986 c) 1984 d) 1982 e) None of these

Q. 4. In which of the following banks did the disbursement of loa
continuously increase over the years?
a) A b) B c) C d) D e) E

Q. 5. If the minimum target in the preceding years was 20% of the
disbursement of loans, how many banks reached the target in 19
a) 1 b) 3 c) 2 d) 4 e) None of th

Q. 6. In which bank was loan disbursement more than 25% of the
disbursement of all banks together in 1986?
a) A b) B c) C d) D e) E

Solutions

Q. 1: (a). For explanation, refer to the discussion above.

Q. 2: (d). All banks together means sum of columns. Thus we hav
find out the percentage increase in 140 to 154. You should be ab
do the remaining mentally.

Q. 3: (e). [**Note :-** No shortcut. Calculation is needed for all the ca

Q. 4: (e). [**Note :-** Only a visual glance is needed.]

Q. 5: (c). [B and C achieved the target. Target was 20% of 118 = 2

Q. 6: (a). [**Note :-** Since this question asks you to find out the bank w
disbursed **more** than a given ratio of the total disbursement, it mean
it must be the bank having the **maximum** disbursement in that year. Y
had, therefore, only to find the bank which disbursed the largest am
in 1986 and this required nothing more than a look. If you used (e
mental) calculations for this question, you wasted your time].

Ex. 2. Study the table carefully and answer the questions given below :

Financial Statement of A Company Over The Years
(Rupees in Lakhs)

Year	Gross Turnover Rs.	Profit before int. and depr.	Interest Rs.	Deprecia-tion Rs.	Net Profi Rs.
1980-81	1380.00	380.92	300.25	69.90	10.67
1981-82	1401.00	404.98	315.40	71.12	18.46
1982-83	1540.00	520.03	390.85	80.02	49.16
1983-84	2112.00	599.01	444.44	88.88	65.69
1984-85	2520.00	811.00	505.42	91.91	212.78
1985-86	2758.99	920.00	600.20	99.00	220.80

Q. 1. During which year did the 'Net Profit' exceed Rs. 1 crore for the first time?
a) 1985-86 b) 1984-85 c) 1983-84
d) 1982-83 e) None of these

Q. 2. During which year was the "Gross Turnover" closest to thrice the 'Profit before Interest and Depreciation' ?
a) 1985-86 b) 1984-85 c) 1983-84
d) 1982-83 e) 1981-82

Q. 3. During which of the given years did the 'Net Profit' form the highest proportion of the 'Profit before Interest and Depreciation'?
a) 1984-85 b) 1983-84 c) 1982-83
d) 1981-82 e) 1980- 81

Q. 4. Which of the following registered the lowest increase in terms of rupees from the year 1984-85 to the year 1985-86 ?
a) Gross Turnover b) Profit before Interest and Depreciation
c) Depreciation d) Interest e) Net profit

Q. 5. The 'Gross Turnover' for 1982-83 is about what per cent of the 'Gross Turnover' for 1984-85?
a) 61 b) 163 c) 0.611 d) 39 e) 0.006

Solutions :-

Q. 1: (b) [only a look is needed]

Q. 2: (a) The ratio of 'Gross turnover' to the 'Profit before Interest and Depreciation':

in 1980-81 is $\frac{1380}{380.92} = 3.62$.

in 1981-82 is $\frac{1401}{404.98} = 3.46$.

in 1982-83 is $\frac{1540}{520.03} = 2.96$.

in 1983-84 is $\frac{2112}{599.01} = 3.53$.

in 1984-85 is $\frac{2520}{811} = 3.11$.

in 1985-86 is $\frac{2758.99}{920} = 3$.

Q. 3: (a). [We look at the 'Net profit' and 'Profits before interest and depreciation' figures. We need

to find the year in which 'Profits before.....' is the smallest multiple of 'Net profits'. Use approximations, $38 \div 1$, $40 \div 2$, $52 \div 5$, $60 \div 6.5$, $80 \div 20$, $92 \div 22$ and make quick mental calculations. Obviously any one of the last two is the answer. We have $80 \div 20 = 4$, $92 \div 22 > 4$, and hence $80 \div 20$ is the minimum. Hence 1984-85 is the answer.]

Q. 4: (c). [Mental calculation with approximation is sufficient. Among 2700 - 2500, 900 - 800, 600 - 500, 99 - 92 and 220 - 212, the fourth is a single digit figure and it is the least.]

Q. 5: (a). [Approximately $\frac{15}{25} \times 100 = 60$. Hence (a) is the answer.]

Ex 3. Study the following table carefully and answer the questions given below :

Number of boys of standard xi pariticipating in different games

↓Games Classes→	XI A	XI B	XI C	XI D	XI E	Total
Chess	8	8	8	4	4	32
Badminton	8	12	8	12	12	52
Table Tennis	12	16	12	8	12	60
Hockey	8	4	8	4	8	32
Football	8	8	12	12	12	52
Total No. of Boys	44	48	48	40	48	228

Note: 1. Every student (boy or girl) of each class of standard XI participates in a game.
2. In each class, the number of girls participating in each game is 25% of the number of boys participating in each game.
3. Each student (boy or girl) participates in one and only one game.

Q. 1. All the boys of class XI D passed the annual examination but a few girls failed. If all the boys and girls who passed and entered XII D are in the ratio of boys to girls as 5:1, what would be the number of girls who failed in class XI D?
a) 8 b) 5 c) 2 d) 1 c) none of these

Q. 2. Girls playing which of the following games need to be combined to yield a ratio of boys to girls of 4:1, if all boys playing Chess and Badminton are combined ?
(a) Table Tennis and Hockey (b) Badminton and Table Tennis
(c) Chess and Hockey (d) Hockey and Football
(e) None of these

Q. 3. What should be the total number of students in the school if all the boys of class XI A together with all the girls of class XI B and class XI C were to be equal to 25% of the total number of students ?
a) 272 b) 560 c) 656
d) 340 e) None of these

Q. 4. Boys of which of the following classes need to be combined to equal four times the number of girls in class XI B and Class XI C?
a) XI D & XI E b) XI A & XI B c) XI A & XI C
d) XI A & XI D e) None of these

Q. 5. If boys of class XI E participating in Chess together with girls of class XI B and class XI C participating in Table Tennis & Hockey respectively are selected for a course at the college of sports, what percent of the students will get this advantage approximately ?
a) 4.38 b) 3.51 c) 10.52
d) 13.5 e) None of these

Q. 6. If for social work every boy of class XI D and class XI C is paired with a girl of the same class, what percentage of the boys of these two classes cannot participate in social work ?
a) 33 b) 66 c) 50
d) 75 e) None of these

Solutions

Before proceeding to solve this question, let us analyse the significance of the 'Note' given at the end of the table. 1 and 3 together imply that each boy or girl must participates in one and only one game. Now, therefore, 2 implies that (a) the number of girls participating in each game is one-fourth and (b) the number of girls in each class is one-fourth of the number of boys. (Can you understand the conclusion (b) here ?).

Q. 1. Total number of boys in XI D = 40
Number of girls in XI D = 25% of 40 = 10
Since all boys of XI D passed, so the number of boys in XII D = 40.
Ratio of boys & girls in XII D is 5 : 1.

Number of girls in XII D = $\frac{1}{5} \times 40 = 8$.

$\therefore$ Number of girls who failed = (10-8) = 2. (Answer: c)

Q. 2. Total number of boys playing Chess & Badminton = (32 + 52) = 84.

Number of girls playing Hockey & Football = 25% of 84

$= \left(\frac{1}{4} \times 84\right) = 21$.

Since 84 : 21 is 4 : 1, so the girls playing hockey and football are combined to yield a ratio of boys to girls as 4 : 1.

So, answer (d) is correct.

Q. 3. Number of boys in XI A = 44;

Number of girls in XI B = 25% of 48 = 12;

Number of girls in XI C = 25% of 48 = 12;

$\therefore$ (44 − 12 + 12) = 68

Let x be the total number of students.

Then 25% of x = 68

or, $x = \frac{68 \times 100}{25} = 272$.

Total number of students in the school = 272 i.e. (a) is correct.

Q. 4. 4 times the number of girls in XI B & XI C = 4 (12 + 12) = 96.

But, none of the pairs of classes given through (A) to (D) has this as the number of boys. So, (c) is correct.

Q. 5. Number of boys of XI E playing chess = 4;

Number of girls of XI B playing table tennis = 25% of 16 = 4;

Number of girls of XI C playing hockey = 25% of 8 = 2

$\therefore$ Number of students selected for a course at the college of sports = (4 + 4 + 2) = 10.

Total number of students

= (228 + 25% of 228) = 285.

Let x% of 285 = 10

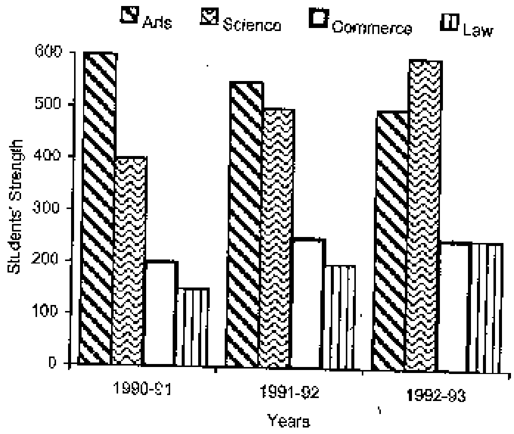
or $x = \left(\frac{10 \times 100}{285}\right) = 3.51$.

So, answer (b) is correct.

Q. 6. Since the number of girls = 25% of the number of boys, so only 25% of the boys can participate in social work.

$\therefore$ Answer (d) is correct.

Ex 4. Shown below is the multiple bar diagram depicting the changes in the student's strength of a college in four faculties from 1990-91 to 1992-93.



Study the above multiple bar chart and mark a tick against the correct answer in each of the following questions.

Q. 1. The percentage of students in science faculty in 1990-91 was :

a) 26.9% b) 27.8%
c) 29.6% d) 30.2%

Q.2. The percentage of students in law faculty in 1992-93 was :

a) 18.5% b) 15.6%
c) 16.7% d) 14.8%

Q. 3. How many times the total strength was of the strength of commerce students in 1991-92 ?

a) 3 times b) 4 times
c) 5 times d) 6 times

Q. 4. During which year the strength of arts faculty was minimum ?

a) 1990-91 b) 1991-92 c) 1992-93

Q. 5. How much percent was the increase in science students in 1992-93 over 1990-91 ?

a) 50% b) 150% c) $66\frac{2}{3}$% d) 75%

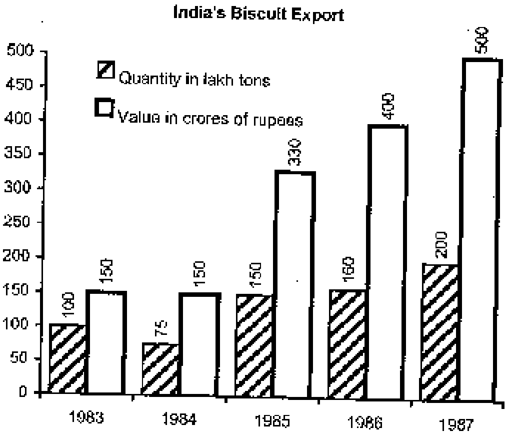
Q. 6. A regular decrease in students, strength was in the faculty of ?
a) arts b) science c) commerce d) law

Solutions

1. c; Total number of students in 1990-91 = (600+400+200−150)
= 1350.
Number of science students in 1990-91 was 400.
Percentage of science students in 1990-91 = $\left(\frac{400}{1350} \times 100\right)\%$
= 29.6%.
∴ Answer (c) is correct.
2. b; Total number of students in 1992-93 = (500+600+250+250)
= 1600.
Number of law students in 1992-93 is 250.
Percentage of law students in 1992-93 = $\left(\frac{250}{1600} \times 100\right)\%$
= 15.6%.
∴ Answer (b) is correct.
3. d; Total strength in 1991-92
(550+ 500+250+200) = 1500.
$\therefore \frac{\text{Total strength}}{\text{Strength of commerce sutdents}} = \frac{1500}{250} = 6.$
So, answer (d) is correct.
4. c; A slight look indicates that the strength in arts faculty in 1990-91, 1991-92 & 1992-93 was 550, 600 and 500 respectively. So, it was minimum in 1992-93.
∴ Answer (c) is correct.
5. a; Number of science students in 1990-91 was 400.
Number of science students in 1992-93 was 600.
Percentage increase = $\left(\frac{200}{400} \times 100\right)\% = 50\%.$
∴ Answer (a) is correct.
6. a; [Just a look is sufficient.]

Ex. 5. Study the following graph carefully and answer the following questions :

India's Biscuit Export



Q.1. In which year was the value per ton minimum ?
a) 1983 b) 1984 c) 1985
d) 1986 e) 1987

Q. 2. What was the difference between the biscuits exported in 1985 and 1986 ?
a) 10 b) 1000 c) 100000
d) 1000000 e) none of these

Q.3. What was the approximate percent increase in export value from 1984 to 1987 ?
a) 350 b) 300 c) 43
d) 24 e) none of these

Q. 4. What was the percentage drop in export quantity from 1983 to 1984?
a) 75 b) nil c) 25
d) 50 e) none of these

Q. 5. If in 1986 the goods were exported at the same rate per ton as that in 1985, what would be the value in crores of rupees of export in 1986?
a) 400 b) 352 c) 375
d) 330 e) none of these

Solutions :-

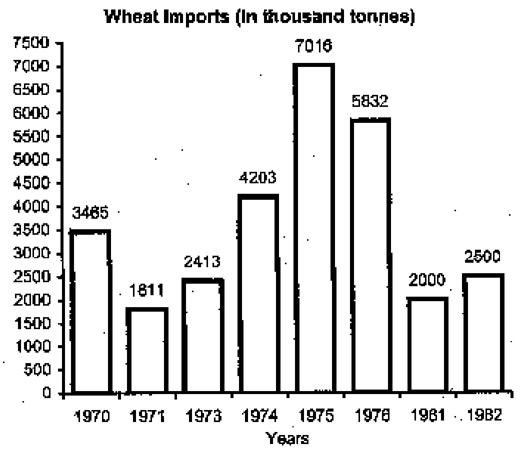
1. (a); To evaluate the value per ton, we need to divide the value of dark bar by the value of the other bar. A quick mental calcula enables us to find that the ratio is $150 \div 100 = 1.5$, $150 \div 75 = 2$, $\div 150 > 2$, $400 \div 160 = 2.5$, $500 \div 200 = 2.5$. It is the least in first c Answer : a.
2. (d); 160 - 150 = 10 lakh tons.
3. (e); Increase from 150 to 500 is an increase of 350. It is (350 ÷ 15 approximately 2.3 or 230%.
4. (c); from 100 to 75 there is a drop of 25 which is 25% of 100.
5. In 1985, the cost of 150 lakh tons = Rs. 330 crores.

$\therefore$ In 1985, the cost of 1 ton = Rs. $\left(\frac{330 \text{ crores}}{150 \text{ lakhs}}\right)$

= Rs. $\left(\frac{330}{1.50}\right)$ = Rs. 220.

In 1986, the export value = Rs. (160 lakh x 220)
= Rs. (1.60 x 220) crores
= Rs. 352 crores.
Hence, answer (b) is correct.

Ex. 6. Study the graph carefully and answer the questions given belo it :



1. In which year did the imports register highest increase over its preceding year ?
a) 1973 b) 1974 c) 1975
d) 1982 e) none of these
2. The imports in 1976 were approximately how many times that of the year 1971 ?
a) 0.31 b) 1.68 c) 2.41 d) 3.22 e) 4.5
3. What is the ratio of the years which have above average imports to those which have below average imports ?
a) 5:3 b) 2:6 c) 8:3 d) 3:8 e) none of these
4. The increase in imports in 1982 was what percent of the imports in 1981 ?
a) 25 b) 5 c) 125 d) 80 e) none of these
5. The imports in 1974 is approximately what percent of the average imports for the given years?
a) 125 b) 115 c) 190 d) 85 e) 65

olutions

1. (c). See explanation in the introductory discussion.
2. (d). See explanation in the introductory discussion.
3. (e). Average of the imports

$= \frac{1}{8}(3465 + 1811 + 2413 + 4203 + 7016 + 5832 + 2000 + 2500)$
= 3655.

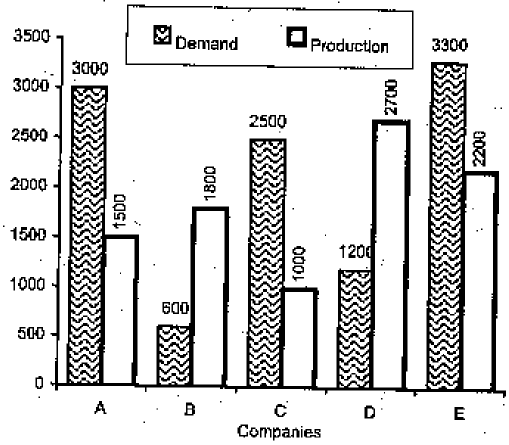
The years in which the imports are above average are 1974, 1975 & 1976, i.e. there are 3 such years.
The years in which the imports are below average are 1970, 1971, 1973, 1981 & 1982, i.e. there are 5 such years.
$\therefore$ Required ratio is 3 : 5.

4. (a) (500 of 2000 is 25%. You should have done it mentally.)
5. Average import = 3655 thousand tonnes.
Import in 1974 = 4203 thousand tonnes.
Let x% of 3655 = 4203.
Then, x = $\left(\frac{4203 \times 100}{3655}\right)$

= 115%.

$\therefore$ Answer (b) is correct.

Ex. 7. Study the following graph carefully and answer the followin questions.



Q. 1. What is the ratio of companies having more demand than produ ction to those having more production than demand ?

a) 2:3 b) 4:1 c) 2:2 d) 3:2

Q. 2. What is the difference between average demand and average production of the five companies taken together ?

a) 1400 b) 400 c) 280 d) 138 e) none of these

Q. 3. The production of company D is approximately how many time of the production of the company A ?

a) 1.8 b) 1.5 c) 2.5 d) 1.11 e) none of these

Q. 4. The demand for company 'B' is approximately what per cent of the demand for company 'C' ?

a) 4 b) 24 c) 20 d) 60

Q. 5. If company 'A' desires to meet the demand by purchasing surplu T.V. sets from a single company, which one of the following compa nies can meet the need adequately ?

a) B b) C c) D d) none of these

Solution

1. d; A simple inspection is enough to tell that.

2. c; Average demand

$= \frac{1}{5}(3000+600+2500+1200+3300) = 2120.$

Average production

$= \frac{1}{5}(1500+1800+1000+2700+2200) = 1840.$

$\therefore$ Difference between average demand and average production $= (2120 - 1840) = 280.$

So, answer (c) is correct.

3. a; Let $k(1500) = 2700$ or $k = \frac{2700}{1500} = 1.8.$

So, answer (a) is correct.

4. b; Let x% of (demand for C) = (demand for B).

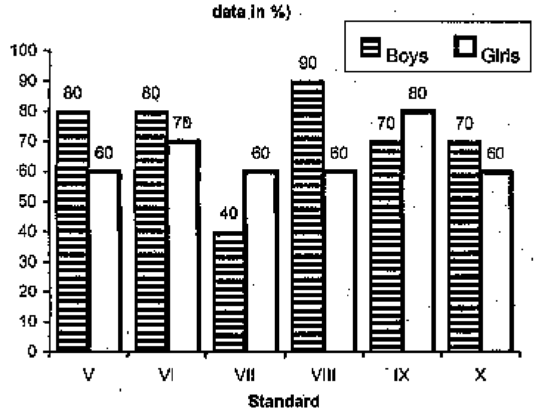
i.e. $\frac{x}{100} \times 2500 = 600$ or $x = \left(\frac{600 \times 100}{2500}\right) = 24\%.$

$\therefore$ Answer (b) is correct.

5. (c). See the explanation in the introductory discussion.

Ex. 8. Study the following graph and answer the questions given below:

Result of Annual Examination in a High School (all data in %)



Q.1. In which standard is the difference between the result of girls and that of boys maximum ?
a) V b) VII
c) X d) VIII

Q. 2. In which standard is the result of boys less than the average result of the girls ?
a) VII b) IX c) VI
d) VIII e) V

Q. 3. In which pair of standards are the results of girls and boys in inverse proportion ?
a) V & X b) V & VI c) VI & VIII
d) V & IX e) VI & IX

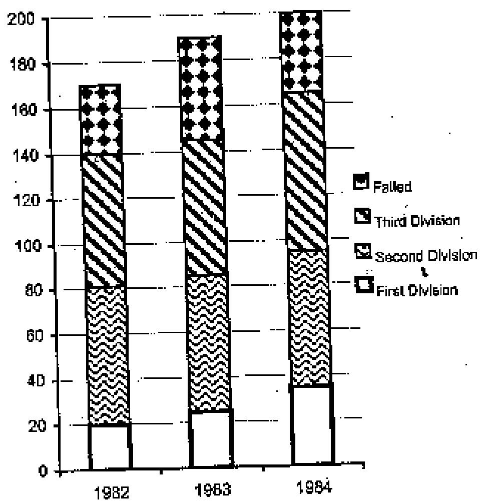
Q. 4. In which standard is the result of the girls more than the average result of the boys for the school ?
a) IX b) VIII c) VI
d) X e) VII

Q. 5. In which standard is the failure of girls the lowest ?
a) X b) VII c) VIII
d) V e) none of these

Solutions

1. (d). (Visual inspection is sufficient.)
2. Average result of girls
$=\frac{1}{6}(60+70+60+60+80+60)=\frac{390}{6}=65\%.$
So, in VII standard the result of boys is less than the average result of the girls. Therefore, (a) is correct.
3. In VI standard, the results of boys and girls are in the ratio 8:7; while in IX standard, the results of boys and girls are in the ratio 7:8. So, answer (e) is correct.
4. (a): [You should not have done any calculation for this question. Obviously, the only answer possible is that in which the result of girls is the best. In other words, the tallest dark bar. Hence class IX.]
5. (e). [Same explanation as in previous example.]

Ex 9. The sub-divided bar diagram given below depicts the result of B.Sc. students of a college for three years.



Study the above bar diagram and mark a tick against the correct answer in each question.

Q. 1. How many percent passed in 1st division in 1982 ?
a) 20% b) 34% c) $14\frac{2}{7}\%$ d) $11\frac{13}{17}\%$

Q. 2. What was the pass percentage in 1982 ?
a) 65% b) 70% c) 74.6% d) 82.3%

Q.3. In which year the college had the best result for B. Sc. ?
a) 1982 b) 1983 c) 1984

Q.4. What is the number of third divisioners in 1984 ?
a) 165 b) 75 c) 70 d) 65

Q. 5. What is the percentage of students in 1984 over 1982 ?

a) 30% b) $17\frac{11}{17}$% c) $117\frac{11}{17}$% d) 85%

Q. 6. What is the aggregate pass percentage during three years ?

a) $51\frac{2}{3}$% b) 82.7% c) 80.5% d) 77.6%

Solutions

Ans. 1. Percentage of 1st divisioners $= \left(\frac{20}{170} \times 100\right) = 11\frac{13}{17}\%$

$\therefore$ Answer (d) is correct.

Ans. 2. Total students passed = 140. Total students appeared = 170.

Pass percentage $= \left(\frac{140}{170} \times 100\right)\% = 82.3\%$.

$\therefore$ Answer (d) is correct.

Ans. 3. Pass percentage in 1982 $= \left(\frac{140}{170} \times 100\right)\% = 82.3\%$.

Pass percentage in 1983 $= \left(\frac{150}{195} \times 100\right)\% = 76.9\%$.

Pass percentage in 1984 $= \left(\frac{165}{200} \times 100\right)\% = 82.5\%$.

So the college had best result in 1984. $\therefore$ Answer (c) is correct.

Ans. 4. Third divisioners in 1984 = (165 - 95) = 70.

$\therefore$ Answer (c) is correct.

Ans. 5. Students in 1984 = 200. Students in 1982 = 170.

Required percentage $= \left(\frac{200}{170} \times 100\right)\% = 117\frac{11}{17}\%$.

$\therefore$ Answer (c) is correct.

Ans 6. Total number of students appeared during 3 years

= (170 + 195 + 200) = 565.

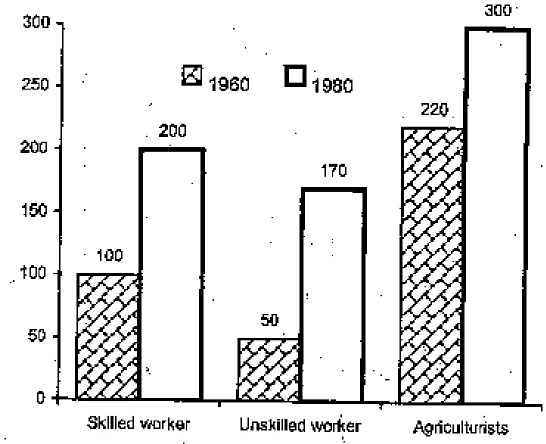
Total number of students passed during 3 years

= (140 + 150 + 165) = 455.

Aggregate pass percentage $= \left(\frac{455}{565} \times 100\right)\% = 80.5\%$.

So, answer (c) is correct.

Ex 10:



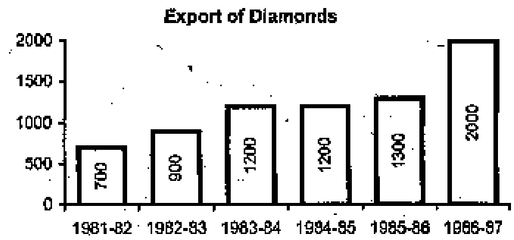
In the above graph, which category shows the highest % increase in the periods shown?

a) Skilled workers b) Unskilled workers

c) Agriculturists d) None of these

Soln: We don't look for calculation. The maximum difference of two bars lies for unskilled worker. Thus, our answer is b.

Ex 11:

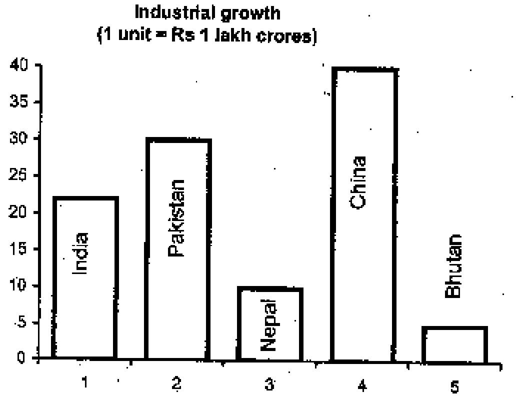


Between which two years the export of diamonds increased by minimum per cent?

a) 1983-85 b) 1984-86 c) 1985-87 d) 1981-83

Soln: There is regular increase in export of diamond except the year from 1983-84 to 1984-85. Thus, our answer (a). Here also we don't need to calculate the percentage increase for different periods.

Ex 12:

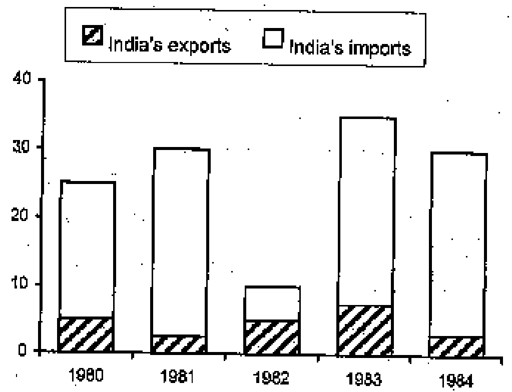


How many of the countries (mentioned in the graph) have th
industrial growth more than Rs 10 lakh crores?

1) 2 2) 3 3) None 4) 4

Soln: 2; India, Pakistan, China.

Ex 13:



Which one of the following conclusions drawn is not correct?

a) There was downward trend in trade between 1981-82

b) After 1983 there was a fall of about 30% in India's exports and imports.

c) In the year 1982 only the trade balance was is favour of Indian economy.

d) None of these

Soln: b; (a) is correct. There is upward trend between 1981-82 because exports increase and imports decrease during this period.

(c) is also currect. Because in none of the other years imports was less than the exports.

Ex 15: Profit earned by different sectors (in Rs crores)

↓Sectors/Years →	80-81	81-82	82-83	83-84	84-85
Private Sectors	30	45	50	48	60
Public Sectors	25	32	40	50	50

Which of the following statements is true?

a) In 1980-81, the gap between the profits of private sectors and public sectors is minimum.

b) Private sectors earned more profit than public sectors in each time period.

c) Private sectors' earnings shows an increasing trend from 1980 to 1985.

d) Public sectors' earnings reaches its highest point.

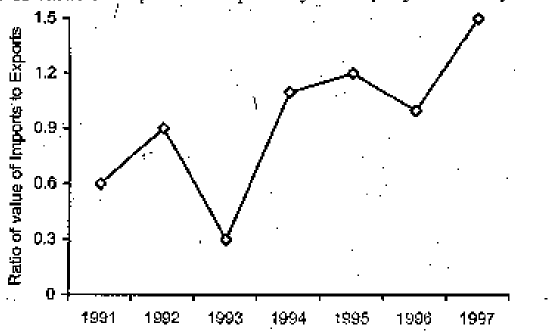
Soln: b; (a) is not true because 1983-84 has the least gap.

(d) Absurd conclusion is made. We can't say that an earning of Rs 50 cr. (in 1985) is the limit for Public Sector unit.

Problems on Line Graph and Tables (asked in previous exams)

Directions (Q. 1-5): Study the following graph carefully and answer the questions given below it:

Ratio of value of Imports to Exports by a company over the years



Q. 1: If the total exports in 1992 and 1993 together was Rs. 600 crore, what was the total of imports in those two years?
1) Rs 800 crore 2) Rs 540 crore
3) Rs 900 crore 4) Rs 450 crore
5) Data inadequate

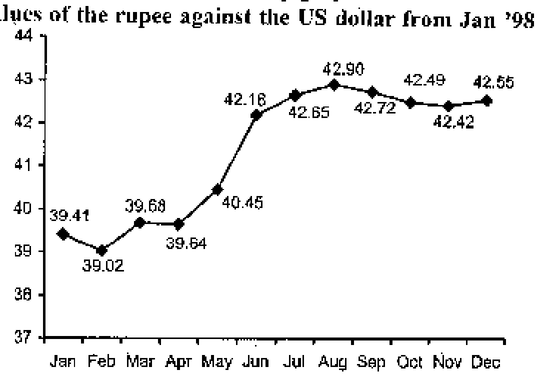
Q. 2: If the imports of the company in 1995 were Rs 270 crore, what was the exports of the company in the same year?
1) Rs 200 crore 2) Rs 240 crore
3) Rs 180 crore 4) Data inadequate
5) None of these

Q. 3: In which of the following years were the imports minimum proportionate to the exports of the company?
1) 1991 2) 1992 3) 1993 4) 1996 5) None of these

Q. 4: What was the percentage increase in imports from 1993 to 1994?
1) $166\frac{2}{3}$ 2) $66\frac{2}{3}$ 3) $133\frac{1}{3}$
4) Data inadequate 5) None of these

Q. 5: In how many of the given years were the imports more than the exports?
1) 1 2) 2 3) 3 4) 4 5) None of these

Directions (Q. 6-8): The following graph shows the monthly average values of the rupee against the US dollar from Jan '98 to Dec '98.



Q. 6: Find the number of months which have shown below average performance of the monthly average values of the rupee against the US dollar in the year 1998.
1) 5 2) 7 3) 6 4) 8 5) None of these

Q. 7: The highest value of the rupee against the US dollar is approximately how much greater than the average value for the given period?
1) 2.28 2) 1.91 3) 1.22 4) 1.62 5) 2.63

Q. 8: Find the percentage increase in the value of the rupee against the US dollar from Jan to Dec.
1) 8.1% 2) 8% 3) 7.8% 4) 7.9% 5) 8.8%

Solutions

1. 5

2. 5; Import in 1995 = 270 cr; $\frac{E}{I} = 1.2$

$\therefore$ Export in 1995 $= \frac{270}{1.2} =$ Rs 225 crore

3. 3; The E/I is minimum in this year.

4. 4; % increase in imports in 1994 over 1993 can't be found in absence of absolute value. 5. 3

6. 2; Average of integral values $= \frac{1}{12}(39 \times 4 + 40 + 42 \times 7)$

$= \frac{1}{12}(156 + 40 + 294) = \frac{1}{12} \times 490 = 40.8$

Clearly, there are 7 months (Jun-Dec) showing below-average performance.

Note: (i). A higher value of the dollar implies poor performance of the rupee.

(ii). Why did we ignore fractional values? Because if we add them, the average will be increased by less than 1. Since no month value lies between 40.8 and 41.8, any such increase is insignificant for us.

7. 4; In Aug, there is the highest value 42.90 of the rupee against the US dollar.

Average $\approx 40.8 + 0.5$ (approx. ave. of fractional values)

≈ 41.3 $\therefore$ Required value $\approx 42.9 - 41.3 = 1.6$

8. 3; Required percentage increase

$= \frac{42.55 - 39.41}{39.41} \times 100\%$

$\approx$ slightly greater than $3.14 \times \frac{100}{40}$

i.e. 3.14×2.5 i.e. 7.85% ≈ 7.9%

Directions (Q. 9-10): The following table shows India's r export.

Year	Quantity (in lakh kg)	Value (in Rs crore)
1993	110	230
1994	95	210
1995	125	340
1996	130	430
1997	200	450

Q. 9: In which year was the value per kg maximum?
1) 1993 2) 1994 3) 1995
4) 1996 5) 1997

Q. 10: If the price of rice were to go up by 40% in the year 1997 and quantity of rice exported to fall by 30%, what would have be India's earning from rice export in this condition?
1) 442 cr 2) 438 cr 3) 460 cr
4) 441 cr 5) None of these

Directions (Q. 11-15): The following table shows the producti of different types of two-wheelers from 1993 to 1998. Study the tab carefully and then answer the questions.

No. of two-wheelers (in '000)

Year → / Types ↓	1993	1994	1995	1996	1997	1998
A	36	34	40	35	37.5	40
B	20	22	25	23	19.5	18
C	14	22	16	25	29	35
D	60	62	67.5	75	76	80
E	40	45	48	50	80	105
F	45	52	55	60	57.5	56
Total	215	237	251.5	268	299.5	334

Q. 11: In which year was the total production of A and D together eq to the total production of E and F together?
1) 1997 2) 1993 3) 1994
4) 1995 5) None of these

Q. 12: In which year was the total production of all types of two-whee taken together equal to the approximate average of the to production of the two-wheelers during the given period?

1) 1996 2) 1997 3) 1995 4) 1994 5) None of these

Q. 13: How many types of two-wheelers have shown a continuous growth in the production for the given period?
1) 2 2) 3 3) 4 4) 1 5) None of these

Q. 14: The approximate percentage increase in the total production of all types of two-wheelers in 1997 in comparison to 1995 was
1) 16% 2) 20% 3) 23% 4) 25% 5) 28%

Q. 15: What is the maximum value of the difference in the production of any two types of two-wheelers for the years 1997 and 1998?
1) 14750 2) 87500 3) 157500 4) 147500 5) None of these

Solutions

9. 4; Value per kg in 1993 $= \frac{230}{110} = 2.09$

$1994 = \frac{210}{95} = 2.21 \qquad 1995 = \frac{340}{125} = 2.72$

$1996 = \frac{430}{130} = 3.3$ maximum value

$1997 = \frac{450}{200} = 2.25$

10. 4; India's earning in 1997 $= 450 \times \frac{100+40}{100} \times \frac{100-30}{100} = 441$ cr.

11. 5; 1996 is such a year.

12. 1; Approximate average of the total production

$= \frac{215 + 237 + 251.5 + 268 + 299.5 + 334}{6}$

$= \frac{1605}{6} = 267.5 \approx 268$ which is equal to prod. in 1996.

13. 1; D and E have shown continuous increase in the production.

14. 2; $\frac{299.5 - 251.5}{251.5} = \frac{48}{251.5} \approx \frac{50}{250} = \frac{1}{5} = 20\%$

15. 4; For the maximum difference of the production of two types for 1997 and 1998, we have to select the types one having the highest production for 1997 and 1998 and the other having the lowest production for 1997 and 1998.

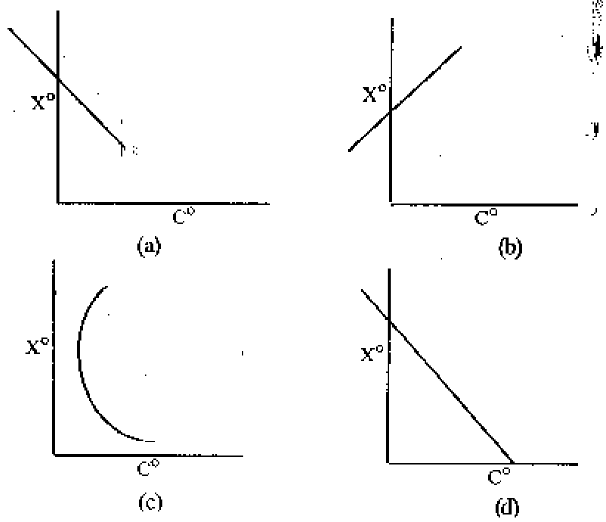
E → $80 + 105 = 185 \Rightarrow 185000$

B → $19.5 + 18 = 37.5 \Rightarrow 37500$

∴ Required difference = 185000 - 37500 = 147500

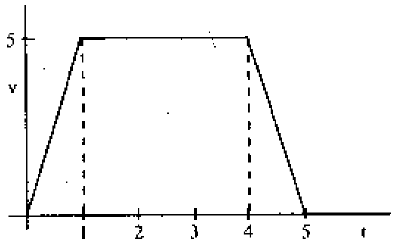
SOME OTHER ILLUSTRATIONS OF GRAPHS

Ex 16: If 0° C is given by 4° x and 100° C is given by 24° x, which of the following gives roughly the relationship between C and x?



Soln: Ans is (b). The graph should be increasing because with increase in C, the value of x° also increases.

Ex 17: If a body follows the motion as shown is the following figure, what is the total distance covered by the body? (V is velocity in M/sec and T is the time in seconds)



a) 10 metres b) 50 metres c) 80 metres d) 20 metres

Soln: d; Distance = Velocity × time

$\therefore$ Total enclosed area $= \frac{1}{2} \times 5 \times 1 + 5 \times (4 - 1) + \frac{1}{2} \times 5 \times 1$

$= 20$ metres.

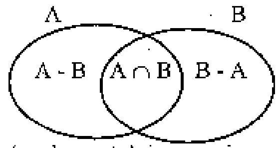
Venn Diagrams

Pictorial representation of sets gives most of the ideas about sets and their properties in a much easier way than the representation of sets given in language form. This pictorial representation is done by means of diagrams, known as **Venn Diagrams.**

The objects in a set are called the **members or elements** of the set.

If A = {1, 2, 3, 4, 5, 6}, then 1, 2, 3, 4, 5 and 6 are the members or elements of the set A.

If B = {x : x is a positive integer divisible by 5 and x < 25} or, B = {5, 10, 15, 20}, then 5, 10, 15 and 20 are the elements of the set B.



$A \cap B$ (read as set A intersection set B) is the set having the common elements of both the sets A and B. $A \cup B$ (read as set A union set B) is the set having all the elements of the sets A and B. A - B (read as set A minus set B) is the set having those elements of A which are not in B.

In other words, A - B represents the set A exclusively, i.e. A - B have the elements which are only in A. Similarly, B - A represents the set B exclusively. We keep it in mind that $n(A \cup B) = n(B \cup A)$ and $n(A \cap B) = n(B \cap A)$.

The number of elements of a set A is represented by n(A), but $n(A - B) \neq n(B - A)$

Now, by the above Venn diagram it is obvious that

$n(A) = n(A - B) + n(A \cap B)$ ..... (1)

$n(B) = n(B - A) + n(A \cap B)$ ..... (2)

$n(A \cup B) = n(A - B) + n(A \cap B) + n(B - A)$ .... (i)

Adding (1) and (2) we get,

$n(A) + n(B) = n(A - B) + n(B - A) + n(A \cap B) + n(A \cap B)$

or, $n(A) + n(B) - n(A \cap B) = n(A - B) + n(B - A) + n(A \cap B)$ ... (ii)

From (i) and (ii), we have

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$ .... (3)

Solved examples

Ex. 1: In a class of 70 students, 40 like a certain magazine and 37 li another certain magazine. Find the number of students who li both the magazines simultaneously.

Soln: $n(A \cup B) = 70 \quad n(A) = 40, n(B) = 37$

Now, $70 = 40 + 37 - n(A \cap B) \quad \therefore \ n(A \cap B) = 77 - 70 = 7$,

Ex. 2: In a group of 64 persons, 26 drink tea but not coffee and 34 dri tea. Find how many drink (i) tea and coffee both, (ii) coffee b not tea.

Soln: (i) $n(T \cup C) = 64, n(T - C) = 26, n(T) = 34$

We have, $n(T) = n(T - C) + n(T \cap C)$

or, $34 = 26 + n(T \cap C) \quad \therefore \ n(T \cap C) = 34 - 26 = 8$

(ii) Again, we have

$n(T \cup C) = n(T) + n(C) - n(T \cap C)$

or, $64 = 34 + n(C) - 8 \quad \therefore \ n(C) = 38$

Now, $n(C) = n(C - T) + n(T \cap C)$

or, $38 = n(C - T) + 8 \quad \therefore \ n(C - T) = 38 - 8 = 30$

Ex. 3: In a class of 30 students, 16 have opted Mathematics and 12 hav opted Mathematics but not Biology. Find the number of studen who have opted Biology but not Mathematics.

Soln: $n(M \cup B) = 30, n(M) = 16, n(M - B) = 12, n(B - M) = ?$

We have, $n(M) = n(M - B) + n(M \cap B)$

or, $16 = 12 + n(M \cap B) \quad \therefore \ n(M \cap B) = 16 - 12 = 4$

Again, we have, $n(M \cup B) = n(M) + n(B) - n(M \cap B)$

or, $30 = 16 + n(B) - 4 \quad$ or, $n(B) = 30 - 12 = 18$

Now, $n(B) = n(B - M) + n(M \cap B)$

or, $18 = n(B - M) + 4 \quad \therefore \ n(B - M) = 18 - 4 = 14$

Ex. 4: In a class of 70 students, 40 like a certain magazine and 37 li another while 7 like neither.

(i) Find the no. of students who like at least one of the tw magazines.

(ii) Find the no. of students who like both the magazines simu taneously.

Soln: We have, total no. of students = 70 in which 7 do not like any the magazines.

For our consideration regarding liking of magazines, we are l with 70 - 7 = 63 students.

Thus, $n(A \cup B) = 63, n(A) = 40, n(B) = 37$

(i) The no. of students who like at least one of the two magazines $= n(A \cup B) = 63$.

(ii) The no. of students who like both the magazines simultaneously $= n(A \cap B) = ?$

We have, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

or, $63 = 40 + 37 - n(A \cap B) \quad \therefore \ n(A \cap B) = 77 - 63 = 14$

Ex. 5: In a school 45% of the students play cricket, 30% play hockey and 15% play both. What per cent of the students play neither cricket nor hockey?

Soln: $n(C) = 45, n(H) = 30, n(C \cap H) = 15$

$\therefore \ n(C \cup H) = 45 + 30 - 15 = 60$

i.e., 60% of the students play. They play either cricket or hockey or both.

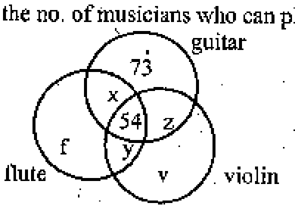
So, the remaining 100 - 60 = 40% students play neither cricket nor hockey.

Ex. 6: Out of a total of 360 musicians in a club 15% can play all the three instruments — guitar, violin and flute. The no. of musicians who can play two and only two of the above instruments is 75. The no. of musicians who can play the guitar alone is 73.

(i) Find the total no. of musicians who can play violin alone and flute alone.

(ii) If the no. of musicians who can play violin alone be the same as the number of musicians who can play guitar alone, then find the no. of musicians who can play flute.

Soln:



Total no. of musicians = 360

15% of 360 = 54 musicians can play all the three instruments.

Given that $x + y = z = 75$

Now, $73 + f + v + (x + y + z =) 75 + 54 = 360$

$\therefore \ v + f = 360 - (73 + 75 + 54) = 158$

(ii) Now we have $v = 73$

The no. of musicians who can play flute alone,

$f = (v + f) - v = 158 - 73 = 85$

and the no. of musicians who can play flute

$= f + x + y + 54 = 85 + 54 + (x + y)$

We have $x + y + z = 75$, $x + y = 75 - z$.

As either $x + y$ or z is unknown, we cannot find out the no. o musicians who can play flute. Hence, data is inadequate

Ex. 7: Out of a total 85 children playing badminton or table tennis o both, total number of girls in the group is 70% of the total numb of boys in the group. The number of boys playing only badminto is 50% of the number of boys and the total number of boys playin badminton is 60% of the total number of boys. The number o children playing only table tennis is 40% of the total number o children and a total of 12 children play badminton and table tenni both. What is the number of girls playing only badminton?

1) 16 2) 14 3) 17 4) Data inadequate 5) None of these

Soln: Let the number of boys = x

then $x + \frac{7x}{10} = 85 \Rightarrow x = 50$

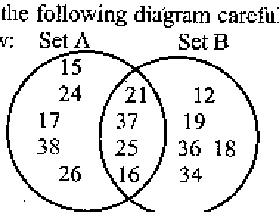
No. of girls = 85 - 50 = 35

Number diagram

In such type of questions there are certain numbers in the sets and keeping the principles of Venn diagram in mind we have to answer the questions.

Solved Example

Study the following diagram carefully and answer the questions given below:



Q. 1: What is the difference between the sum of the numbers of the two sets?

Q. 2: What is the difference between the sum of the numbers of set A and the sum of the numbers which are exclusively in set A?

Q. 3: Find the difference between the sum of the numbers of set A and the sum of the numbers which are exclusively in set B.

Q. 4: What is the minimum difference between the total of all the numbers of a set and the total of all the numbers which are common in both the sets?

Q. 5: What is the sum of the numbers which are in set B but not in A?

Soln: The sum of the numbers which are exclusively in set A

$= 15 + 24 + 17 + 38 + 26 = 120$

The sum of the numbers which are exclusively in set B

$= 12 + 19 + 36 + 18 + 34 = 119$

The sum of the numbers which are common the both the sets

$= 21 + 37 + 25 + 16 = 99$

Now,

1. $(120 + 99) - (119 + 99) = 120 - 119 = 1$
2. The required difference = the sum of the numbers which are common to both the sets = 99
3. The required difference $= (120 + 99) - 119 = 100$
4. We have to find out simply the sum of the numbers which are exclusively in set A and set B separately and select the lower value. Clearly, 119 is the answer.
5. The sum of the numbers which are in set B but not in A = the sum of the numbers which are exclusively in set B = 119.

Number table

	Column 1	Column 2	Column 3	Column 4	Column 5
Row 1	24	13	35	26	14
Row 2	30	16	20	11	27
Row 3	17	27	19	33	28
Row 4	31	28	23	21	29
Row 5	15	26	32	18	12

In examinations, rows and columns may or may not be mentioned. So, in any table you must keep in mind that horizontal lines represent rows whereas vertical lines represent columns.

So, 31, 28, 23, 21 and 29 are the numbers of row 4; 35, 20, 19, 23 and 32 are the numbers of column 3; etc.

Also, 20 is the number of row 2 and column 3, 21 is the number of row 4 and column 4.

If R $\Rightarrow$ Row and C $\Rightarrow$ Column then R5 C4 implies the number 18, R3 C3 implies the number 19 etc.

Solved Example

Study the above table carefully and answer accordingly.

(R ⇒ Row and C ⇒ Column)

Q. 1: Which of the following pair is the same?

(i) R4 C2 and R2 C4 (ii) R3 C3 and R2 C5
(iii) R5 C2 and R4 C1 (iv) R3 C5 and R4 C2
(v) None of these

Soln: R4 C2 ⇒ 28, R2 C4 ⇒ 11, 28 ≠ 11. So we go for the next optio
R3 C3 ⇒ 19, R2 C5 ⇒ 27, 19 ≠ 27. So we again go for the ne
option and get (iv) as the answer as R3 C5 ⇒ 28, R4 C2 ⇒ 28.

Q. 2: The total of the number of which of the following combinations
the maximum?

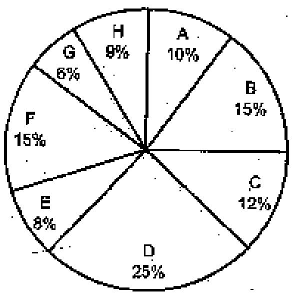
(i) R3 C3 and R4 C4 (ii) R2 C4 and R4 C2
(iii) R5 C3 and R4 C2 (iv) R4 C5 and R2 C3
(v) R3 C4 and R2 C2

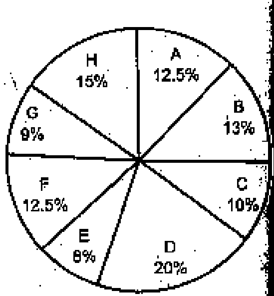
Soln: We have the option (i) → 19 + 21 = 40
option (ii) → 11 + 28 = 39
option (iii) → 32 + 28 = 60
option (iv) → 29 + 20 = 49
and option (v) → 33 + 16 = 49
Clearly, (iii) is the answer.

Solved Example

The following graph shows the no. of workers of different cate-
gories A, B, C, D, E, F, G and H of a factory for the two different years.

Total no. of workers in 1997 = 1900 **In 1998 = 1800**





Q. 1: What is the total no. of increased workers for the categories in
which the no. of workers has been increased?

Q. 2: Find the percentage decrease in the no. of workers for the categories
D and F taken together?

Q. 3: Which categories have shown the decrease in the no. of workers from 1997 to 1998?

Q. 4: Find the maximum possible difference of the no. of workers of any two categories taken together for one year and any two for the other year.

Q. 5: What is the difference between the no. of workers of the category F for the two years and the angular values of the same category for the two years?

Soln:

	1997	1998
A →	(10% of 1900 =) 190	(12.5% of 1800 =) 225
B →	285	234
C →	228	180
D →	475	360
E →	152	144
F →	285	225
G →	114	162
H →	171	270

Solutions

1. The no. of workers has been increased in the category A (from 190 to 225 = 35), G (from 114 to 162 = 48) and H (from 171 to 270 = 99).
∴ Total no. of increased workers = 35 + 48 + 99 = 182

2: Reqd. percentage decrease

$$= \frac{(475-360)+(285-225)}{475+285} \times 100\% = \frac{175}{760} \times 100\% = 23\%$$

3: Reqd. categories are B, C, D, E and F.

4: For the reqd. purpose, we have to select **(1)** the two categories having the highest no. of workers in 1997 and simultaneously the two categories having the least no. of workers in 1998 and **(2)** the two categories having the highest no. of workers in 1998 and simultaneously the two categories having the least no. of workers in 1997.
Now, considering (1), we get
the difference = (475 + 285) - (144 + 162) = 454
Now, considering (2), we get
the difference = (360 + 270) - (114 + 152) = 364
454 > 364 So, the reqd. difference = 454

5: The difference between the no. of workers of the category F for the two years = 285 − 225 = 60

and the percentage difference (15 - 12.5 =) 2.5%

We have 100% ≡ 360°

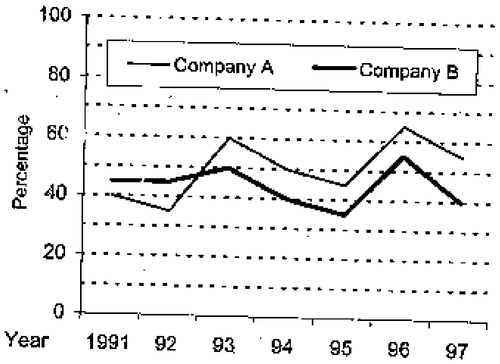
$\therefore\ 2.5\% \equiv \left(\frac{360}{100} \times 2.5\right) = 9°$

Questions asked in previous exams

SBI PO held on February 1999

Directions (Q. 1-5): Study the following graph carefully and answer the questions given below.

Percentage net profit of two companies over the years



1. If the total income in 1992 for Company B was 140 crores, what was the total expenditure in that year?
   1) 100 cr 2) 110 cr 3) 98 cr
   4) Data inadequate 5) None of these
2. If the total expenditure of 1993 and 1994 together of Company B was Rs 279 crores, what was the total income in these years?
   1) Rs 121.5 cr 2) Rs 135 cr 3) Rs 140 cr
   4) Data inadequate 5) None of these
3. In how many of the given years the percentage of expenditure to the income of Company A was less than fifty?
   1) One 2) Two 3) Three 4) Four 5) None of these
4. If the total expenditue of Company B in 1994 was Rs 200 crores, what was the total income?
   1) Rs 160 cr 2) Rs 280 cr 3) Rs 260 cr
   4) Data inadequate 5) None of these

5. In which of the following years was the total income more than double the total expenditure in that year for Company B?
   1) 1995 2) 1993 3) 1997 4) 1992 5) None of these

Directions (Q. 6-10): Study the following table carefully and answer the questions given below it.

Number of candidates from different locations appeared and passed in a competitive examination over the years

Year	Rural		Semi-urban		State capitals		Metropolises	
	App.	Passed	App.	Passed	App.	Passed	App.	Passed
1990	1652	208	7894	2513	5054	1468	9538	3214
1991	1839	317	8562	2933	7164	3248	10158	4018
1992	2153	932	8139	2468	8258	3159	9695	3038
1993	5032	1798	9432	3528	8529	3628	11247	5158
1994	4915	1658	9784	4015	9015	4311	12518	6328
1995	5628	2392	9969	4263	1725	4526	13624	6419

6. For the candidates from which of the following locations was there continuous increase both in appeared and passed?
   1) Semi-urban 2) State capital
   3) State capital & Rural 4) Metropolises
   5) None of these
7. In which of the following years was the percentage passed to appeared candidates from Semi-urban area the least?
   1) 1991 2) 1993 3) 1990
   4) 1992 5) None of these
8. What **approximate** value was the percentage drop in the number of Semi-urban candidates appeared from 1991 to 1992?
   1) 5 2) 10 3) 15 4) 8 5) 12
9. In 1993 percentage of candidates passed to appeared was **approximately** 35 from which location?
   1) Rural 2) Rural and Metropolises
   3) Semi-urban and Metropolises 4) Rural and Semi-urban
   5) None of these
10. The total number of candidates passed from Rural in 1993 and Semi-urban in 1990 was exactly equal to the total number of candidates passed from State capital in which of the following years?
    1) 1990 2) 1993 3) 1994
    4) 1992 5) None of these

Directions (Q. 11-15): Study the following table carefully and answer the questions given below:

Marks (out of 50) obtained by Five students P, Q, R, S and T in Five Subjects in Five periodical examination of each subject

Sub	Students														
	P					Q					R				
	Periodicals														
	I	II	III	IV	V	I	II	III	IV	V	I	II	III	IV	V
Math	40	30	45	20	35	30	20	35	45	40	30	35	40	45	40
Scien	30	40	25	30	20	25	45	30	37	28	48	46	31	40	80
His	35	25	15	30	40	33	27	40	34	26	35	45	40	30	35
Geog	45	47	32	39	37	42	43	30	40	25	25	35	48	37	25
Eng	24	28	36	39	43	30	28	37	34	31	26	28	31	30	40

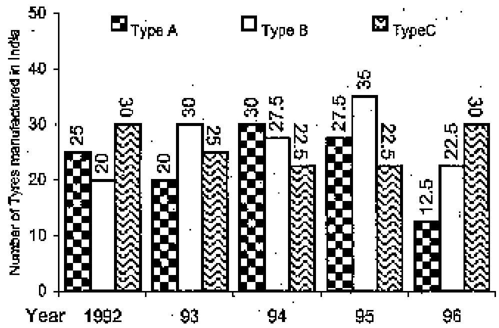
Sub	Students									
	S					T				
	Periodicals									
	I	II	III	IV	V	I	II	III	IV	V
Math	25	35	40	45	30	29	31	39	41	40
Scien	31	34	38	27	30	44	36	40	30	40
His	34	40	36	42	48	37	43	35	45	40
Geog	39	37	44	40	30	38	39	33	40	40
Eng	31	34	35	45	40	30	30	35	45	40

11. What was the average marks of the five subjects of student Q in the 1st periodical?
    1) 32 2) 34 3) 40 4) 30 5) None of these
12. What was the total of marks of student T in Science in all the periodicals together?
    1) 160 2) 180 3) 190 4) 140 5) None of these
13. The average percentage of marks obtained by student P in Maths in the five periodicals was exactly equal to the average percentage of marks obtained by student R in the five periodicals in which of the following subjects?
    1) English 2) Geography
    3) Science and Geography 4) Maths
    5) None of these

14. In which of the following subjects was the average percentage of marks obtained by student S the highest?
    1) Maths 2) Science 3) History
    4) Geography 5) English
15. In which of the periodicals the student P obtained highest percentage of marks in Geography?
    1) I 2) II 3) III 4) IV 5) V

Directions (Q. 16-20): Study the following graph carefully and answer the questions given below.

Production of three types of tyres by a company over the year (in lakh)



16. What was the percentage drop in the number of C type tyres manufactured from 1993 to 1994?
    1) 25 2) 10 3) 15 4) 25 5) None of these
17. What was the difference between the number of B type tyres manufactured in 1994 and 1995?
    1) 1,00,000 2) 20,00,000 3) 10,00,000
    4) 15,00,000 5) None of these
18. The total number of all the three types of tyres manufactured was the least in which of the following years?
    1) 1995 2) 1996 3) 1992 4) 1994 5) 1993
19. In which of the following years was the percentage production of B type to C type tyres the maximum?
    1) 1994 2) 1992 3) 1996 4) 1993 5) 1995

20. The total production of C type tyres in 1992 and 1993 together was wha[t] percentage of production of B type tyres in 1994?
1) 50 2) 100 3) 150 4) 200 5) None of these

Solutions:

1. 5; % profit $= \dfrac{\text{Income} - \text{Expenditure}}{\text{Expenditure}} \times 100$

or, $45 = \dfrac{140 - E}{E} \times 100$

or, $\dfrac{140}{E} = \dfrac{45}{100} + 1 = \dfrac{9}{20} + 1 = \dfrac{29}{20}$

$\therefore E = 140\left(\dfrac{20}{29}\right) = 96.6$ cr.

Direct Formula:

$$\text{Expenditure} = \text{Income}\left[\frac{100}{\%\text{ profit} + 100}\right]$$

$$= 140 \times \frac{100}{100 + 45} = 140 \times \frac{100}{145} \approx 96.6 \text{ cr}$$

Note: 1. Because when we purchase something, we pay some amoun[t] or we expend some amount so we can consider the Cost Price a[s] Expenditure. The same logic can be applied with Selling Pric[e] and Income.

2. Understand the above logic and you can derive the dire[ct] formula yourself. See the logic: To make profit, Expenditur[e] should be less than Income. So our multiplying fraction is les[s] than one. Since there is profit the concerned numbers are 100 an[d] 100 + 45 (See the chapter Profit & Loss). If you don't want to g[o] in detail, remember the formula.

Now, Expenditure is the Cost Price and Income is the Sellin[g] Price.

Mark that if we use the rule of fraction (see chapter Profit & Los[s]) Selling Price (or Income)

$$= \text{Cost Price (or Expenditure)}\left[\frac{100 + \%\text{ profit}}{100}\right]$$

or, Cost Price (or Income)

$$= \text{Selling Price (or Expenditure)}\left[\frac{100}{100 + \%\text{ profit}}\right]$$

2. 4; $I_{93} = E_{93}\left(\dfrac{100 + 50}{100}\right) = \dfrac{3}{2}E_{93}$

$I_{94} = E_{94}\left[\dfrac{100 + 40}{100}\right] = \dfrac{7}{5}E_{94}$

$E_{93} + E_{94} = 279$

But we can't find $\dfrac{3}{2}E_{93} + \dfrac{7}{5}E_{94}$. Hence we can't solve it.

3. 5; $E = I\left(\dfrac{100}{100 + P}\right)$

or, $\dfrac{E}{I} = \dfrac{100}{100 + P}$ ..... (1)

We require $\dfrac{E}{I} \le 50\%$

or, $\dfrac{E}{I} \le \dfrac{1}{2}$

Now from (1), $\dfrac{100}{100 + P} \le \dfrac{1}{2}$

So, the value of P should be more than 100, which is not correct for any of the given years.

Quicker Approach:

% of Expenditure to the Income less than 50

$\Rightarrow$ Income is more than double the Expenditure

$\Rightarrow$ Profit % more than 100, which is not correct for any of the given years.

4. 2; $I = E\left[\dfrac{100 + \%\text{ Profit}}{100}\right] = 200\left(\dfrac{100 + 40}{100}\right)$ cr $= 280$ cr

5. 5; $I > 2E$

$\Rightarrow$ Profit % is more than 100. Which is not correct for any of the given years.

6. 5

7. 4; Our intelligent observation says that the required year can't be 1993, 1994, 1995. Why? Because see the following conclusions:

% passed to appear $= \dfrac{\text{Passed}}{\text{Appeared}} \times 100$

% of passed to appear is least when $\dfrac{\text{Passed}}{\text{Appeared}}$ is the least

or, $\dfrac{\text{Appear}}{\text{Passed}}$ is the most. Now we do the further calculations mentally. See the following conclusions:

For 1990: $\frac{7894}{2513} \Rightarrow$ Quotient = 3 & Remainder $\approx$ 300

For 1991: $\frac{8562}{2933} \Rightarrow Q = 3$ & $R \approx 400$

For 1992: $\frac{8139}{2468} \Rightarrow Q = 3$ & $R \approx 800$

Similarly for 1993, 1994, 1995, Q is 2.

So, 1992 gives the highest value.

Note: When R is close for two or three years you should go for furt calculations and find the exact possible values. But larger diff ence in R for almost equal divisors gives the option to stop further calculations, as happened in this case.

8. 1; $\frac{8562 - 8139}{8562} \times 100 = \frac{423}{8562} \times 100 \approx \frac{42}{84} \times 10 = 5$

9. 1; We don't need to calculate the values for each year. Follow as:
   For Rural area: 35% of 5032 $\approx 35 \times 50 \approx 1750 \approx 1798$
   For Semi-urban area: 35% of 9500 $\approx 35 \times 95 \approx 3300$
   Which can't be approximated to 3500.
   For State capitals: $35 \times 85 \approx 3000$
   For Metropolises: $35 \times 110 \approx 3850$

10. 3; 1798 + 2513 = 4311

11. 1; Average marks of Q in 1st periodical = $\frac{30 + 25 + 33 + 42 + 30}{5}$
   $= \frac{160}{5} = 32$

12. 3; Total marks of T in Science = 44 + 36 + 40 + 30 + 40 = 190

13. 2; Average percentage of marks obtained by P in Maths
   $= \frac{80 + 60 + 90 + 40 + 70}{5} = 68\%$ = percentage of marks obtai by student R in Geography.

14. 3; Our observation finds two options which are close to each oth These are History & Geography. When we find the actual val we find that our answer is History.

Note: You can decide the answer with totalling only. You don't need calculate the percentage value.

15. 2

16. 2; Required percentage drop = $\frac{25 - 22.5}{25} \times 100 = 10\%$

17. 5; Required difference = (35 - 22.5) lakh = 12,50,000

18. 2; Only by visual observation you can find the answer. You don't need to calculate the values.

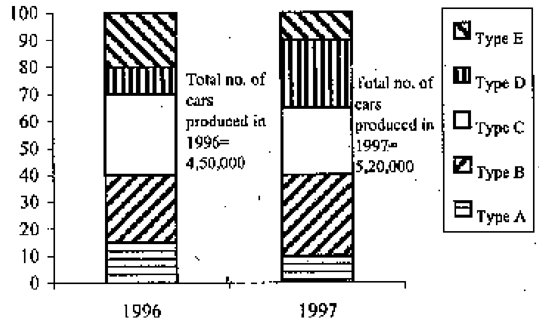
19. 5; Production of B type cars is more than the production of C type cars only in 1993, 1994 and 1995. We see the largest difference exists in 1995. So, the answer is 1995.

20. 4; Total production of C type tyres in 1992 and 1993 together = (30 + 25 =) 55 lakh and that of B in 1994 = 27.5 lakh.
   $\therefore$ Reqd percentage = $\frac{55}{27.5} \times 100 = 200$

BSRB Baroda PO held on 21st March 1999

Directions (Q. 1-5): *Study the following graph carefully and then answer the questions based on it.*

The percentage of five different types of cars produced by the company during two years is given below.



1. What was the difference in the production of C type cars between 1996 and 1997?
   1) 5,000 2) 7,500 3) 10,000 4) 2,500 5) None of these

2. If 85% of E type cars produced during 1996 and 1997 are being sold by the company, then how many E type cars are left unsold by the company?
   1) 1,42,800 2) 21,825 3) 29,100 4) 21,300 5) None of these

3. If the number of A type cars manufactured in 1997 was the same as that of 1996, what would have been its approximate percentage share in the total production of 1997?

1) 11 2) 13 3) 15 4) 9 5) None of these

4. In the case of which of the following types of cars was the percenta increase from 1996 to 1997 the maximum?

1) A 2) E 3) D 4) B 5) C

5. If the percentage production of B type cars in 1997 was the same a that of 1996, what would have been the number of cars produced i 1997?

1) 1,12,500 2) 1,20,000 3) 1,30,000
4) Data inadequate 5) None of these

Directions (Q. 6-10): *Read the following table carefully an answer the questions given below it:*

Average marks obtained by 20 boys and 20 girls in five subjects from five different schools

Subject	Max Marks	P B	P G	Q B	Q G	R B	R G	S B	S G	T B	T G
Eng	200	85	90	80	75	100	110	65	60	105	110
Hist	100	40	50	45	50	50	55	40	45	65	60
Geo	100	50	40	40	45	60	55	50	55	60	65
Math	200	120	110	95	85	135	130	75	80	130	135
Scien	200	105	125	110	120	125	115	85	90	140	135

B = Boys, G = Girls

6. What was the total marks obtained by boys in History from school Q?

1) 900 2) 1000 3) 800 4) 1300 5) None of these

7. In which of the following subjects did the girls have highest average percentage of marks from all the schools?

1) Science 2) Geography 3) English
4) History 5) Mathematics

8. The pooled average marks of both boys and girls in all the subjects was minimum from which of the following schools?

1) Q 2) P 3) T 4) S 5) R

9. In the case of which of the following schools was total marks obtained by girls in Mathematics 100% more than the total marks obtained by boys in History?

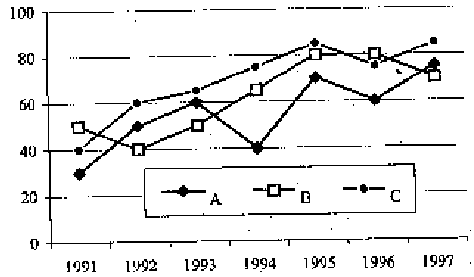
1) R 2) S 3) P 4) Q 5) T

10. What was the difference between the total marks obtained in Mathematics by boys from school R and the girls from school S?

1) Nil 2) 1100 3) 100 4) 1200 5) None of these

Directions (Q. 11-15): *Study the following graph carefully and answer the questions given below it:*

Imports of 3 companies over the years (in Rs crores)



11. In which of the following years, the imports made by Company A was exactly equal to average imports made by it over the given years?

1) 1992 2) 1993 3) 1994 4) 1995 5) None of these

12. In which of the following years was the difference between the imports made by Company B and C the maximum?

1) 1995 2) 1994 3) 1991 4) 1992 5) None of these

13. In which of the following years was the imports made by Company A exactly half of the total imports made by Company B and C together in that year?

1) 1992 only 2) 1993 only 3) 1992 and 1993
4) 1995 only 5) None of these

14. What was the percentage increase in imports by Company B from 1992 to 1993?

1) 10 2) 25 3) 40 4) 20 5) None of these

15. In which of the following years was the total imports made by all the three companies together the maximum?

1) 1996 only 2) 1997 only 3) 1995 only
4) 1995 and 1997 only 5) None of these

Solutions:

1. 1; Production of C type cars in 1996 = (70 - 40)% of 4,50,000
= 30% of 4,50,000 = 1,35,000
Production of C type cars in 1997
= (65 - 40)% of 5,20,000 = 25% of 5,20,000 = 1,30,000

$\therefore$ Required difference = 5,000

2. 4; Production of E type cars in 1996 = (100 - 80)% of 4,50,000
= 20% of 4,50,000 = 90,000

And in 1997 = 10% of 5,20,000 = 52,000

$\therefore$ Total production = 90,000 + 52,000 = 1,42,000

$\therefore$ Required no. of cars = 15% of 1,42,000 = 21,300

3. 2; Production of A type cars in 1997 = production of A type cars i
1996 (given) = (100 - 85 =)15% of 4,50,000 = 67,500

$\therefore$ Reqd percentage $= \frac{67,500}{5,20,000} \times 100 \approx 13$

4. 3; Clearly, by visual inspection D is the desired option.

5. 3; Percentage production of B type cars in 1997 = that in 1996 (given
= (40 - 15 =) 25% of 5,20,000 = 1,30,000

6. 1; Average marks obtained by 20 boys in History from school Q = 45

$\therefore$ Total marks = 20 × 45 = 900

7. 1; From visual inspection it is clear that Science is the desired subject

Note: Our visual observation says that it is either Math or Science i which maximum marks has been obtained. So, compare the tota of Maths and Science only.

8. 4; Total marks obtained by boys and girls in all the subjects:
For school P = (85 + 40 + 50 + 120 + 105) + (90 + 50 + 40 + 110 + 125) = 815
Similarly, for Q = 745, for R = 935, for S = 645 and for T = 1005
645 is the minimum, so S is the desired school.

Note: From careful observation we find that our answer is school S. The other school nearest to it is either P or Q. But if you compare the marks, P and Q also take lead of at least 100 marks. So, only visual observation gives the result.

9. 2; As the no. of boys and girls in the different schools are the same, so for the desired purpose we have to select a certain school in which the average marks of girls in Mathematics be exactly double the average marks of boys in History. By visual inspection (as 80 = 2 × 40) we get that S is the desired school.

10. 2; In Mathematics total marks obtained by boys from school R = 135 × 20

By girls from school S = 80 × 20

$\therefore$ Reqd difference = (135 - 80 =) 55 × 20 = 1100.

11. 5; Average imports made by company A

$$= \frac{30 + 50 + 60 + 40 + 70 + 60 + 75}{7} = \frac{385}{7} = 55$$

In none of the given years the imports is exactly equal to 55 (crores). Hence, the answer is 5.

12. 4; By visual inspection it is clear that 1992 is the desired year (as the distance between two points is the maximum in 1992.)

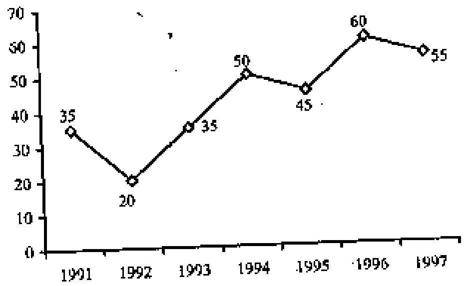
13. 1; By mental observation $\left(\text{as } 50 = \frac{40 + 60}{2}\right)$, 1992 only is the desired year. You don't need any calculation. See the year where the point of A lies exactly in the middle of points of B and C.

14. 2; Reqd percentage increase $= \frac{50 - 40}{40} \times 100 = 25\%$

15. 3; The total imports (in crores) made by all the three companies together: From the heights of the points we observe that the total heights of three points is the maximum either in 1995 or 1997. If you observe carefully, our clear answer is 1995, but to be sure we find actual values for the two years.
In 1995 = 70 + 80 + 85 = 235. In 1997 = 75 + 70 + 85 = 230.
Clearly, 1995 is the desired year.

BSRB Bangalore PO held on 7th March 1999

Directions (Q. 1-5): *The following graph shows the percentage net profit of a certain company during the given period. Study it carefully and answer the questions given below.*



1. If the expenditure in 1993 was 20% more than the expenditure in 19 by what per cent the income in 1993 was more/less than the inco in 1991?
   1) 25% less 2) 20% more 3) 27% more
   4) Data inadequate 5) None of these
2. During which of the following years was the ratio of income expenditure the minimum?
   1) 1991 2) 1994 3) 1995
   4) 1996 5) None of these
3. During which year the ratio of percentage net profit earned to that the previous year is the minimum?
   1) 1993 2) 1994 3) 1996
   4) 1993 and 1994 both 5) 1993, 1994 and 1996
4. If the expenditure in 1995 and 1997 was equal and income in 199 was 15.5 lakhs, what was the income in 1995?
   1) 12.5 lakhs 2) 13.5 lakhs 3) 14 lakhs
   4) Data inadequate 5) None of these
5. If the income in 1994 was Rs 25 lakhs, what was the expenditure in that year?
   1) 16 lakhs 2) 16.33 lakhs 3) 16.67 lakhs
   4) 15.67 lakhs 5) None of these

Directions (Q. 6-10): *Refer to the table and answer the questions given below:*

Distribution of marks obtained by 100 students in two papers (I & II) in Mathematics

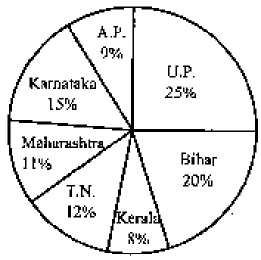
Marks out of 50 → ↓ Paper	40 & above	30 & above	20 & above	10 & above	0 & above
I	5	22	67	82	100
II	8	31	79	91	100
Aggregate (Average)	6	27	71	88	100

6. What should be the passing marks if minimum 80 students are required to be qualified with compulsory passing only in Paper I?
   1) Below 20 2) Above 20 3) Below 40
   4) Above 40 5) None of these
7. What will be the difference between the number of students passed with 30 as cut-off marks in paper II and the no. of students passed with same cut-off marks in aggregate?
   1) 2 2) 4 3) 8 4) 3 5) None of these

8. How many students have scored less than 40% marks in aggregate?
   1) 30 2) 12 3) 17 4) 29 5) None of these
9. What is the **approximate** percentage of students who have obtained 60% and more marks in paper II over the number of students who obtained 40% and more marks in aggregate?
   1) 44 2) 40 3) 48
   4) Data inadequate 5) None of these
10. How many students will pass if there is compulsory passing of minimum 40% marks only in paper I?
    1) 5 2) 31 3) 67 4) 8 5) None of these

Directions (Q. 11-17): *Study the given graphs and table and answer the following questions given below.*

The total population of the different states in 1993 is 25 lakhs



Sex-and-literacywise population ratio

States	Sex M — F	Literacy Literate — Illiterate
U.P.	5 : 3	2 : 7
Bihar	3 : 1	1 : 4
A.P.	2 : 3	2 : 1
Karnataka	3 : 5	3 : 2
Maharashtra	3 : 4	5 : 1
Tamil Nadu	3 : 2	7 : 2
Kerala	3 : 4	9 : 4

M = Male, F = Female

11. Approximately what is the total number of literate people in Maharashtra and Karnataka together?

1) 4.5 lakh 2) 6.5 lakh 3) 3 lakh
4) 3.5 lakh 5) 6 lakh

12. Approximately what will be the percentage of total male in UP, Maharashtra & Kerala of the total population of the given states?
1) 20% 2) 18% 3) 28% 4) 30% 5) 25%

13. If in the year 1993 there was an increase of 10% population of U.P. and 12% of Bihar compared to the previous year, then what was the ratio of the population of UP to Bihar in year 1992?
1) 50 : 40 2) 40 : 50 3) 48 : 55
4) 55 : 48 5) None of these

14. What was the approximate percentage of women of Andhra Pradesh to the women of Tamil Nadu?
1) 90% 2) 110% 3) 120% 4) 85% 5) 95%

15. What is the ratio of the number of females in Tamil Nadu to the number of females in Kerala?
1) 1 : 2 2) 1 : 1 3) 2 : 1 4) 2 : 3 5) None of these

16. In Tamil Nadu if 70% of the females are literate and 75% of the males are literate, what is the total number of illiterates in the state?
1) 75,000 2) 85,000 3) 71,000
4) 81,000 5) None of these

17. What is the ratio of literates in Andhra Pradesh to the literates in Bihar?
1) 2 : 5 2) 3 : 5 3) 3 : 2
4) 2 : 3 5) None of these

Solutions:

1. 2; **Quicker Approach:** From the graph we see that percentage profits for the years 1991 and 1993 are the same (35%). So, if expenditure in 1993 was 20% more than the expenditure in 1991, then income in 1993 should also be 20% more than the income in 1991.

Detail Soln: Suppose Income = I and Expenditure = E

Then, from the graph: $\frac{I_{91}-E_{91}}{E_{91}} = \frac{I_{93}-E_{93}}{E_{93}}$ (Since % income for the two years are the same)

$\frac{I_{91}}{E_{91}} - 1 = \frac{I_{93}}{E_{93}} - 1$

or, $\frac{I_{91}}{E_{91}} = \frac{I_{93}}{E_{93}}$ or, $\frac{I_{91}}{E_{91}} = \frac{I_{93}}{120\% \text{ of } E_{91}}$

$\therefore I_{93} = 120\%$ of $I_{91} \Rightarrow I_{93}$ was 20% more than I_{91}

2. 5; Clearly, when % profit is the minimum the ratio of income to expenditure is also the minimum.

Note: Because we know that

$\%\text{ Profit} = \frac{I-E}{E} \times 100 = \left(\frac{I}{E} - 1\right) \times 100.$

In the above relationship % profit is directly proportional to $\frac{I}{E}$.

That is, % profit totally depends on $\frac{I}{E}$ and they vary in the same direction.

3. 3; From the choices it is clear that we have to find the ratio for the years 1993, 1994 and 1996 only.

Ratio in 1993 $= \frac{35}{20} = 1.75$

Ratio in 1994 $= \frac{50}{35} = 1.43$

Ratio in 1996 $= \frac{60}{45} = 1.33$

So, the answer is 1996, i.e. (3).

4. 5; We are given: $E_{95} = E_{97} = x$ and $I_{97} = 15.5$ lakh

From the graph we have,

$\frac{I_{95} - E_{95}}{E_{95}} = \frac{45}{100} = \frac{9}{20}$

$\frac{I_{95}}{x} = 1 + \frac{9}{20} = \frac{29}{20}$ .... (1)

And $\frac{I_{97} - E_{97}}{E_{97}} = \frac{55}{100} = \frac{11}{20}$

or, $\frac{I_{97}}{E_{97}} = \frac{11}{20} + 1 = \frac{31}{20}$ ... (2)

Dividing (1) by (2), we get

$\frac{I_{95}}{x} \times \frac{x}{I_{97}} = \frac{29}{20} \times \frac{20}{31}$

or, $I_{95} = \frac{29}{31} \times 15.5 = 14.5$ lakh

Quicker Method: (By the rule of fraction)

$I_{95} = \left(\frac{100+45}{100+55}\right) I_{97} = \frac{145}{155} \times 15.5 \text{ lakh} = 14.5 \text{ lakh}$

The above relationship has been defined on the following basis:

(1) Expenses are the same for 1995 & 1997.

(2) Since % profit is less in 1995 than in 1997, income in 199[illegible] will also be less than the income in 1997. So we used less-than one fraction (145/155).

5. 3; We have,

$$\frac{I_{94}-E_{94}}{E_{94}} = 50\% = \frac{50}{100} = \frac{1}{2}$$

$$\frac{I_{94}}{E_{94}} = 1 + \frac{1}{2} = \frac{3}{2}$$

$$\therefore E_{94} = \frac{2}{3} \times I_{94} = \frac{2}{3} \times 25 = 16.67 \text{ lakh}$$

Quicker Method (Direct Formula):

We should remember the direct relationship as:

Income = Expenditure × $\left[\frac{100 + \%\text{ profit}}{100}\right]$ has been defined earlier

$$\therefore I_{94} = E_{94}\left(\frac{100+50}{100}\right) \ldots\ldots (*)$$

$$\therefore E_{94} = \frac{100}{150} \times 25 \text{ lakh} = \frac{2}{3} \times 25 \text{ lakh} = 16.67 \text{ lakh.}$$

Note: (*) Can be defined by the rule of fraction. The logic is: for a profit Income should be more than Expenditure, so our multiplying factor is a more-than-one fraction $\left(\text{ie, } \frac{100+50}{100}\right)$.

6. 5; When the pass marks is 10 and above then a total of 82 students qualify, and when 0 and above then all the 100 students qualify. As the question implies that there is no limit of only 82 students passing, this no. should be merely higher than 80 (it may be 85 or 92 or even all the 100 students). So, as regards the context, the correct answer is either 10 and above, or 0 and above.

For a moment, considering option (1), we see that when the pass marks is 19 then the no. of qualified students is 82 only when all the (82 - 67 =) 15 students have scored 19 marks, which cannot be said with certainly. Hence, we conclude that the answer is (5).

7. 2; Required difference = 31 - 27 = 4

8. 4; Full marks = 50. Now, 40% of 50 = 20

Now we see that 71 students have scored 20 and above marks in aggregate. So, the remaining (100 - 71 =) 29 students have scored less than 20 marks in aggregate.

9. 1; $\frac{31}{71} \times 100 = 43.66\% \approx 44\%$

10. 3; 40% of 50 = 20

So, 67 students will pass as they obtain 20 and above marks.

11. 1; Total no. of literate people in Maharashtra and Karnaaka

$$= \left[\frac{5}{6} \times 11\% + \frac{3}{5} \times 15\%\right] \text{ of } 25 \text{ lakh}$$

$$= \left[\frac{55}{6} + 9\right] \text{ of } \frac{25}{100} \text{ lakh}$$

$$= \frac{109}{6} \times \frac{25}{100} \approx = 4.50 \text{ lakh}$$

12. 5; Required percentage

$$= \left[25\% \text{ of } \frac{5}{8} + 11\% \text{ of } \frac{3}{7} + 8\% \text{ of } \frac{3}{7}\right] \times 100$$

$$= \frac{125}{8} + \frac{33}{7} + \frac{24}{7} \approx 25\%$$

13. 5; Required ratio $= \dfrac{\frac{100}{110} \times 25\% \text{ of } 25 \text{ lakh}}{\frac{100}{112} \times 20\% \text{ of } 25 \text{ lakh}}$

$$= \frac{112 \times 25}{110 \times 20} = \frac{14}{11} = 14 : 11$$

14. 2; Required percentage $= \dfrac{\frac{3}{5} \text{ of } 9\% \text{ of } 25 \text{ lakh}}{\frac{2}{5} \text{ of } 12\% \text{ of } 25 \text{ lakh}} \times 100$

$$= \frac{3 \times 9}{2 \times 12} \times 100 = \frac{9 \times 100}{8} = 9 \times 12.5 \approx 110(\%)$$

15. 5; Required ratio $= \dfrac{\frac{2}{5} \text{ of } 12\% \text{ of } 25 \text{ lakh}}{\frac{4}{7} \text{ of } 8\% \text{ of } 25 \text{ lakh}}$

$$= \frac{\frac{2}{5} \times 12}{\frac{4}{7} \times 8} = \frac{2 \times 12 \times 7}{5 \times 4 \times 8} = \frac{3 \times 7}{5 \times 4} = \frac{21}{20}$$

16. 4; The total no. of illiterates in Tamil Nadu = (100 - 70 =) 30% of females + (100 - 75 =) 25% of males in the state

$$= \left(\frac{30}{100} \times \frac{2}{5} + \frac{25}{100} \times \frac{3}{5}\right) \text{ of } 12\% \text{ of } 25 \text{ lakh}$$

$$= (6 \times 2 + 5 \times 3 =) \frac{27}{100} \times \frac{12}{100} \times 2500000$$

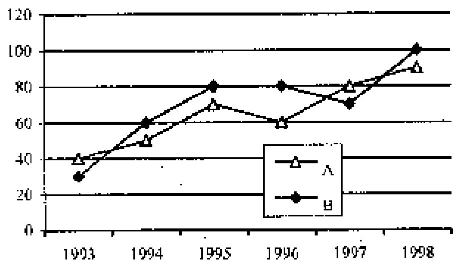
$$= 27 \times 12 \times 250 = 81{,}000$$

17. 3; Required ratio $= \dfrac{\frac{2}{3} \text{ of } 9\% \text{ of } 25 \text{ lakh}}{\frac{1}{5} \text{ of } 20\% \text{ of } 25 \text{ lakh}} = \dfrac{\frac{2}{3} \times 9}{\frac{1}{5} \times 20} = \dfrac{2 \times 3}{4} = 3 : 2$

SBI Associates PO held on 18th July 1999

Directions (Q. 1-5): Study the graph carefully and answer th questions given below it.

Per cent profit earned by the two companies A & B over the year



1. If income for company A in the year 1994 was 35 lakhs what was the expenditure for company B in the same year?
   1) 123.5 lakhs 2) 128 lakhs 3) 132 lakhs
   4) Data inadequate 5) None of these
2. The income of company A in 1996 and the income of company B i 1997 are equal. What will be the ratio of expenditure of company A in 1996 to the expenditure of company B in 1997?
   1) 26 : 7 2) 17 : 16 3) 15 : 17
   4) 16 : 17 5) None of these

3. During which of the following years the ratio of per cent profit earned by company A to that of company B was the maximum?
   1) 1993 & 1996 both 2) 1995 & 1997 both
   3) 1993 only 4) 1998 only
   5) None of these
4. If the expenditure of company B increased by 20% from 1995 to 1996, the income in 1996 will be how many times the income in 1995?
   1) 2.16 times 2) 1.2 times 3) 1.8 times
   4) equal 5) None of these
5. If the income of company A in 1996 was Rs 36 lakhs, what was the expenditure of company A in 1996?
   1) 22.5 lakhs 2) 28.8 lakhs 3) 20 lakhs
   4) 21.6 lakhs 5) None of these

Directions (Q. 6-10): *Study the following table carefully and answer the questions given below it:*

Statewise and Disciplinewise Number of Candidates Appeared (App.) and Qualified (Qual.) at a competitive Examination)

State	A.P.		U.P.		Kerala	
Discipline	App.	Qual.	App.	Qual.	App.	Qual.
Arts	5420	1840	4980	1690	2450	845
Commerce	8795	2985	6565	2545	3500	2040
Science	6925	2760	8750	3540	4250	2500
Engg	1080	490	2500	1050	1200	450
Agri.	840	850	1085	455	700	200
Total	23060	8425	23880	9280	12100	6035

Orissa		M.P.		W.B.		Total	
App.	Qual.	App.	Qual.	App.	Qual.	App.	Qual.
3450	1200	7500	2000	4800	1500	28600	9075
4800	2200	8400	2400	7600	2700	39660	14870
4500	1950	6850	3000	8500	3200	39775	16950
1850	850	2500	750	3400	1400	12530	4990
450	150	1500	475	1200	500	5775	2130
15050	6350	26750	8625	25500	9300	126340	48015

6. For which of the following disciplines the proportion of qualifying candidates to the appeared candidates from U.P. State is the lowest?
   1) Arts 2) Commerce 3) Science 4) Engineering 5) Agriculture
7. For which of the pair of States, the qualifying percentage from Agriculture discipline is exactly the same?

1) A.P. & U.P. 2) A.P. & West Bengal
3) U.P. & West Bengal 4) Kerala & Orissa
5) None of these

8. For which of the following states the percentage of candidates qualified to appeared is the minimum for commerce discipline?
1) AP 2) UP 3) Kerala 4) Orissa 5) MP

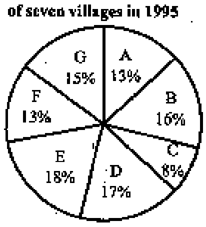
9. Approximately what is the ratio between total qualifying percentage of UP and that of MP?
1) 15 : 16 2) 13 : 14 3) 14 : 13 4) 19 : 16 5) 17 : 16

10. The qualifying percentage for which of the following states is the lowest for Science discipline?
1) AP 2) UP 3) West Bengal 4) Kerala 5) None of these

Directions (Q. 11-15): *Study the following chart to answer the question given below:*

Villago	% population below poverty line
A	45
B	52
C	38
D	58
E	46
F	49
G	51

Proportion of population of seven villages in 1995



11. In 1996, the population of villages A as well as B is increased by 10% from the year 1995. If the population of village A in 1995 was 5000 and the percentage of population below poverty line in 1996 remains same as in 1995, find approximately the population of village B below poverty line in 1996.

1) 4000 2) 4500 3) 2500 4) 3000 5) 3500

12. If in 1997 the population of village D is increased by 10% and the population of village G is reduced by 5% from 1995 and the population of village G in 1995 was 9000, what is the total population of villages D and G in 1997?
1) 19770 2) 19200 3) 18770 4) 19870 5) None of these

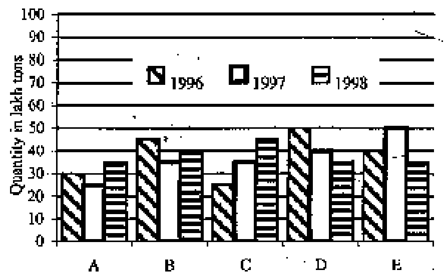
13. If in 1995 the total population of the seven villages together was 55,000 approximately, what will be population of village F in that year below poverty line?
1) 3000 2) 2500 3) 4000 4) 3500 5) 4500

14. If the population of village C below poverty line in 1995 was 1520, what was the population of village F in 1995?
1) 4000 2) 6000 3) 6500 4) 4800 5) None of these

15. The population of village C is 2000 in 1995. What will be the ratio of population of village C below poverty line to that of the village E below poverty line in that year?
1) 207 : 76 2) 76 : 207 3) 152 : 207
4) Data inadequate 5) None of these

Directions (Q. 16-20): *Study the following graph carefully to answer these questions.*

The production of fertilizer in lakh tons by different companies for three years 1996, 1997 & 1998



16. The total production by five companies in 1998 is what per cent of th total production by companies B & D in 1996?
 1) 100% 2) 150% 3) 95%
 4) 200% 5) None of these
17. What is the ratio between average production by Company B in thre years to the average production by company C in three years?
 1) 6 : 7 2) 8 : 7 3) 7 : 8
 4) 7 : 6 5) None of these
18. For which of the following companies the rise or fall in production o fertiliser from 1996 to 1997 was the maximum?
 1) A 2) B 3) C 4) D 5) E
19. What is the per cent drop in production by Company D from 1996 t 1998?
 1) 30 2) 43 3) 50 4) 35 5) None of these
20. The average production for three years was maximum for which of the following companies?
 1) B only 2) D only 3) E only
 4) B & D both 5) D & E both

Solutions:

1. 4; Incomes-Expenditures of company A and B cannot be correlated.
2. 2; Expenditure of Company A in 1996 =

$$E_{96(A)} = I_{96(A)}\left[\frac{100}{100+60}\right] = \frac{5}{8} I_{96(A)}$$

Expenditure of Company B in 1997 =

$$E_{97(B)} = I_{97(B)}\left(\frac{100}{100+70}\right) = \frac{10}{17} I_{97(B)}$$

Now, $\frac{E_{96(A)}}{E_{97(B)}} = \frac{5}{8} \div \frac{10}{17}$ (Since $I_{96(A)} = I_{97(B)}$)

$$= \frac{5}{8} \times \frac{17}{10} = \frac{17}{16} \neq 17 : 16$$

3. 3; Ratio A : B is greater than 1 in only 1993 and 1997. It is 1.33 in 1993 and 1:1 in 1997.
4. 2; Suppose $E_{95(B)} = x$

Then $E_{96(B)} = 1.2x$ (Since $x + 20\%$ of $x = 1.2x$)

Now, $I_{95(B)} = E_{95(B)}\left[\frac{100+80}{100}\right] = 1.8x$

$I_{96(B)} = E_{96(B)}\left[\frac{100+80}{100}\right] = 1.2x(1.8)$

$\therefore \frac{I_{96(B)}}{I_{95(B)}} = \frac{1.2 \times 1.8x}{1.8x} = 1.2$ times

Quicker Approach: % profits are the same for two years. So if expenditure increases by 20% the income should also increase by 20%.

Hence the required ratio $= \frac{100+20}{100} = 1.2$

5. 1; $E_{96(A)} = I_{96(A)}\left[\frac{100}{100+60}\right]$

$$= \frac{36 \text{ lakh} \times 100}{160} = \text{Rs } 22.5 \text{ lakh}$$

6. 1; UP (Qua/App)

Arts	Commerce	Science	Engg.	Agr.
0.33	0.38	0.40	0.42	0.41

Quicker Approach: $\frac{\text{Qual.}}{\text{App.}}$ should be the least.

$\Rightarrow \frac{\text{App.}}{\text{Qual.}}$ should be the maximum.

Now, for Arts, if we divide (4980 ≈) 5000 by (1690 ≈) 1700 we find the value of quotient near about 3. But in other cases the quotient is just more than 2. So, our answer is Arts.

7. 2
8. 5; Percentage of students qualified in commerce

A.P.	U.P.	Kerala	Orissa	M.P.
33.9	38.7	58.2	45.8	28.5

Quicker Approach: Follow the same as in Soln. 6. Only for MP $\frac{\text{App.}}{\text{Qual.}} = 3.5$ (more than 3). In other cases $\frac{\text{App.}}{\text{Qual.}} < 3$.

9. 4; Qualifying percentage of UP $= \frac{9280}{23880} \times 100 = 38.86$

Qualifying percentage of MP $= \frac{8625}{26750} \times 100 = 32.24$

Ratio = 38 : 32 = 19 : 16

10. 3; Qualifying percentage for Science

A.P.	U.P.	W.B.	Kerala
39.8	40.4	37.6	58.8

Quicker Approach: Follow the same as in Soln. 6 & 8.

11. 5; Population of village B in 1995 = $5000 \times \frac{16}{13} \approx 6150$

Population of B in 1996 $\approx 6150 \times \frac{110}{100} = 6750$

Pop. below poverty line = 52% of 6750 $\approx$ 3500

12. 1; Population of village D in 1995 = $9{,}000 \times \frac{17}{15} = 10{,}200$

Population of village D in 1997 = $10{,}200 \times \frac{110}{100} = 11{,}220$

Population of village G in 1997 = $9{,}000 \times \frac{95}{100} = 8{,}550$

$\therefore$ Total population of villages D and G in 1997 $= 11{,}220 + 8{,}550 = 19{,}770$

13. 4; Population of village F below poverty line

$= 55000 \times \frac{13}{100} \times \frac{49}{100} \approx 3500$

14. 3; Population of village F in 1995

$= 1520 \times \frac{100}{38} \times \frac{13}{8} = 6500$

15. 2; Population of village C below poverty line = $2000 \times \frac{38}{100} = 760$

Population of village E below poverty line

$= \frac{2000}{8} \times 18 \times \left(\frac{46}{100}\right) = 2070$

$\therefore$ Required ratio $= \frac{760}{2070} = 76 : 207$

16. 4; Required percentage $= \frac{35 + 40 + 45 + 35 + 35}{45 + 50} \times 100$

$= \frac{190}{95} \times 100 = 200\%$

17. 2; Average production by B $= \frac{45 + 35 + 40}{3} = 40$

Average production by C $= \frac{25 + 35 + 45}{3} = 35$

Ratio = (40 : 35 =) 8 : 7

18. 3; **Quicker Approach:** Maximum difference is 10 lakh tonnes for three companies C, D & E. So, our answer should be the company for which the production is least in 1996. (Why?) Because to

calculate the % increase or decrease our denominator is the production in 1996.

19. 1; Percentage drop $= \frac{50 - 35}{50} \times 100 = 30\%$

20. 5; You should not calculate the values to get answer. You can decide by mere visual observation.

BSRB Guwahati PO held on 8th August 1999

Directions (Q. 1-5): *Study the following table carefully to answer the questions given below it.*

Number of candidates appeared, qualified and selected in a competitive examination from five states A, B, C, D and E over the years 1994 to 1998.

State →	A			B		
Year ↓	App.	Qual.	Sel.	App.	Qual.	Sel.
1994	4500	600	75	6400	540	60
1995	5700	485	60	7800	720	84
1996	8500	950	80	7000	650	70
1997	7200	850	75	8800	920	86
1998	9000	800	70	9500	850	90

C			D			E		
App.	Qual.	Sel.	App.	Qual.	Sel.	App.	Qual.	Sel.
5200	350	55	7100	650	75	6400	700	75
6500	525	65	6800	600	70	8200	680	85
4800	400	48	5600	620	85	7500	720	78
7400	560	70	7500	800	65	7800	810	82
7500	640	82	4800	500	48	8000	850	94

1. What is the average number of candidates appeared over the years for State B?

1) 8900 2) 7900 3) 7400 4) 8100 5) None of these

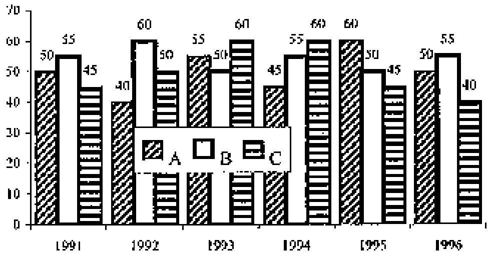
2. What approximately is the percentage of total number of candidates selected to the total number of candidates qualified for all the five states together during the year 1996?

1) 11% 2) 15% 3) 12% 4) 16% 5) 14%

3. For which of the following years is the percentage of candidates selected over the number of candidates qualified the **highest** for state 'C'?
   1) 1997 2) 1995 3) 1996 4) 1994 5) 1998
4. For which of the following states the average number of candidates selected over the years is the **maximum**?
   1) A 2) E 3) C 4) D 5) B
5. For which of the following states is the percentage of candidates qualified to appeared the highest during the year 1997?
   1) A 2) B 3) C 4) D 5) E

Directions (Q. 6-10): *Study the following graph carefully to answer the questions given below it.*

Production of paper (in lakh tons) by 3 different companies A, B & C over the years

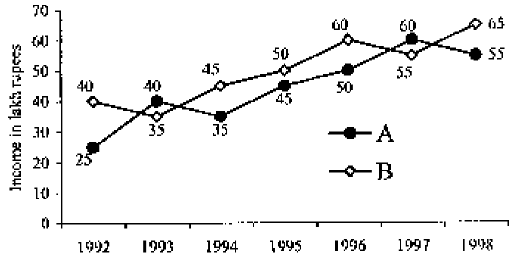


6. What is the difference between the production of company C in 1991 and the production of company A in 1996?
   1) 50,000 tonnes 2) 5,00,00,000 tonnes
   3) 50,00,000 tonnes 4) 5,00,000 tonnes
   5) None of these
7. What is the percentage increase in production of company A from 1992 to 1993?
   1) 37.5 2) 38.25 3) 35 4) 36 5) None of these

8. For which of the following years the percentage of rise/fall in production from the previous year the **maximum** for company B?
   1) 1992 2) 1993 3) 1994 4) 1995 5) 1996
9. The total production of company C in 1993 and 1994 is what percentage of the total production of company A in 1991 and 1992?
   1) 95 2) 90 3) 110 4) 115 5) None of these
10. What is the difference between the average production per year of the company with highest average production and that of the company with lowest average production in lakh tonnes?
   1) 3.17 2) 4.33 3) 4.17 4) 3.33 5) None of these

Directions (Q. 11-15): *Study the following graph carefully and answer the questions given below:*

Income of two plastic manufacturing Companies A & B over the years (in lakh rupees)



11. If the per cent profit earned by both the companies A and B in 1997 is equal and the expenditure of company B in 1997 is Rs 50 lakhs, what **approximately** is the amount of profit earned by company A in 1997?
   1) Rs 4.5 lakhs 2) Rs 5 lakhs 3) Rs 6.2 lakhs
   4) Data inadequate 5) Rs 5.5 lakhs
12. For which of the following combinations of company and year is the percentage increase in income from the previous year the **maximum** among all such given combinations?
   1) Company B — 1994 2) Company B — 1996

3) Company A — 1993 4) Company A — 1994
5) Company A — 1997

13. If company A had a loss of 15% in the year 1992, what **approximately** was its expenditure in that year?
1) Rs 22 lakhs 2) Rs 29 lakhs 3) Rs 21 lakhs
4) Rs 28 lakhs 5) Rs 23 lakhs

14. Average income of company A per year is **approximately** what percentage of the average income of company B per year?
1) 80% 2) 110% 3) 115% 4) 90% 5) 75%

15. Income of company B in 1994 is what per cent of income of company A in 1997?
1) 75% 2) 133.33% 3) 63.64%
4) 150% 5) None of these

Directions (Q. 16-20): *Study the following table carefully and answer the questions given below it.*

Fare in rupees for three different types of vehicles

Vehicle	Fare for distance upto					
	2 km	4 km	7 km	10 km	15 km	20 km
Type A	Rs 5.00	Rs 9.00	Rs 13.50	Rs 17.25	Rs 22.25	Rs 26.00
Type B	Rs 7.50	Rs 14.50	Rs 24.25	Rs 33.25	Rs 45.75	Rs 55.75
Type C	Rs 10.00	Rs 19.00	Rs 31.00	Rs 41.50	Rs 56.50	Rs 69.00

Note: Fare per km for intermittent distance is the same.

16. Shiv Kumar has to travel a distance of 15 kms in all. He decides to travel equal distance by each of the three types of vehicles. How much money is to be spent as fare?
1) Rs 51.75 2) Rs 47.50 3) Rs 47.25
4) Rs 51.25 5) None of these

17. Ajit Singh wants to travel a distance of 15 kms. He starts his journey by Type A vehicle. After travelling 6 kms, he changes the vehicle to Type B for the remaining distance. How much money will he be spending in all?
1) Rs 42.25 2) Rs 36.75 3) Rs 40.25
4) Rs 42.75 5) None of these

18. Mr X wants to travel a distance of 8 kms by Type A vehicle. How much more money will be required to be spent if he decides to travel by Type B vehicle instead of Type A?
1) Rs 16 2) Rs 12.50 3) Rs 14
4) Rs 13.50 5) None of these

19. Rita hired a Type B vehicle for travelling a distance of 18 kms. After travelling 5 kms, she changed the vehicle to Type A. Again after travelling 8 kms by Type A vehicle, she changed the vehicle to Type C and completed her journey. How much money did she spend in all?
1) Rs 50 2) Rs 45.50 3) Rs 55
4) Rs 50.50 5) None of these

20. Fare for 14th km by Type C vehicle is equal to the fare for which of the following?
1) Type B — 11th km 2) Type B — 9th km
3) Type A — 4th km 4) Type C — 8th km
5) None of these

Solutions:

1. 2; Average no. of candidates appeared for state B

$$= \frac{6400 + 7800 + 7000 + 8800 + 9500}{5} = 7900$$

2. 1; Required percentage $= \frac{361}{3340} \times 100 \approx 11\%$

3. 4; Percentage of candidates selected for state C can be seen in the following table:

1994	1995	1996	1997	1998
15.7%	12.3%	12%	12.5%	12.8%

Quicker Approach: We have to find the year for which $\frac{\text{Sel.}}{\text{Qual.}}$ is the highest; ie, $\frac{\text{Qual.}}{\text{Sel.}}$ is the least. Clearly, only for the year 1994 it is below 7. In other cases it is more than 7. Hence our answer is (4).

4. 2; Average no. of candidates selected

A	B	C	D	E
72	78	64	68.6	82.8

5. 1; Percentage of candidates qualified in 1997

A	B	C	D	E
11.8%	10.4%	7.5%	10.6%	10.3%

Quicker Approach: Follow the same as in Soln. 3. We have to find the highest value of $\frac{\text{Qual.}}{\text{App.}}$, ie the least value of $\frac{\text{App.}}{\text{Qual.}}$. It is for state A,

which is less than 9. In other cases it is more than 9. So, our answer is (1).

6. 4; Difference of production of C in 1991 and A in 1996 = 5,00,000 tonnes.

7. 1; Percentage increase of A from '92 to '93 $= \frac{55-40}{40} \times 100 = 37.5\%$

8. 2; Percentage rise/fall in production for B

1992	1993	1994	1995	1996
9%	-16.6%	10%	-9%	10%

Quicker Approach: The maximum difference is from 1992 to 1993, which is 10. And the second nearest to it is fall or rise of 5. So, undoubtedly the answer is 1993.

9. 5; Percentage production $= \frac{120}{90} \times 100 = 133.3\%$

10. 3; Average production of A = 50
Average production of B = 54.17
Average production of C = 50
Difference of production = 54.17 - 50 = 4.17

11. 5; According to the question,

$$\frac{I_{97(A)} - E_{97(A)}}{E_{97(A)}} = \frac{I_{97(B)} - E_{97(B)}}{E_{97(B)}}$$

or, $\frac{I_{97(A)}}{E_{97(A)}} - 1 = \frac{I_{97(B)}}{E_{97(B)}} - 1$

or, $\frac{I_{97(A)}}{E_{97(A)}} = \frac{I_{97(B)}}{E_{97(B)}}$

$\therefore E_{97(A)} = \frac{60 \times 50}{55} = 54.50$ lakh

$\therefore$ Profit earned by company A in 1997

$I_{97(A)} - E_{97(A)} = 60 - 54.50 = 5.50$ lakh

12. 3; Percentage increase in income

B-94	B-96	A-93	A-94	A-97
28.5%	20%	60%	(-) 12.5	20%

Quicker Approach: The maximum increase is in 1993 for company A, which is 15. So our answer is (3). We don't need to calculate anything.

13. 2; Expenditure of A in 1992 $= 25 \times \frac{100}{85} \approx 29$ lakh

14. 4; Required percentage $= \frac{310}{350} \times 100 \approx 90\%$

15. 1; Required percentage $= \frac{45}{60} \times 100 = 75\%$

16. 4; Distance to be travelled by each type of vehicle $= \frac{15}{3} = 5$ km

Since, to travel 5 km by vehicle A, he will pay Rs 9 for 4 km and for the next 1 km he will have to pay Rs $\frac{13.5 - 9.00}{(7-4)} \times 1$.

Similarly for other cases.

Fare by A = Rs $9 + \frac{13.50 - 9}{7-4} = 9 + 1.50 =$ Rs 10.50

Fare by B = $14.50 + \frac{24.25 - 14.50}{7-4} = 14.50 + 3.25 = 17.75$

Fare by C = $19 + \frac{31-19}{3} = 19 + 4 = 23$

Total fare = 10.50 + 17.75 + 23 = Rs 51.25

17. 1; Fare by A = $9 + \frac{4.50}{3} \times 2 =$ Rs 12

Fare by B = $24.25 + \frac{33.25 - 24.25}{3} \times 2 =$ Rs 30.25

Total fare = 30.25 + 12 = Rs 42.25

18. 2; Fare for 8 km by A = $13.50 + \frac{17.25 - 13.50}{10-7}$

$= 13.50 + \frac{3.75}{3} =$ Rs 14.75

Fare by B = $24.25 + \frac{33.25 - 24.25}{3} =$ Rs 27.25

Difference = 27.25 - 14.75 = Rs 12.50

19. 5; Fare by B for 5 km = 14.50 + 3.25 = Rs 17.75

Fare by A for 8 km = $13.50 + \frac{17.25 - 13.50}{3} =$ Rs 14.75

Fare by C for 5 km = $19 + \frac{31-19}{3} =$ Rs 23

Total fare = 17.75 + 14.75 + 23 = 55.50

20. 2; Fare for 14th km by C $= \frac{56.50 - 41.50}{15-10} =$ Rs 3

Fare for 9th km by B = $\frac{33.25 - 24.25}{10 - 7}$ = Rs 3

BSRB Mumbai PO held on 25th July 1999

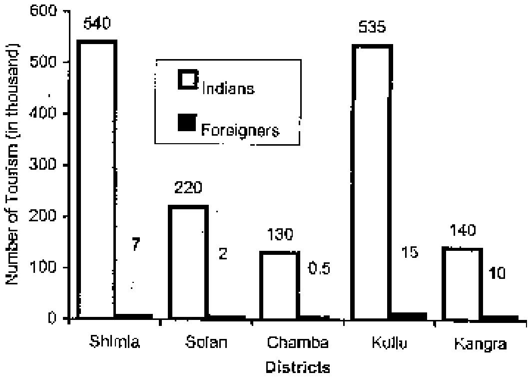
Directions (Q. 1-5): *Read the following table carefully and answer the questions given below it.*

Details of leading openers' performance in 20 one-day cricket matches

Openers	Total Runs	Highest Runs	No. of matches with runs		
			100 or more	50-99	0's
A	994	141	5	3	1
B	751	130	1	8	2
C	414	52	–	2	2
D	653	94	–	4	1
E	772	85	–	7	–

1. What is the difference between the **average runs** of top two openers in terms of **highest runs**, if matches having 0's were ignored?
   1) 4.7 2) 13.7 3) 11.1
   4) 16.62 5) None of these
2. If matches having zero runs and the one with highest runs is ignored, what will be the average runs for opener C?
   1) 21.29 2) 21.79 3) 20.7
   4) 21.17 5) 20.19
3. By how much the difference between the two highest total runs differs from the difference between the two lowest total runs?
   1) Lower by 18 2) More by 18 3) Lower by 4
   4) More by 4 5) None of these
4. Which of the given pairs of openers have ratio 3 : 2 in their highest runs?
   1) B and D 2) B and C 3) A and D
   4) D and C 5) None of these
5. Excluding the match with the highest runs and matches with 50-99 runs, what will be the **approximate** average runs for opener B?
   1) 25 2) 15 3) 10
   4) 30 5) Data inadequate

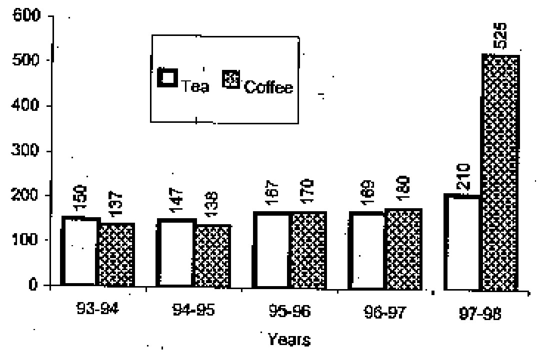
Directions (Q. 6-10): *Study the following graph carefully and answer the questions given below it.*



6. How many districts in Himachal Pradesh were visited by more than 10% of the total Indian tourists?
   1) 5 2) 3 3) 4
   4) 2 5) None of these
7. By what percentage the Indian tourists visiting Chamba were less than those visiting Shimla?
   1) 50 2) 55 3) 60
   4) 70 5) 75
8. **Approximately** what percentage was shared by total foreign tourists among all the tourists visiting Himachal Pradesh?
   1) 2 2) 8 3) 4 4) 5 5) 6
9. What was the ratio between the Indian tourists and the foreign tourists visiting Kullu?
   1) 105 : 3 2) 70 : 3 3) 107 : 3
   4) 35 : 1 5) None of these
10. Which of the following districts were visited by less than 10% of the total foreign tourists who visited Himachal Pradesh?
    1) Chamba, Kullu 2) Solan, Kangra
    3) Solan, Chamba, Shimla 4) Solan, Chamba
    5) None of these

Directions (Q. 11-15): *Study the following graph carefully and answer the questions given below it.*

Export of Tea and Coffee (in million kgs)



11. By what **approximate percentage** the export of coffee increased from 1996-97 to 1997-98?
1) 205 2) 185 3) 195
4) 200 5) 190

12. What is the ratio of the export of coffee in 1994-95 to that in 1996-97?
1) 69 : 85 2) 30 : 23 3) 85 : 69
4) 23 : 30 5) None of these

13. What was the percentage increase in the export of tea in 1997-98 from that in 1993-94?
1) 40 2) 90 3) 20
4) 35 5) None of these

14. By what per cent did the export of tea fall in 1994-95 from that in the previous duration?
1) 1 2) 2 3) 3
4) 4 5) None of these

15. What was the ratio between export of coffee to tea in 1997-98?
1) 5 : 14 2) 2 : 5 3) 5 : 2
4) 14 : 5 5) 14 : 35

Solutions:

1. 5; Avg. runs of A $= \frac{994}{19} = 52.31$

Avg. runs of B $= \frac{751}{18} = 41.72$

Difference $= 52.31 - 41.72 = 10.59$

2. 1; Avg. runs of C $= \frac{414-52}{17} = 21.29$

3. 5; Difference between two highest runs $= 994 - 772 = 222$

Difference between two lowest runs $= 653 - 414 = 239$

Difference $= 239 - 222 = 17$

4. 3; Ratio of A & D $= 141 : 94 = 3 : 2$

5. 5; Without knowing the individual runs of 8 openers, we can't find the average runs of remaining batsmen.

6. 2; Total Indian tourists $= 540 + 220 + 130 + 535 + 140$
$= 1565$ thousand

10% of Indian tourists $= \frac{1565 \times 10}{100} = 156.5$ thousand

7. 5; Required % $= \frac{540 - 130}{540} \times 100 \approx 75\%$

8. 1; Percentage share of foreign tourists visiting HP
$= \frac{34.5}{1600} \times 100 \approx 2\%$

9. 3; Ratio $= 535 : 15 = 107 : 3$

10. 4; 10% of foreign tourists $= \frac{10}{100} \times 34.5 = 3.45$

11. 5; Percentage increase in export of coffee in 1997-98
$= \frac{525 - 180}{180} \times 100 \approx 190\%$

12. 4; Ratio of export of coffee in '94-95 and '96-'97 $= 138 : 180 = 23 : 30$

13. 1; Percentage increase in export of tea in '97-98
$= \frac{210 - 150}{150} \times 100 = 40\%$

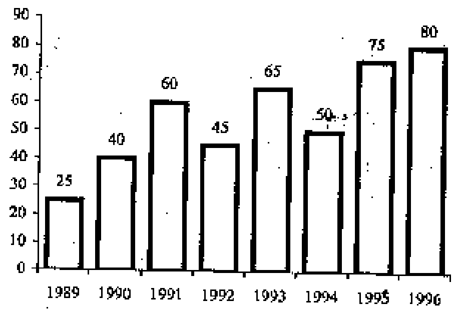
14. 2; Fall in export of tea in '94-95
$= \frac{150 - 147}{150} \times 100 = 2\%$

15. 3; Ratio of coffee to tea in '92 - '98 $= 525 : 210 = 5 : 2$

BSRB Calcutta PO held on 4th July 1999

Directions (Q. 1-5): *Study the following graph carefully a answer the questions given below it.*

Production of foodgrain by a state over the years (1000 tons)



1. The average production of 1990 and 1991 was exactly equal to th average production of which of the following pairs of years?
   1) 1991 and 1992 2) 1992 and 1994 3) 1993 and 1994
   4) 1994 and 1995 5) None of these
2. What was the difference in the production of foodgrains between 199 and 1994?
   1) 10,000 tons 2) 15,000 tons 3) 500 tons
   4) 5,000 tons 5) None of these
3. In which of the following years was the percentage increase i production from the previous year the maximum among the give years?
   1) 1991 2) 1993 3) 1995
   4) 1990 5) None of these
4. In how many of the given years was the production of foodgrain mo than the average production of the given years?
   1) 2 2) 3 3) 4 4) 1 5) None of these
5. What was the percentage drop in the production of foodgrain fro 1991 to 1992?
   1) 15 2) 20 3) 25 4) 30 5) None of these

Directions (Q. 6-13): *Read the following table carefully and answer the questions given below.*

Highest marks and average marks obtained by students in subjects over the years

The maximum marks in each subject is 100.

	Subjects									
	English		Hindi		Maths		Science		History	
	High	Avg	High	Avg	High	Avg	High	Avg	High	Avg
1992	85	62	75	52	98	65	88	72	72	46
1993	80	70	80	53	94	60	89	70	65	55
1994	82	65	77	54	85	62	95	64	66	58
1995	71	56	84	64	92	68	97	68	68	49
1996	75	52	82	66	91	64	92	75	70	58
1997	82	66	81	57	89	66	98	72	74	62

6. What was the grand average marks of the five subjects in 1996?
   1) 63 2) 64 3) 65
   4) 68 5) None of these
7. The difference in the average marks in History between 1994 and 1995 was exactly equal to the difference in the highest marks in Hindi between which of the following pairs of years?
   1) 1992 and 1995 2) 1993 and 1995 3) 1992 and 1996
   4) 1993 and 1997 5) None of these
8. What was the approximate percentage increase in average marks in History from 1992 and 1993?
   1) 20 2) 25 3) 24
   4) 16 5) 18
9. The average highest marks in English in 1992, 1993 and 1996 was exactly equal to the highest marks in Hindi in which of the following years?
   1) 1996 2) 1997 3) 1994
   4) 1996 5) 1993
10. The difference between the highest marks and the average marks in Hindi was maximum in which of the following years?
    1) 1994 2) 1997 3) 1995
    4) 1996 5) 1993

11. The highest marks in Hindi in 1993 was what per cent of the a marks in Mathematics in 1996?
 1) 135 2) 130 3) 125 4) 140 5) None of the
12. If there were 50 students in 1993, what was the total marks obtain them in Mathematics?
 1) 2400 2) 3000 3) 2500
 4) 3200 5) None of these
13. The difference between the highest marks in science was maxi between which of the following pairs of years among the given ye
 1) 1992 and 1993 2) 1992 and 1996
 3) 1996 and 1997 4) 1992 and 1995
 5) None of these

Solutions:

1. 5
2. 1; Required difference = 60 - 50 = 10,000 tonnes.
3. 4; Percentage increase in production = $\frac{15}{25} \times 100 = 60\%$
4. 3; Average production

$$= \frac{25 + 40 + 60 + 45 + 65 + 50 + 75 + 80}{8}$$

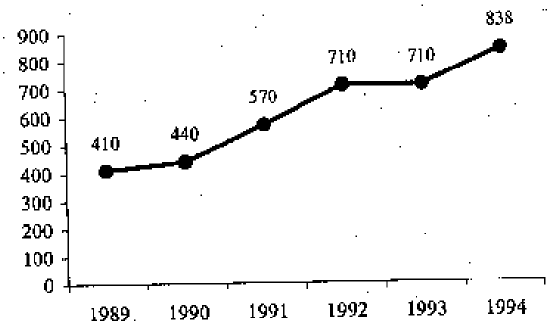
$$= \frac{440}{8} = 55$$

5. 3; Required percentage drop = $\frac{60 - 45}{60} \times 100 = 25\%$
6. 1; Average = $\frac{52 + 66 + 64 + 75 + 58}{5} = \frac{315}{5} = 63.$
7. 1; The difference is 9.
8. 1; Percentage increase = $\frac{55 - 46}{46} \times 100 \approx 20\%$
9. 5; Average highest marks = $\frac{85 + 80 + 75}{3} = \frac{240}{3} = 80.$
10. 5
11. 3; Required percentage = $\frac{80}{64} \times 100 = 125\%$
12. 2; Marks obtained by students = 50 × 60 = 3000
13. 5; The maximum difference is in the years 1992 & 1997. Since the least value is in 1992 and the highest value is in 1997.

BSRB Delhi PO held on 1st August, 1999

Directions (Q. 1-5): *Study the given chart carefully and then answer the questions accordingly?*

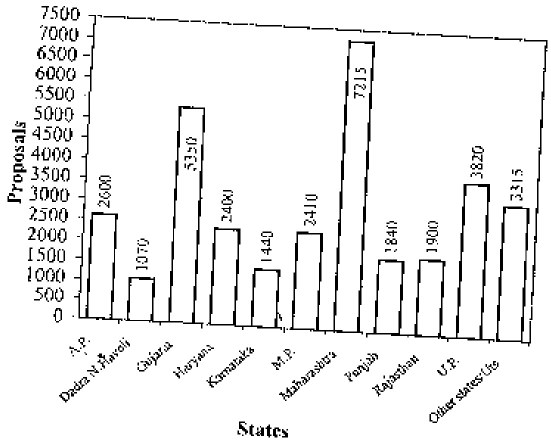
Number of hotels in a state



1. The approximate percentage increase in hotels from year 1989 to 1994 was
 1) 75 2) 100 3) 125 4) 150 5) 175
2. If the number of newly made hotels in 1991 was less by 10 then what is the ratio of the number of hotels in 1991 and that in 1990?
 1) 14 : 11 2) 3 : 4 3) 4 : 5 4) 5 : 4 5) 1 : 4
3. If the percentage increase in the number of hotels from 1993 to 1994 continued up to 1995 then what is the number of hotels built in 1995?
 1) minimum 75 2) minimum 70 3) minimum 50
 4) minimum 139 5) minimum 80
4. In which of the given years increase in hotels in comparison to the previous year is the maximum?
 1) 1990 2) 1991 3) 1992 4) 1993 4) 1994
5. If increase in hotels from 1991 to 1992 is P% and increase in hotels from 1992 to 1994 is Q%, then which of the following relations between P and Q is true?
 1) Data is inadequate 2) P < Q 3) P = Q
 4) P > Q 5) None of these

Directions (Q. 6-10): *Study the following chart carefully and answer accordingly.*

No. of industrial investment proposals in a state/UT during 1991-1998



6. How many more investment proposals will be in Gujarat so that the ratio of investment proposals in Gujarat and those in Maharashtra becomes 5 : 4?
   1) 1024 2) 1862 3) 1042 4) 422 5) None of these
7. What is the combined percentage of investment proposals of Gujarat and Maharashtra in the given states/UTs between the years 1991-1998?
   1) More than 25% but less than 30%.
   2) More than 35% but less than 40%
   3) More than 30% but less than 35%
   4) Less than 25%
   5) None of these
8. **Approximately** by what per cent investment proposals received from Karnataka are more in comparison to those received from D and N Haveli?
   1) 25 2) 30 3) 40 4) 45 5) 35
9. The ratio of number of proposals from Punjab to those from Karnataka is
   1) 23 : 30 2) 30 : 23 3) 23 : 18
   4) 18 : 23 5) None of these
10. How many states other than Other States/UTs have contributed to less than 10% of the total investment proposals received during 1991-1998?
    1) 7 2) 8 3) 6 4) 5 5) None of these

Directions (Q. 11-15): *Study the following table carefully and answer accordingly.*

Year	Deaths	Type of accident		
		Seriously wounded	Wounded	Simply wounded
1994	6,000	6,600	20,300	31,700
1995	7,000	7,700	22,100	36,000
1996	8,600	8,500	23,400	38,000
1997	7,900	8,600	32,000	37,000
1998	6,500	7,300	18,200	29,800

11. In which of the following pairs of years, no. of deaths is less than 10% of all the accidents occured in those years?
    1) 1994 and 1997 2) 1994 and 1996 3) 1995 and 1997
    4) 1997 and 1998 5) None of these
12. In which of the following pair of years the ratio of simply wounded people was 18 : 19?
    1) 1998 and 1994 2) 1995 and 1997 3) 1996 and 1997
    4) 1997 and 1998 5) None of these
13. What is percentage decrease in deaths from 1997 to 1998?
    1) more than 10 2) more than 20 3) more than 15
    4) more than 5 5) None of these
14. In which of the following years total number of accidents is more than that in other years?
    1) 1994 2) 1995 3) 1996
    4) 1997 5) 1998
15. What ws the ratio of wounded people between 1994 and 1998?
    1) 13 : 26 2) 29 : 26 3) 17 : 14
    4) 13 : 14 5) None of these

Solutions:

1. 2; Reqd % increase $= \frac{838 - 410}{410} \times 100 \approx 100\%$

Note: You should not calculate the exact values. Since the no. of hotels approximately doubles, the % increase is approx 100.

2. 1; Required ratio = $\frac{(570-10)}{440} = 14:11$

3. 4; Percentage increase in 1994 = $\frac{838-710}{710} \times 100 \approx 18\%$

$\therefore$ Hotels in 1995 $\approx$ 150 which is not in option, but the nearest option is minimum 139.

4. 2; From the graph we may conclude that our answer is 1990. You may confirm from the following table.

1990	1991	1992	1993	1994
7.31%	29.54%	24.56%	No change	18.05%

5. 4; P% = 24.56% and Q% = 18.02% $\therefore$ P > Q

6. 5; Let x be the additional investment proposals in Gujarat

i.e. $\frac{5350+x}{7215} = \frac{5}{4}$ $\therefore$ x = 3668.75

7. 2; Total no. of proposals = 2600 + 1070 + 5350 + 2400 + 1440 + 2410 + 7215 + 1840 + 1900 + 3820 + 3315 = 33360

$\therefore$ Required percentage = $\frac{12565}{33360} \times 100 = 37.66\%$

8. 5; Required percentage = $\frac{1440-1070}{1070} \times 100 \approx 35\%$

9. 3; Required ratio = $\frac{1840}{1440} = 23:18$

10. 1; Total no. of investment proposals = 33360

10% = 3336. 7 states are less than 3336.

11. 5;

Year	1994	1995	1996	1997	1998
Total no. of accidents	58,600	65,800	69,900	77,600	55,300
Percentage	10.23%	1063%	12.30%	10.18%	11.75%

12. 5; The ratio will be obtained in 1995 and 1996, which is not in option.

13. 3; Required percentage = $\frac{7900-6500}{7900} \times 100 = 17.72\%$

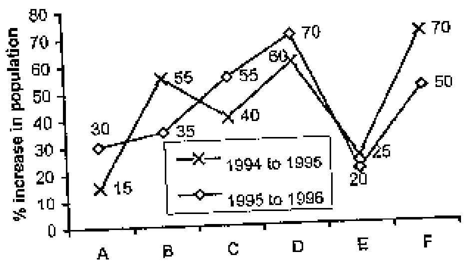
14. 4; Taking the help from the table in question no. 16.

15. 2; Required ratio = $\frac{203}{182} = \frac{29}{26} = 29:26$

BSRB Hyderabad PO held on 29th August, 1999

Directions (Q. 1-5): *Study the following graph carefully and answer the questions given below.*

Percentage growth in population of six states A, B, C, D, E and F from 1994 to 1995 and 1995 to 1996



1. Population of state 'F' in 1995 was approximately what per cent of its population in 1996?
   1) 60 2) 67 3) 75 4) 55 5) Data inadequate
2. If the population of state 'B' in the year 1994 was 5 lakh, what was approximately its population in the year 1996?
   1) 9.50 lakh 2) 8 lakh 3) 10.50 lakh
   4) 14.50 lakh 5) 11 lakh
3. If the population of states C and D in 1995 are in the ratio of 2 : 3 respectively and the population of state 'C' in 1994 was 2.5 lakh, what was the population of state 'D' in 1995?
   1) 5.25 lakh 2) 4.75 lakh 3) 3.50 lakh 4) 6 lakh 5) None of these
4. In 1994 the population of states B and D are equal and the population of state B in 1996 is 4 lakh; what approximate was the population of state 'D' in 1996?
   1) 3 lakh 2) 3.50 lakh 3) 6 lakh 4) 5 lakh 5) 4.50 lakh
5. Population of state 'E' in 1994 was what fraction of its population in 1996?
   1) $\frac{4}{5}$ 2) $\frac{3}{2}$ 3) $\frac{5}{8}$ 4) $\frac{3}{4}$ 5) $\frac{2}{3}$

Direction (Q. 6-10): *Read the following information carefully and answer the questions based on it:*

In 6 educational years, number of students taking admission and leaving from the 5 different schools which are founded in 1990 are given below

School	A		B		C		D		E	
	Ad	L	Ad	L	Ad	L	Ad	L	Ad	L
1990	1025	—	950	—	1100	—	1500	—	1450	—
1991	230	120	350	150	320	130	340	150	250	125
1992	190	110	225	115	300	150	300	160	280	130
1993	245	100	185	110	260	125	295	120	310	120
1994	280	150	200	90	240	140	320	125	340	110
1995	250	130	240	120	310	180	360	140	325	115

In the above table shown Ad = Admitted, L = Left

6. What is the average number of students studying in all the five schools in 1992?
   1) 1494   2) 1294   3) 1590
   4) 1640   5) None of these
7. What was the number of students studying in school B in 1994?
   1) 2030   2) 1060
   3) 1445   4) 1150
   5) None of these
8. Number of students leaving school C from the year 1990 to 1995 is approximately what percentage of number of students taking admission in the same school and in the same year?
   1) 50%   2) 25%   3) 48%
   4) 36%   5) 29%
9. What is the difference in the number of students taking admission between the years 1991 and 1995 in school D and B?
   1) 514   2) 1065   3) 965
   4) 415   5) None of these
10. In which of the following schools, percentage increase in the number of students from the year 1990 to 1995 is maximum?
   1) A   2) B   3) C
   4) D   5) E

Directions (Q. 11-15): *Study the following table and answer the following questions carefully.*

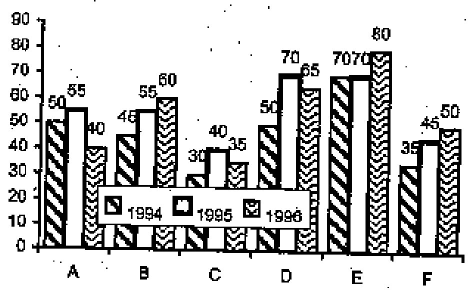
Following table shows the percentage population of six states below poverty line and the proportion of male and female

State	Percentage population below poverty line	Proportion of male and female	
		Below poverty line	Above poverty line
		M : F	M : F
A	12	3 : 2	4 : 3
B	15	5 : 7	3 : 4
C	25	4 : 5	2 : 3
D	26	1 : 2	5 : 6
E	10	6 : 5	3 : 2
F	32	2 : 3	4 : 5

11. The total population of state A is 3000, then what is the approximate no. of females above poverty line in state A?
   1) 1200   2) 2112   3) 1800
   4) 1950   5) 2025
12. If the total population of C and D together is 18000, then what is the total no. of females below poverty line in the above stated states?
   1) 5000   2) 5500   3) 4800
   4) Data inadequate   5) None of these
13. If the population of males below poverty line in state A is 3000 and that in state E is 6000, then what is the ratio of the total population of state A and E?
   1) 3 : 4   2) 4 : 5   3) 1 : 2
   4) 2 : 3   5) None of these
14. If the population of males below poverty line in state B is 500 then what is the total population of that state?
   1) 14400   2) 6000   3) 8000
   4) 7600   5) None of these
15. If in state E population of females above poverty line is 19800 then what is the population of males below poverty line in that state?
   1) 5500   2) 3000   3) 2970
   4) Data inadequate   5) None of these

Directions (Q. 16-20): *Study the following graph carefully answer the questions that follow.*

Production of steel by different companies in three consecutiv years (in lakh tonnes)



16. What is the difference between average production of the six companie in 1995 and average production of the same companies in 1994?
 1) 7,05,000 tonnes 2) 7,50,000 tonnes 3) 75,000 tonnes
 4) 75,00,000 tonnes 5) None of these
17. What is the percentage decline in production by company C from 1995 to 1996?
 1) 5% 2) 20% 3) 15% 4) 12.50% 5) None of these
18. Which of the following companies recorded the minimum percentage growth from 1994 to 1995?
 1) A 2) B 3) C 4) D 5) F
19. Production of company 'C' in 1995 and production of company 'F' in 1994 together is what per cent of production of B in 1996?
 1) 80% 2) 75% 3) 133% 4) 120% 5) None of these
20. In which of the following pairs of companies the difference between average production for the three years is maximum?
 1) E and F 2) D and C 3) E and C
 4) A and E 5) None of these

Solutions:

1. 2; Let the population of state 'F' in 1995 = 100
 Then population of state 'F' in 1996 = 150 (Since growth is 50%)

$\therefore$ required % $= \frac{100}{150} \times 100 \approx 67\%$

2. 3; Population of state 'B' in the year 1996
 $= 5 \times \frac{155}{100} \times \frac{135}{100} \approx 10.50$ lakh

3. 1; $\frac{C_{95}}{D_{95}} = \frac{2}{3}$ and $C_{94} = 2.50$ lakh
 $\therefore C_{95} = 2.5 \times \frac{140}{100} = 3.50$ lakh
 $\therefore$ The population of D in 1995 $= \frac{3 \times 3.50}{2} = 5.25$ lakh

4. 4; Population of state 'B' in 1994
 $= \frac{4 \times 100 \times 100}{155 \times 135}$ = Population of D in 1994
 $\therefore$ Population of state 'D' in 1996
 $= \frac{4 \times 100 \times 100 \times 160 \times 170}{100 \times 100 \times 155 \times 135} \approx 5$ lakh

Note: We have followed the rule of fraction.

5. 5; Suppose population of state E in 1994 = 100
 Then population of state E in 1995 = 125 (since 25% growth)
 and population of state E in 1996 $= 125\left(\frac{120}{100}\right) = 150$
 (since 20% growth)
 $\therefore$ Required ratio $= \frac{100}{150} = \frac{2}{3}$

6. 1; Total no. of students studying in all schools in 1992
 $= (1025 + 230 + 190 + 950 + 350 + 225 + 1100 + 320 + 300 + 1500 + 340 + 300 + 1450 + 250 + 280) - (120 + 110 + 150 + 115 + 130 + 150 + 150 + 160 + 125 + 130)$
 $= 8810 - 1340 = 7470$ $\therefore$ Average $= \frac{7470}{5} = 1494$

7. 3; Number of students studying in school B in 1994
 $= 950 + (350 - 150) + (225 - 115) + (185 - 110) + (200 - 90)$
 $= 950 + 200 + 110 + 75 + 110 = 1445$

8. 5; Number of students leaving school 'C'
 from 1990 to 1995 $= 130 + 150 + 125 + 140 + 180 = 725$
 Number of students admitted during the period
 $= 1100 + 320 + 300 + 260 + 240 + 310 = 2530$

$\therefore$ Required percentage $= \frac{725}{2530} \times 100 \approx 29\%$

9. 4; Required difference $= (340 + 300 + 295 + 320 + 360) - (350 + 22$
$+ 185 + 200 + 240) = 1615 - 1200 = 415$

10. 2; Increase in no. of students in school A
$= (230 - 120) + (190 - 110) + (245 - 100) + (280 - 150)$
$+ (250 - 130) = 585$

$\therefore$ % increase from 1990 (1025) to 1995 $= \frac{585}{1025} \times 100 = 57.07\%$

Similarly, we can calculate for other schools.

Percentage increases in all schools are given in the following table:

A	B	C	D	E
57.07%	64.73%	64.09%	61.33%	62.40%

11. 1; No. of females above poverty line in state A
$= 3000 \times (100 - 12)\% \times \frac{3}{7} \approx 1200$

12. 4; Since we cannot find the population of states C and D separately, we can't find the required value.

13. 5; Population of state A below poverty line $= 3000 \times \frac{5}{3} = 5000$

$\therefore$ Total population of state A $= \frac{5000}{12} \times 100$

and the population of state E below poverty line
$= 6000 \times \frac{11}{6} = 11000$

$\therefore$ Total population of state E $= \frac{11000}{10} \times 100$

$\therefore$ Required ratio $= \frac{5}{12} \times \frac{10}{11} = \frac{25}{66}$

14. 3; Total population of state B $= 500 \left(\frac{12}{5}\right)\left(\frac{100}{15}\right) = 8000$

15. 2; Population of state E $= 19800 \left(\frac{5}{2}\right)\left(\frac{100}{100 - 10}\right) = 55000$

$\therefore$ Population of males below poverty line
$= 55000 \left(\frac{10}{100}\right)\left(\frac{6}{11}\right) = 3000$

16. 5; Required difference $= \frac{335 - 280}{6} = 916666$ tonnes

17. 4; Percentage decline $= \frac{40 - 35}{40} \times 100 = 12.50\%$

18. 1; It is clear from the graph. But see the table which shows the percentage growth of six companies from 1994 to 1995.

A	B	C	D	F
10%	22.22%	33.33%	40%	28.57%

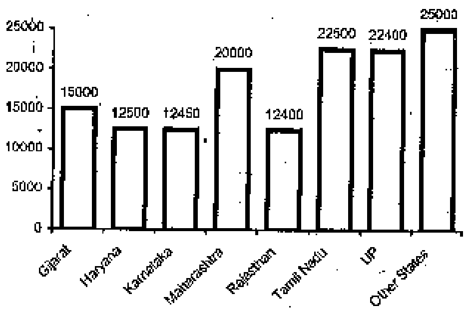
19. 5; Required percentage $= \frac{40 + 35}{60} \times 100 = 125\%$

20. 3; It is clear that the highest average production is in E and lowest average production is in C. So, the maximum difference would be in E & C.

NABARD held on 18th July, 1999

Directions (Q. 1-5): *Study the following graph carefully and answer the questions given below it.*

Statewise Production of Roses



1. Which of the following state(s) contribute(s) less than 10 per cent in the total rose production?
   1) Only Rajasthan
   2) Rajasthan, Karnataka
   3) Rajasthan, Karnataka, Haryana

4) Rajasthan, Karnataka, Haryana and Gujarat
5) None of these

2. By what percentage rose production of other states is more than that of the Maharashtra?
1) 25 2) 30 3) 20 4) 15 5) None of these

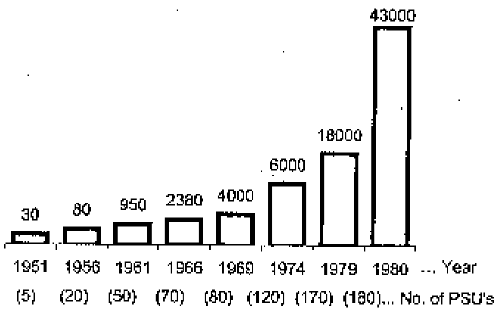
3. What is the approximate average production of roses (in thousands) across all the states?
1) 21 2) 20 3) 19 4) 18 5) 17

4. Approximately what percentage of the total rose production is shared by the other states?
1) 10 2) 20 3) 30 4) 40 5) 35

5. If total percentage contribution of the states having production of roses below twenty thousand is considered, which of the following statements is true?
1) It is little above 40% 2) It is exactly 35%
3) It is below 35% 4) It is little below 30%
5) None of these

Directions (Q. 6-10): Study the following graph carefully and answer the questions given below it:

Growth in PSU investment (Amount in Rs crores)



6. By what percentage the PSU investment in 1974 was more than that in the year 1969?
1) 200 2) 100 3) 150 4) 50 5) None of these

7. In which year increase in the average PSU investment was the highest as compared to the earlier year?
1) 1979 2) 1969 3) 1961
4) 1966 5) None of these

8. What is the ratio between the PSU investment made in 1966 to that made in 1979?
1) 119 : 300 2) 1 : 4 3) 19 : 320
4) 900 : 119 5) None of these

9. Which year onwards the average PSU investment became Rs 50 crore or more?
1) 1961 2) 1966 3) 1969
4) 1974 5) None of these

10. For how many of the given years the average PSU investment was less than Rs 10 crores?
1) 1 2) 2 3) 3
4) 4 5) None of these

Directions (Q. 11-15): *Read the following table carefully and answer the questions given below it:*

Export of agricultural food products (in Rs crores)

	Years		
	1993-94	1994-95	1995-96
Groundnut	17,000	10,000	23,000
Basmati rice	1,06,000	87,000	85,000
Non-basmati rice	23,000	34,000	3,72,000
Wheat	18,00	4,200	37,00
Other cereals	3,400	2,800	1,700

11. During the year 1994-95, which two products together constituted around 70 per cent of the total export in that year?
1) Basmati rice, Wheat 2) Basmati rice, Other cereals
3) Basmati rice, None-basmati rice 4) Groundnut, Basmati rice
5) None of these

12. During the period 1993-94 what was the approximate average export of the given products in crores?
1) 30,000 2) 29,000 3) 27,000
4) 28,000 5) 25,000

13. In case of which of the following products the percentage export against the total export of the year has shown continuous increase over the three-year period?
1) Wheat, Other cereals

2) Wheat, Non-basmati rice
3) Non-basmati rice, Other cereals
4) Non-basmati rice, Groundnut
5) None of these

14. In case of which of the following food products the exports in th earlier year and the consecutive next year are exactly in the rat 14 : 17?
1) Other cereals 2) Wheat 3) Non-Basmati rice
4) Basmati rice 5) None of these

15. Export of which of the following food products in 1995-96 was mo than 200% of the export in 1993-94 and 1994-95 together?
1) Groundnut 2) Non-Basmati rice
3) Basmati rice 4) Other cereals
5) None of these

Solutions

1. 3; Total rose production = (15 + 12.5 + 12.45 + 20 + 12.4 + 22.5 + 22.4 + 25) × 1000 = 142250

Percentage production of rose in the states (the lowest four states)

Rajasthan	Karnataka	Haryana	Gujarat
8.71	8.75	8.78	10.54

2. 1; Required percentage $= \frac{25-20}{20} \times 100 = 25\%$ (more)

3. 4; Total production of rose by all the states = 142250

$\therefore$ Average $= \frac{142250}{8 \times 1000} \approx 18$ thousand

4. 2; Required percentage $= \frac{25}{142.25} \times 100 \approx 20\%$

5. 5; It is 36.8% approximately.

6. 4; Required percentage $= \frac{6000-4000}{4000} \times 100$

$= \frac{2000}{4000} \times 100 = 50\%$

7. 5; Average PSUs investment in crores

1951	1956	1961	1966	1969	1974	1979	1980
6	4	19	34	50	50	105.88	238.88

Increase in the average PSU investment (crore) in comparison to earlier year

1961	1966	1969	1979	1980
15	15	16	55.88	133

Note: From the graph, we see that difference in PSU investment in 1979 and that in 1980 is very large but difference of no. of PSUs is only 10. This hints us to go for the answer as 1980.

8. 5; Required ratio $= \frac{2380}{18000} = 119 : 900$

9. 3; See the table given in answer 7.

10. 2; See the table given in answer 7.

11. 4; Total exports in the year 1994-95
= 10,000 + 87,000 + 34,000 + 4,200 + 2,800 = 138,000

70% of the total product $= 138{,}000 \times \frac{70}{100} = 96{,}600$. Which is almost equal to the sum of Groundnut and Basmati rice.

Note: You can answer the question by intelligent guessing. Basmati rice is must in the combination. Basmati rice with non-basmati rice will constitute about more than 80%. Basmati rice with wheat or other cereals will constitute about less than 70%. The only possible and suitable answer is Basmati rice with groundnut.

12. 1; Average export in the year 1993-94

$= \frac{17{,}000 + 1{,}06{,}000 + 23{,}000 + 1{,}800 + 3{,}400}{5}$

$= \frac{151200}{5} \approx 30{,}000$ cr.

13. 5; Observe the table carefully. In 1995-96, the export of non-basmati rice is so high that the percentage share of other products in that year is reduced drastically. So, no product other than non-basmati rice can show a continuous increase in % share over the period.

14. 5; Don't confuse with the export of other cereals. The ratio is 3400 : 2800 = 17 : 14, which is reverse of the ratio given in the question.

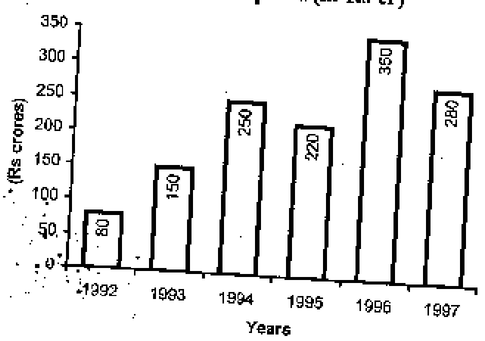
15. 2; From the table it is clear that only non-basmati rice shows such huge increase in export in 1995-96.

BSRB Chennai PO held on 30th January, 2000

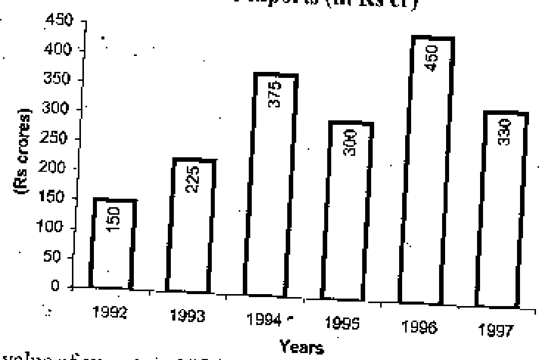
Direction (Q. 1-5): *Study the following graphs carefully and answer the questions given below:*

Value of imports and exports by a company over the years.

Value of imports (in Rs cr)



Value of exports (in Rs cr)



1. The value of exports in 1996 was what percentage of the average value of imports in the years 1994, 1995 and 1997?
   1) 200 2) 100 3) 300 4) 150 5) None of these
2. The value of exports in 1994 was exactly what percentage of the value of imports in the same years?
   1) 125 2) 160 3) 200 4) 75 5) None of these
3. What was the approximate difference between the value of average exports and the value of average imports of the given years?
   1) Rs 85 cr 2) Rs 100 cr 3) Rs 75 cr
   4) Rs 90 cr 5) Rs 80 cr
4. In which of the following years was the difference between the value of exports and the value of imports exactly Rs 100 cr?
   1) 1993 2) 1996 3) 1995 4) 1997 5) None of these
5. What was the percentage increase in the value of exports from 1995 to 1996?
   1) 150 2) 100 3) 75 4) 50 5) None of these

Directions (Q. 6-10): *Study the following table carefully and answer the questions given below:*

Production of main crops in India (in million tonnes)

Crops	91-92	92-93	93-94	94-95	95-96	96-97
Pulses	20.5	22.4	24.6	23.5	27.8	28.2
Oilseeds	32.4	34.6	40.8	42.4	46.8	52.4
Rice	80.5	86.4	88.2	92.6	94.2	90.8
Sugercane	140.8	150.2	152.2	160.3	156.4	172.5
Wheat	130.2	138.4	146.8	141.6	152.2	158.4
Coarse grain	45.6	52.8	60.4	62.4	58.2	62.8
Sum	**450**	**484.8**	**513.2**	**522.8**	**535.6**	**565.1**

6. Production of sugarcane in 1993-94 was approximately what percentage of the production of rice in 1992-93?
   1) 50 2) 75 3) 150 4) 125 5) 175
7. Production of what type of crop was going to increase in each year in the given years?
   1) Rice 2) Pulse 3) Sugarcane
   4) Oilseeds 5) None of these
8. What was the average production of pulse in the given years?
   1) 26.8 million tonnes 2) 20.5 million tonnes
   3) 24.5 million tonnes 4) 22.5 million tonnes
   5) None of these
9. Production of oilseeds was what percentage of the total crops produced in the year 1991-92?
   1) 7.2 2) 8.4 3) 2.7 4) 6.4 5) None of these

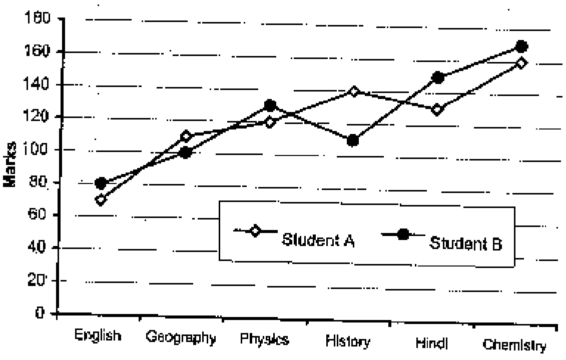
10. In which of the following years the total production of oilseeds in the years 1994-95, 1995-96 and 1996-97 was equal to the production of wheat?
    1) 1993-94 2) 1994-95 3) 1996-97
    4) 1992-93 5) None of these

Directions (Q. 11-15): *Study the following graphs carefully and answer the questions given below it.*

Marks obtained by two students in six subjects in an examination

Maximum marks in Physics, Chemistry and English are 200.

Maximum marks in History, Geography and Hindi are 150.



11. Marks obtained by student B in Hindi was what percentage of the marks obtained by student B in physics?
    1) 25 2) 150 3) 125 4) 105 5) None of these
12. **Approximately** what was the average percentage of marks obtained by A in all the subjects?
    1) 75 2) 60 3) 80 4) 85 5) 70
13. **Approximately** what was the average marks obtained by B in Geography, History and Hindi?
    1) 120 2) 80 3) 140 4) 110 5) 125
14. In how many subjects did student B obtain more than 70 percentage of marks?
    1) 1 2) 2 3) 3 4) 4 5) None of these

15. What was the difference in percentage of marks between A and B in History?
    1) 30 2) 25 3) 40 4) 20 5) None of these

Solutions:

1. 5; Average value of imports in the yrs 1994, 1995 and 1997

$$= \frac{250 + 220 + 280}{3} = \text{Rs } 250 \text{ cr}$$

$$\therefore \text{ Required percentage} = \frac{450}{250} \times 100 = 180\%$$

2. 5; Required percentage $= \frac{375}{250} \times 100 = 150\%$

3. 1; Average import $= \frac{80 + 150 + 250 + 220 + 350 + 280}{6}$

$$= \frac{1330}{6} \approx 222 \text{ cr}$$

$$\text{Average export} = \frac{150 + 225 + 375 + 300 + 450 + 330}{6} = 305 \text{ cr}$$

$\therefore$ Required difference = 83 cr $\approx$ 85 cr

4. 2

5. 4; Required percentage increase $= \frac{450 - 300}{300} \times 100$

$$= \frac{150}{300} \times 100 = 50\%$$

6. 5; Required per cent $= \frac{152.2}{86.4} \times 100 \approx 175\%$

7. 4

8. 3; Average production of pulse

$$= \frac{20.5 + 22.4 + 24.6 + 23.5 + 27.8 + 28.2}{6} = \frac{147.0}{6}$$

= 24.5 million tonnes

9. 1; Required percentage $= \frac{32.4}{450} \times 100 = 7.2\%$

10. 2; Total production of oilseeds in the given yrs
    = 42.4 + 46.8 + 52.4 = 141.6.
    Which is equal to the production of wheat in 1994-95.

11. 5; Required percentage $= \frac{150}{130} \times 100 = 115.38\%$

12. 5; % marks in Eng $= \frac{70}{200} \times 100 = 35$

% marks in Geo $= \frac{110}{150} \times 100 \approx 73$

% marks in Phy $= \frac{120}{200} \times 100 = 60$

% marks in His $= \frac{140}{150} \times 100 = 93$

% marks in Hin $= \frac{120}{150} \times 100 = 80$

% marks in Che $= \frac{160}{200} \times 100 = 80$

$\therefore$ Average % marks $= \frac{421}{6} \approx 70$

13. 1; Marks obtained by B in Geography, History and Hindi
$= 100 + 110 + 150 = 360$

$\therefore$ average $= \frac{360}{3} = 120$

14. 3; Percentage of marks obtained by student B

English	Geography	Physics	History	Hindi	Chemistry
40	66.6	65	73.33	100	85

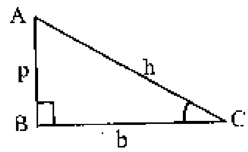
15. 4; Required difference $= \frac{140 - 110}{150} \times 100 = \frac{30}{150} \times 100 = 20\%$

Trigonometry

Pythogoras Theorem

In a right-angled Δ the square of the hypotenuse is sum of the squares of the base and the perpendicular.

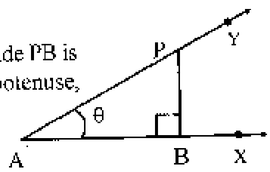
$$h^2 = p^2 + b^2$$



Trigonometric Ratios

The ratios of the sides of a right-angled triangle with respect to its angles are called **trigonometric ratios**. Given any acute angle θ (say angle YAX) in the Fig, we can take a point P on AY and drop perpendicular PB on AX. Then we have a right-angled Δ PAB in which angle PAB = θ. Then the ratio $\frac{PB}{AP}$ is called the **sine of angle** θ and, in short form, it is written as *sin* θ.

Thus $\sin\theta = \frac{PB}{AP}$



Since, in the right-angled Δ PBA, the side PB is opposite to angle θ and AP is the hypotenuse, we actually have:

$$\sin\theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

But you might well ask what happens if we choose P somewhere else on AY? If we take a different position for P, then the lengths PB and AP will change but the ratio $\frac{PB}{AP}$ will remain the same as before, and this can be proved by using similar triangles. We take this result for granted.

Going back to the right-angled Δ PBA in which angle PAB = θ, we define two more trigonometric ratios of θ as follows:

cos θ $= \frac{AB}{AP}$, and **tan** θ $= \frac{PB}{AB}$

$\cos\theta = \dfrac{\text{adjacent side}}{\text{hypotenuse}}$ and $\tan\theta = \dfrac{\text{opposite side}}{\text{adjacent side}}$

There are three other trigonometric ratios, namely, **cosecant, secant** and **cotangent** of an angle θ, which we define as follows: For any acute angle θ,

$\text{cosec}\,\theta = \dfrac{1}{\sin\theta}$, $\sec\theta = \dfrac{1}{\cos\theta}$ and $\cot\theta = \dfrac{1}{\tan\theta}$.

It is obvious that out of the six trigonometric ratios of an angle, if any one is known all the others can be calculated.

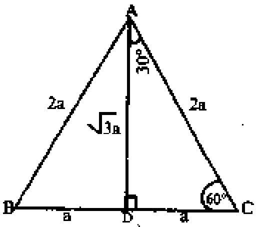
Trigonometric Ratios of Certain Angles

We have defined sin θ, cos θ, etc, for any acute angle θ but we have not yet found the values of sin θ, cos θ etc, for even one specific angle θ. We can use our knowledge of geometry to find the values of the trigonometric ratios of some angles. For other angles, we have to make use of ready-made tables.

Trigonometric Ratio of 30°

We may recall that each angle of an equilateral triangle is of 60°. Thus, the bisector of an angle of such a triangle makes with either side an angle of 30°.

Suppose Δ ABC is equilateral with each side of length 2a (and, of course, each angle 60°), and let AD be perpendicular to BC. Then, as the triangle is equilateral, AD is also the bisector of angle A, and D is the mid-point of BC. Now, BC = 2a.



So, DC = a

and angle CAD = 30°.

In Δ ADC, angle D is a right angle, hypotenuse AC = 2a and DC = a

So, by Pythagoras theorem,

$AD^2 = AC^2 - DC^2 = (2a)^2 - a^2 = 3a^2$

Hence, $AD = \sqrt{3}\,a$

Now in the right-angled Δ ADC, angle DAC = 30°

$\therefore \sin 30° = \dfrac{\text{opposite side}}{\text{hypotenuse}} = \dfrac{DC}{AC} = \dfrac{a}{2a} = \dfrac{1}{2}$;

$\cos 30° = \dfrac{\text{adjacent side}}{\text{hypotenuse}} = \dfrac{AD}{AC} = \dfrac{\sqrt{3}\,a}{2a} = \dfrac{\sqrt{3}}{2}$

$\tan 30° = \dfrac{\text{opposite side}}{\text{adjacent side}} = \dfrac{CD}{AD} = \dfrac{a}{\sqrt{3}\,a} = \dfrac{1}{\sqrt{3}}$;

$\text{cosec}\,30° = \dfrac{1}{\sin 30°} = 2$ $\quad \sec 30° = \dfrac{1}{\cos 30°} = \dfrac{2}{\sqrt{3}}$

$\cot 30° = \dfrac{1}{\tan 30°} = \sqrt{3}$

Note: If in a right-angled triangle, one angle is 30°, then the side opposite to it is half of the hypotenuse.

Trigonometric Ratio of 60°

Referring again to figure in the Δ ADC,

angle DAC = 30° and angle ADC = 90°;

∴ Angle ACD = 60°

∴ In the right-angled Δ ADC, angle ACD = 60°

$\therefore \sin 60° = \dfrac{\text{opposite side}}{\text{hypotenuse}} = \dfrac{AD}{AC} = \dfrac{\sqrt{3}\,a}{a} = \dfrac{\sqrt{3}}{2}$;

$\therefore \cos 60° = \dfrac{\text{adjacent side}}{\text{hypotenuse}} = \dfrac{DC}{AC} = \dfrac{a}{2a} = \dfrac{1}{2}$

$\therefore \tan 60° = \dfrac{\text{opposite side}}{\text{adjacent side}} = \dfrac{AD}{DC} = \dfrac{\sqrt{3}\,a}{a} = \sqrt{3}$;

$\therefore \text{cosec}\,60° = \dfrac{1}{\sin 60°} = \dfrac{2}{\sqrt{3}}$

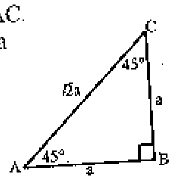
$\therefore \sec 60° = \dfrac{1}{\cos 60°} = 2$

$\therefore \cot 60° = \dfrac{1}{\tan 60°} = \dfrac{1}{\sqrt{3}}$

Trigonometric Ratio of 45°

If in a right-angled Δ ABC with right angle at C, we have angle A = 45°, then obviously, angle B = 45°. So, angle A = angle B. Consequently BC = AC.

Suppose BC = AC = a



Then by Pythagoras theorem, $AB^2 = BC^2 + AC^2 = a^2 + a^2 = 2a^2$ and so, $AB = \sqrt{2}\ a$

Remembering that in Δ ABC, angle A = 45°, we get

$\sin 45° = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{BC}{AB} = \frac{a}{\sqrt{2}\ a} = \frac{1}{\sqrt{2}}$;

$\cos 45° = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{AC}{AB} = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}}$

$\tan 45° = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{BC}{AC} = \frac{a}{a} = 1$

Therefore, cosec 45° = $\sqrt{2}$, sec 45° = $\sqrt{2}$, cot 45° = 1

Trigonometric Ratio of different angles

θ	sin	cos	tan	cosec	sec	cot
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$	$\sqrt{3}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2	$\frac{1}{\sqrt{3}}$

Height and Distance

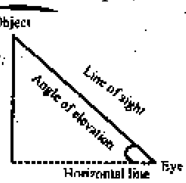
We are now ready to solve this problem in the case of right-angled triangles.

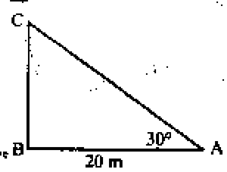
Angles of Elevation and Depression

Suppose we wish to determine the height of a tall tree without climbing to the top of it. We could stand on the ground at a point some distance (say 20 m) from the foot B of the tree.

Suppose we are able to measure angle BAC and suppose we find it to be 30°. Then, just as in Example, we can calculate the height BC of the tree to be

$BC = \frac{20}{\sqrt{3}} = 11.5$ m (approx.)





Suppose we are viewing an object. The **line of sight** or the **line of vision** is a straight line from our eye to the object we are viewing.

If the object is above the horizontal from the eye (i.e. if it is at a higher level than our eyes), we have to lift up our head to view the object. In the process, our eyes move through an angle. This angle is called the **angle of elevation** of the object.

If the object is below the horizontal from the eye (i.e., at a lower level than ourselves), then we have to turn our head downwards to view the object. In the process, our eyes move through an angle. This angle is called the **angle of depression** of the object.

Ex. 1: A man wishes to find the height of a flagpost which stands on a horizontal plane; at a point on this plane he finds the angle of elevation of the top of the flagpost to be 45°. On walking 30 metres towards the tower he finds the corresponding angle of elevation to be 60°. Find the height of the flagpost.

1) 62 m 2) 82 m 3) 71 m 4) $30\sqrt{3}$ m 5) None of these

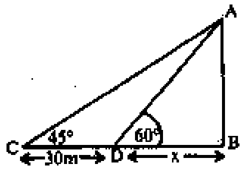
Soln: AB = height of flagpost = x m

In Δ ABD

$\tan 60° = \frac{AB}{BD}$

$\therefore BD = \frac{x}{\sqrt{3}}$ .... (i)

$\tan 45° = \frac{AB}{BD + DC}$



$\therefore \frac{x}{\sqrt{3}} + 30 = x \quad \therefore \frac{x(\sqrt{3}-1)}{\sqrt{3}} = 30 \quad \therefore x = \frac{30\sqrt{3}}{0.732} \approx 71$ m

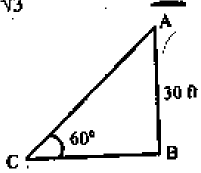
Ex. 2: A small boy is standing at some distance from a flagpost. When he sees the flag the angle of elevation formed is 60°. If the height of the flagpost is 30 ft, what is the distance of the child from the flagpost?

1) $15\sqrt{3}$ ft 2) $10\sqrt{3}$ ft 3) $20\sqrt{3}$ ft 4) $\frac{20}{\sqrt{3}}$ ft 5) None of these

Soln: $\frac{AB}{BC} = \tan 60°$

or, $\frac{30}{BC} = \sqrt{3}$

or, $BC = \frac{\sqrt{3} \times \sqrt{3} \times 10}{\sqrt{3}} = 10\sqrt{3}$ ft

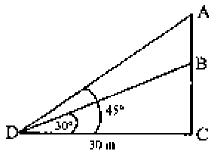


Ex. 3: The angles of elevation of top and bottom of a flag kept on a flagpost from 30 metres distance are 45° and 30° respectively. What is the height of the flag?

1) 17.32 m 2) 14.32 m 3) 12.68 m 4) $12\sqrt{3}$ m 5) None of these

Soln: $\tan 45° = \frac{AC}{30}$ or, AC = 30 m

$\tan 30° = \frac{BC}{30}$ or, $BC = \frac{30}{\sqrt{3}}$

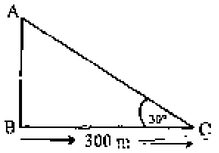


Height of flag AB = $30 - \frac{30}{\sqrt{3}} = 30 - 10\sqrt{3}$

$= 30 - 17.32 = 12.68$m

Ex. 4: 300 m from the foot of a cliff on level ground, the angle of elevation of the top of a cliff is 30°. Find the height of this cliff.

Soln: Let the height of the cliff AB be x m.



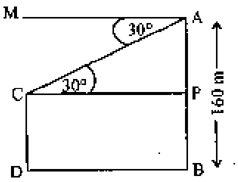
In Δ ABC

$\tan 30° = \frac{AB}{BC} = \frac{x}{300}$

$\therefore x = \frac{300}{\sqrt{3}} = 100\sqrt{3} = 173.20$ m

Ex. 5: The horizontal distance between two towers is $50\sqrt{3}$ m. The angle of depression of the first tower when seen from the top of the second tower is 30°. If the height of the second tower is 160 m, find the height of the first tower.

Soln: Let AB be the tower 160m high.



Let CD be another tower of height x m

Since, AM || PC

∴ angle MAC = angle ACP = 30°

So, in Δ APC

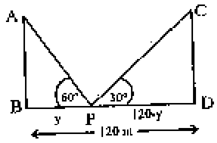
$\tan 30° = \frac{AP}{PC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AP}{50\sqrt{3}}$

∴ AP = 50 m

∴ The height of the other tower = AB - AP = 160 - 50 = 110 m

Ex. 6: Two poles of equal heights stand on either sides of a roadway which is 120 m wide. At a point on the roadway between the poles, the elevations of the tops of the pole are 60° and 30°. Find the heights of the poles and the position of the point.

Soln: Let AB and CD be two poles = x m and P the point on the road. Let BP = y m; then PD = (120 - y) m



In Δ ABP

$\tan 60° = \frac{AB}{BP} = \frac{x}{y} \Rightarrow x = y\sqrt{3}$ ..... (i)

In Δ CDP

$\tan 30° = \frac{CD}{DP} = \frac{x}{120 - y} \Rightarrow x\sqrt{3} = 120 - y$ .... (ii)

Combining equations (i) and (ii), we get

$y\sqrt{3}\,\sqrt{3} = 120 - y$

$\Rightarrow 3y = 120 - y \Rightarrow y = 30$ m

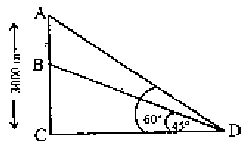
So, from equation (i), $x = y\sqrt{3} = 30\sqrt{3} \approx 52$ m

Ex. 7: An aeroplane when 3,000 m high passes vertically above another at an instant when the angles of elevation at the same observing point are 60° and 45° respectively. How many metres lower is one than the other?

Soln: Let A and B be two aeroplanes, A at a height of 3,000 m from C and B y m lower than A. Let D be the point of observation.

then angle ADC = 60° and angle BDC = 45°

Let DC = x m



In Δ ACD

$\tan 60° = \frac{AC}{CD} = \frac{3,000}{x}$

$\therefore x = \frac{3,000}{\sqrt{3}}$ ..... (i)

Again, in Δ BCD

$\tan 45° = \frac{BC}{CD} \Rightarrow \frac{3000 - y}{x} = 1$

∴ x = 3000 - y .... (ii)

Combining (i) and (ii) we get

$\frac{3,000}{\sqrt{3}} = 3,000 - y$

$\Rightarrow y = 3,000\left(1 - \frac{1}{\sqrt{3}}\right) = \frac{3,000 \times 0.732}{1.732} \approx 1268$ m

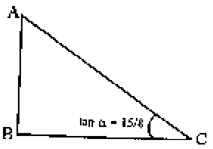
Ex. 8: The length of a string between a kite and a point on the ground is 102 m. If the string makes an angle α with the level ground such that $\tan \alpha = \frac{15}{8}$, how high is the kite?

Soln: C is the point on the ground and the length of the string

CA = 102 m and

$\tan\alpha = \frac{15}{8}$ So, $\sin\alpha = \frac{15}{17}$

In Δ ABC



$\sin\alpha = \frac{AB}{AC} \Rightarrow AB = AC \times \frac{15}{17} = 102 \times \frac{15}{17} = 90$ m

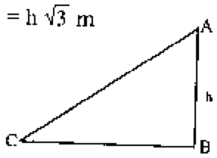
Ex. 9: The shadow of a vertical pole is $\sqrt{3}$ of its height. Find the angle of elevation.

Soln: Let the height of the vertical pole AB be h m.

So, the length of the shadow BC = h $\sqrt{3}$ m

and angle ACB = θ

In Δ ABC



$\tan\theta = \frac{AB}{BC} = \frac{h}{h\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\tan\theta = \tan 30° \quad \therefore \theta = 30°$

Ex. 10: The angles of depression of two ships from the top of a lighthouse are 45° and 30°. If the ships are 100 m apart, find the height of the lighthouse.

Soln: Let AB, the height of the lighthouse = x m

Since MN || PQ $\quad \therefore$ angle MAP = angle APB = 30° and angle NAQ = angle AQB = 45°

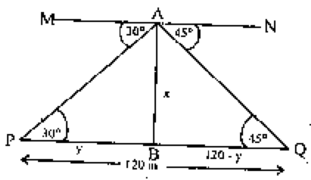
Let the length between P and B be y m.

So, the length between B and Q is (120 - y) m.

In Δ ABP

$\tan 30° = \frac{AB}{BP} \Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{y}$

$\Rightarrow y = x\sqrt{3}$ ..... (i)



Again, in Δ ABQ

$\tan 45° = \frac{AB}{BQ} \Rightarrow 1 = \frac{x}{120 - y}$

$\Rightarrow x = 120 - y$ .... (ii)

Combining equations (i) and (ii), we get

$x = 120 - x\sqrt{3}$ or, $x(1 + \sqrt{3}) = 120 \quad \therefore x = \frac{120}{1 + \sqrt{3}} \approx 44$ m

Ex. 11: The angles of elevation of the top and the foot of a flagstaff fixed on a wall are 60° and 45° to a man standing on the other end of a road 40 m wide. Find the height of the flagstaff.

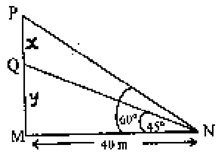
Soln: Let PQ, the height of flagstaff = x m

and QM, the height of wall = y m

In Δ QMN

$\tan 45° = \frac{QM}{MN} \Rightarrow 1 = \frac{y}{40}$

$\therefore y = 40$ ...... (i)



Again, in Δ PMN

$\tan 60° = \frac{PQ + QM}{MN} \Rightarrow = \frac{x + y}{40}$

$\therefore 40\sqrt{3} = x + 40 \quad \therefore x = 40(\sqrt{3} - 1) = 29.28$ m

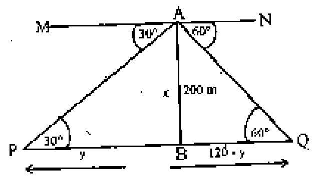
Ex. 12: From the top of a cliff 200 m high the angles of depression of two boats which are due south of observer are 60° and 30°. Find the distance between the two boats.

Soln: Let AB, the height of a cliff = 200 m

In Δ ABP

$\tan 30° = \frac{AB}{BP} \Rightarrow \frac{1}{\sqrt{3}} = \frac{200}{BP}$

$\therefore BP = 200\sqrt{3}$ m ..... (i)



Again, in ABQ

$\tan 60° = \frac{AB}{BQ} \Rightarrow \sqrt{3} = \frac{200}{BQ}$

$\therefore BQ = \frac{200}{\sqrt{3}}$ m ..... (ii)

$\therefore$ distance between the two boats = PB + BQ

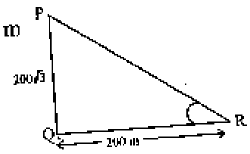
$= 200\sqrt{3} + \frac{200}{\sqrt{3}} \approx 460$ m

Ex. 13: A tower is $200\sqrt{3}$ m high. Find the angle of elevation of its top from a point 200 m away from its roots.

Soln: Let θ be the angle of elevation

and PQ the length of tower $= 200\sqrt{3}$ m

In Δ PQR



$\tan\theta = \frac{PQ}{QR} = \frac{200\sqrt{3}}{200} = \sqrt{3}$

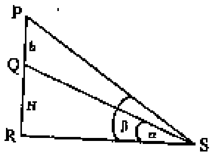
$\tan\theta = \tan 60° \quad \therefore \theta = 60°$

Ex. 14: A vertical tower stands on a horizontal plane and is surmounted by a vertical flagstaff of height h. At a point on the plane, the angles of elevation of the bottom of the flagstaff is α and that of the top of the flagstaff is β. Find the height of the tower.

Soln: Let QR, the height of tower = H

and PQ, the height of flagstaff = h

In Δ QRS



$\tan\alpha = \frac{QR}{RS} = \frac{H}{RS}$

$\therefore RS = \frac{H}{\tan\alpha}$ ..... (i)

Again, in Δ PRS

$\tan\beta = \frac{PQ + QR}{RS} = \frac{(h+H)\tan\alpha}{H}$ .... [from (i)]

or, $H\tan\beta = h\tan\alpha + H\tan\alpha$

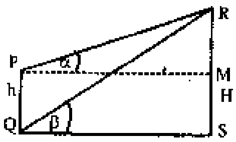
$\therefore H = \frac{h\tan\alpha}{\tan\beta - \tan\alpha}$

Ex. 15: From the top and bottom of a building of height h, the angles of elevation of the top of a tower are α and β respectively. Find the height of the tower.

Soln: Let PQ, the height of building = h

and RS, the height of tower = H

In Δ RMP



$\tan\alpha = \frac{RM}{PM} = \frac{H-h}{PM}$

$\therefore PM = \frac{H-h}{\tan\alpha}$ ..... (i)

Again, in Δ RSQ

$\tan\beta = \frac{RS}{SQ} = \frac{H\tan\alpha}{H-h}$ [from (i)]

After solving, we get, $H = \frac{h\tan\beta}{\tan\beta - \tan\alpha}$

Exercise

1. A tower is 30 m high. An observer from the top of the tower makes an angle of depression of 60° at the base of a building and angle of

depression of 45° at the top of the building, what is the height of the building?
1) 18 mts 2) $12\sqrt{2}$ mts 3) $10\sqrt{3}$ mts 4) 15 mts 5) None of these

2. The angle of elevation on the top of a tower from two horizontal points at distances of a and b from the tower are α and $(90°-\alpha)$ respectively. The height of the tower will be
1) $\sqrt{\frac{a}{b}}$ 2) $\sqrt{ab}$ 3) ab 4) $\sqrt{\frac{b}{a}}$ 5) None of these

3. Town B is 14 km south and 16 km west of town A. Find the distance of B from A.
1) 15.6 km 2) 18.8 km 3) 21.2 km 4) 24.4 km 5) 25.8 km

4. The angle of elevation of a lamppost changes from 30° to 60° when a man walks 20 m towards it. What is the height of the lamppost?
1) 8.66 m 2) 10 m 3) 17.32 m 4) 20 m 5) None of these

5. From the top of a cliff, 60 m high, the angles of depression of the top and bottom of a tower are observed to be 30° and 60° respectively. Find the height of the tower.
1) 40 m 2) 50 m 3) 30 m 4) 35 m 5) None of these

6. The angle of elevation of the top of an unfinished tower at a point 120 m from its base is 45°. How much higher must the tower be raised so that its angle of elevation at the same point be 60°?
1) 90m 2) 92 m 3) 97 m 4) 87.84 m

7. What is the angle of elevation of the sun when the length of the shadow of a pole is $\sqrt{3}$ times the height of the pole?
1) 60° 2) 30° 3) 45° 4) 90° 5) None of these

8. Two towers of equal height stand on either side of a wide road which is 100 m wide. At a point on the road between the pillars the elevations of the tops of the pillars are 60° and 30°. Find their heights.
1) $20\sqrt{3}$ 2) $26\sqrt{3}$ 3) $30\sqrt{3}$ 4) $22\sqrt{3}$ 5) None of these

9. At a point A, the angle of elevation of a tower is found to be such that its tangent is 5/12. On walking 240 m nearer the tower the tangent of the angle of elevation is found to be 3/4. What is the height of the tower?
1) 220 m 2) 200 m 3) 225 m 4) 240 m 5) None of these

10. The shadow of a tower standing on a level plane found to be 60 m longer when the angle of the sun is 30° than when it is 45°. Find the height of the tower when it is 45°.

1) $60(\sqrt{3}+1)$ 2) $30(\sqrt{3}+1)$ 3) $\frac{60}{\sqrt{3}+1}$

4) $30(\sqrt{3}-1)$ 5) None of these

11. An observer on the top of a cliff, 200 m above the sea-level, observ the angle of depression of two ships at anchor to be 45° and 3 respectively. Find the distance between the ships, if the line joi them stretches to the base of cliff.

1) 140 m 2) 150 m 3) 156 m

4) 146.4 m 5) None of these

12. The upper part of a tree broken by wind makes an angle of 30° with the ground, and the distance from the root to the point where the top of the tree touches the ground is 10 m. What was the height of the tree?

1) 30 m 2) 40 m 3) 50 m

4) 60 m 5) None of these

13. From an aeroplane vertically over a straight horizontal road, the angles of depression of two consecutive milestones on the opposite sides of the aeroplane are observed to be 30° and 60°. Then find the height in miles of the aeroplane above the road.

1) $\frac{\sqrt{3}}{2}$ 2) $\frac{\sqrt{3}}{4}$ 3) $\frac{\sqrt{5}}{8}$

4) $\frac{2\sqrt{3}}{12}$ 5) None of these

14. From a 125-metre-high tower, the angle of depression of a car is 45°. Find how far the car is from the tower.

1) 60 m 2) 75 m 3) 80 m 4) 95 m 5) None of these

15. The angle of elevation of a ladder leaning against a house is 60° and the foot of the ladder is 6.5 metre from the house. Find the length of the ladder.

1) 3.25 m 2) $\frac{13}{\sqrt{3}}$ m 3) 13 m

4) 15 m 5) None of these

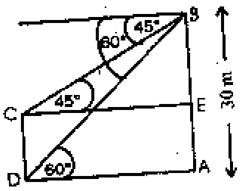
16. From a tower 125 m high, the angles of depression of two rocks which are in horizontal line through the base of the tower are 45° and 30°. Find the distance between the rocks if they are on the same side of the tower.

1) $125\sqrt{3}$ m 2) $\frac{125}{\sqrt{3}}$ m 3) $125(\sqrt{3}-1)$ m

4) $\frac{125}{(\sqrt{3}-1)}$ m 5) None of these

Answers

1. 5; In Δ ABD,



$AD = \frac{AB}{\tan 60°} = \frac{30\text{ mt}}{\sqrt{3}}$

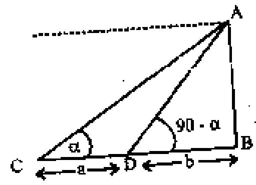
In Δ BCE, BE = CE

$\tan 45° = CE = \frac{30\text{mt}}{\sqrt{3}}$

$\therefore$ CD = AE = AB - BE

$30\left(1-\frac{1}{\sqrt{3}}\right) = \frac{30(\sqrt{3}-1)}{\sqrt{3}} = 10\sqrt{3}(\sqrt{3}-1)$

2. 2; In right-angled Δ ABC



$\tan\alpha = \frac{AB}{BC} = \frac{AB}{a}$

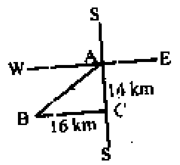
In right-angled Δ ABD

$\tan(90° - \alpha) = \frac{AB}{BD} = \frac{AB}{b}$

$\Rightarrow \cot\alpha = \frac{AB}{b}$

$\Rightarrow \frac{a}{AB} = \frac{AB}{b} \Rightarrow AB = \sqrt{ab}$

3. 3; $AB = \sqrt{AC^2 + BC^2}$



$= \sqrt{(14)^2 + (16)^2}$

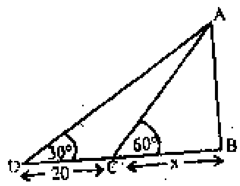
$= \sqrt{196 + 256}$

$= \sqrt{452} = 21.2$ km.

4. 3; From Δ ABC

$\tan 60° = \frac{AB}{x} \Rightarrow AB = \sqrt{3}x$ .... (i)

From Δ ABD



$\tan 30° = \frac{AB}{20+x} \Rightarrow \frac{1}{\sqrt{3}}$

$= \frac{AB}{20+x}$ ..... (ii)

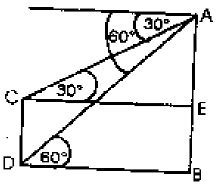
From (i) & (ii), we have

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{\sqrt{3}\,AB}{20\sqrt{3}+AB} \Rightarrow 20\sqrt{3} + AB = 3AB$

$\Rightarrow AB = 10\sqrt{3} = 17.32$ m

5. 1; Let AB = Cliff = 60 m
CD = Tower = ?
In Δ ABD

$\tan 60° = \frac{AB}{BD}$ ∴ $BD = \frac{60}{\sqrt{3}}$ m ..... (i)



In Δ AEC

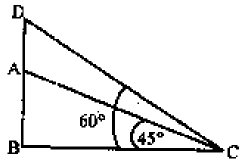
$\tan 30° = \frac{AE}{EC}$

∴ $AE = \frac{60}{\sqrt{3}} \times \frac{1}{\sqrt{3}}$

AE = 20 m

∴ CD = AB - AE = 60 - 20 = 40 m

6. 4; AB = unfinished tower
BC = 120 m
In Δ ABC



$\tan 45° = \frac{AB}{BC}$

∴ AB = BC = 120 m

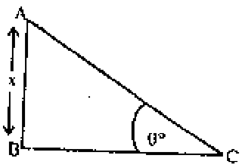
In Δ DBC

$\tan 60° = \frac{DA + AB}{BC}$

$120\sqrt{3} = DA + 120$

∴ $DA = 120(\sqrt{3} - 1) = 120 \times 0.732 = 87.84$ m

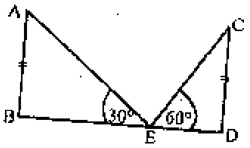
7. 2; Let AB = x m
then, $BC = x\sqrt{3}$ m
In Δ ABC



$\tan\theta = \frac{AB}{BC} = \frac{x}{x\sqrt{3}} = \frac{1}{\sqrt{3}}$

∴ $\tan\theta = \tan 30° \Rightarrow \theta = 30°$

8. 5; AB = CD = Towers = x(say)
BD = Road = 100 m
In Δ ABE



$\tan 30° = \frac{ab}{BE} = \frac{x}{BE}$

∴ $BE = x\sqrt{3}$ ..... (i)

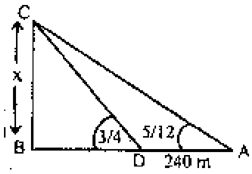
In Δ CDE

$\tan 60° = \frac{CD}{DE} = \frac{x}{100 - x\sqrt{3}}$

∴ $\sqrt{3} = \frac{x}{100 - x\sqrt{3}}$; ∴ $100\sqrt{3} - 3x = x$

∴ $x = \frac{100\sqrt{3}}{4} = 25\sqrt{3} = 43.3$ m

9. 3; Height of the tower = BC = x m
In Δ ABD



tan (angle CDB) = $\frac{BC}{BD}$

$\frac{3}{4} = \frac{x}{BD}$

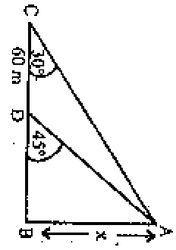
∴ $BD = \frac{4}{3}x$ ..... (i)

In Δ ABC

tan (angle CAB) = $\frac{BC}{BA} = \frac{x}{\frac{4}{3}x + 240}$

$= \frac{5}{12} = \frac{3x}{4x + 720}$ ∴ x = 225 m

10. 2; Height of tower = AB = x m
In Δ ABD



∴ $\tan 45° = \frac{AB}{BD}$ ∴ x = BD

In Δ ABC

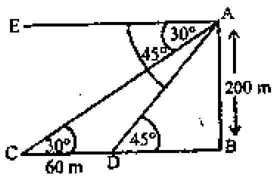
$\tan 30° = \frac{AB}{BD + DC} = \frac{x}{x + 60} = \frac{1}{\sqrt{3}}$

∴ $x\sqrt{3} = x + 60$

∴ $x = \frac{60}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$

∴ $x = 30(\sqrt{3} + 1)$

11. 4; Cliff = AB = 200 m
In Δ ABD



$\tan 45° = \frac{AB}{BD}$

∴ BD = 200 m

In Δ ABC

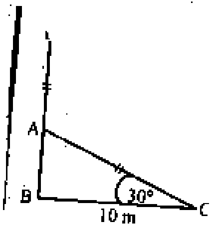
$\tan 30° = \dfrac{AB}{BD + DC}$

$\dfrac{1}{\sqrt{3}} = \dfrac{200}{200 + DC}$

$\therefore\ 200 + DC = 200\sqrt{3}$

$\therefore\ DC = 200(\sqrt{3} - 1) = 200 \times 0.732 = 146.4$ m

12. 5; In Δ ABC



$\tan 30° = \dfrac{AB}{BC}$

$\therefore\ AB = \dfrac{10}{\sqrt{3}}$ .... (i)

Again, in Δ ABC, $\cos 30° = \dfrac{BC}{AC}$

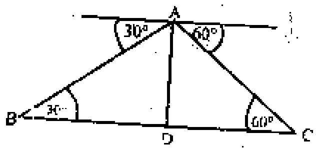
$\therefore\ AC = \dfrac{2}{\sqrt{3}} \times 10$ or, $AC = \dfrac{20}{\sqrt{3}}$;

$\therefore$ Height of the tree = AB + AC

$= \dfrac{10}{\sqrt{3}} + \dfrac{20}{\sqrt{3}} = \dfrac{30}{\sqrt{3}} = 10\sqrt{3} = 17.32$ m.

13. 2; Given BD + DC = 1 ..... (i)

AD = ?



In Δ ABD

$\tan 30° = \dfrac{AD}{BD}$

$\therefore\ BD = \sqrt{3}\ AD$ and

Δ ADC $\tan 60° = \dfrac{AD}{DC}$; $\therefore\ DC = \dfrac{AD}{\sqrt{3}}$

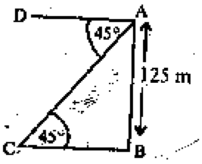
From equation (i), we have

BD + DC = 1

$\sqrt{3}\ AD + \dfrac{AD}{\sqrt{3}} = 1 \Rightarrow AD(3 + 1) = \sqrt{3}$

$\therefore\ AD = \dfrac{\sqrt{3}}{4}$

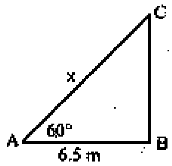
14. 5; In Δ ABC



$\tan 45° = \dfrac{AB}{BC}$

$\therefore\ BC = \dfrac{125}{1} = 125$

15. 3;



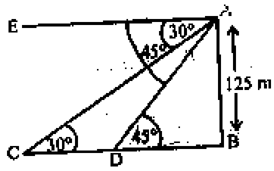
$\cos 60° = \dfrac{6.5}{x} \Rightarrow \dfrac{1}{2} = \dfrac{6.5}{x} \Rightarrow x = 13$ m.

16. 3; $\tan 45° = \dfrac{AB}{DB}$

$DB = \dfrac{125}{\tan 45°} = 125$ m and, $\tan 30° = \dfrac{AB}{CB}$

$\Rightarrow CB = \dfrac{AB}{\tan 30°} = 125\sqrt{3}$ m

$\therefore\ CD = CB - DB = 125(\sqrt{3} - 1)$ m



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